

BOUNDARY EFFECTS ON CONFINED SWIRLING FLOWS WITH VORTEX BREAKDOWN

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Résumé

Ce travail propose une étude numérique des écoulements confinés, générés par la rotation des disques co-axiaux d'une enceinte cylindrique / tronçonnée. La topologie de l'écoulement résultant dépend fortement de la direction et du rapport de rotation des disques. Pour des couples de paramètres de contrôle, les calculs ont révélé l'occurrence de zones de recirculation, sous forme de bulbes, caractérisées par des points de stagnation sur l'axe de rotation. Pour suivre l'évolution de ces éclatements tourbillonnaires, l'étude propose des moyens de contrôle basés sur une modification des conditions cinématiques et géométriques à l'amont de l'éclatement. Les résultats révèlent que ces conditions peuvent soit éliminer ces bulbes ou favoriser leur apparition.

Mots clés: *Ecoulement stationnaire, éclatement tourbillonnaire et contrôle, disques en rotation, simulation numérique.*

Abstract

Confined steady swirling flows, driven by the end disks of a cylindrical/truncated conical enclosure have been numerically studied. Particular attention is focused on combined kinematics and geometric conditions of generation and control of the vortex breakdown phenomenon. First, the basic steady flow topology in a truncated conical cavity is described, which is shown to depend strongly on the direction as well as the rate of rotation of the end disks. For a set of governing flow parameters, the computations revealed the occurrence of bubble-like reverse flows, characterised by on-axis stagnation points. The present work, explores means of controlling the evolution of this physical phenomenon, by modifying the boundary conditions upstream the vortex breakdown. These means are found to either suppress or enhance the occurrence and size of the bubbles.

Keywords: *steady swirl flow; vortex breakdown control; rotating disks; numerical investigation*

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ملخص

نقترح في هذا العمل دراسة عددية لسيلان مغلق، لسانل لزوج غير قابل للانضغاط ناتج عن دوران الأقراص الملساء متوازية المحور لتجوييف أسطواني أو مخروطي. بنية السيلان المحصل عليها تتعلق أساسا باتجاه و نسبة دوران الأقراص (S)، من أجل جملة من عوامل المراقبة، النتائج أثبتت ظهور مناطق رجوع السيلان تظهر على شكل فقاعات، تتميز بنقاط ركود على محور الدوران، لمتابعة تطور الانتفاخ الدوراني، الدراسة تقترح وسائل مراقبة تعتمد على تغيير في الشروط الحركية و الهندسية قبل الانتفاخ، كما أن النتائج بينت أن هذه الشروط قد تزيل الفقاعات أو تفضل ظهورها. الكلمات المفتاحية:

الكلمات المفتاحية: سيلان مستمر، مراقبة الانتفاخ الدوراني، أقراص دورانية، نموذج عددية.

Swirling flows in cylindrical cavities, driven by the independent rotation of the boundaries, have been the subject of numerous numerical and experimental works, primarily motivated by their widespread engineering applications [2]. Particular interest has been devoted to the phenomenon of vortex breakdown, characterised by an abrupt change in the flow topology, ever since it was observed by Vogel [1], in the model flow driven by the rotation of a single disk of a cylindrical cavity. In this model set up, the flow is governed by only two parameters, namely, the rotational Reynolds number Re and the aspect ratio of the enclosure (height/radius) and characterised by a concentrated vortex core along the axis. Beyond a threshold rotation ratio, the core breaks and gives rise to bubble-like recirculation regions with on axis stagnation points; commonly defined as vortex breakdown [2]. Most subsequent research topics [2,3,5], have adopted Vogel's setup, as it presents well defined boundary conditions and provides direct and effective comparison between numerical and experimental simulations.

Escudier[3], extended Vogel's work and uncovered, experimentally, much wider parameters range to map regions of occurrence of up to three vortex bubbles as well as steady and unsteady flow regions. Escudier[3] also reported that the vortex phenomena appeared highly axisymmetric; which subsequently motivated extensive 2D numerical investigations, giving accurate steady results comparable with experiments. A noteworthy feature of vortex breakdown, as reported by Tsiberblit [7], is that its onset does not result from a hydrodynamic instability or bifurcation, but is a continuous process with increasing Re. In practical situations, vortex breakdown may be harmful, as observed, for instance, on the tip vortices of a delta-winged aircraft; causing a loss of its control [7,8,10]. On the other hand, it may be beneficial and desired, for example, in bioreactors where it can constitute an ideal environment for cell growth [13]. More over, it enhances mixing in vortex chambers and stabilises flames in burners. With these considerations in mind, it appeared necessary to investigate appropriate means of vortex breakdown control. In the present work, the primary objective is to explore numerically, intrusive and non intrusive methods of controlling on-axis as well as off-axis vortex breakdown. These methods are based, essentially, on modifying kinematics and/or geometric conditions upstream the breakdown. First, the basic steady flow topology, in a conical cavity is described. Then, interest is focused on the steady flow regimes which display on-axis vortex bubbles. By varying the radial aspect ratio parameter, we explored effects of the sidewall inclination. Furthermore, the sensitivity of the on-axis bubbles, to weak differential rotation of a top end conical lid, is studied. Finally, we numerically confirm Husain's et al. [10] experimental findings, based on the concept of adding a near-axis swirl.

I. Formulation and numerical approach

Consider the flow in a truncated conical cavity, driven by the top and bottom end walls of radii R_b and R_t , which rotate with constant, but different, angular velocities Ω_b and Ω_t , respectively. We note that $R_b=R_t$ corresponds to the cylindrical enclosure. Using as the timescale $1/\Omega_b$ and R_b as a length scale, this configuration introduces the following dimensionless parameters, which govern the dynamics; namely, the Reynolds number, the rotation ratio, the axial and radial aspect ratios, defined respectively by:

$$\text{Re} = R_b^2 \Omega_b / \nu, \quad S = \Omega_b / \Omega_t, \quad \Lambda_h = H / R_b, \quad \Lambda_r = R_t / R_b$$

The flow is described using axisymmetric Navier-Stokes equations, expressed in cylindrical coordinates expressed in a conventional stream function-vorticity formulation. Let $\psi(r, z)$ denote the stream function, Γ the circulation (angular momentum) such that the velocity and the corresponding vorticity fields may be written, respectively:

$$(u, v, w) = \left(\frac{1}{r} \frac{\partial \psi}{\partial z}, \frac{\Gamma}{r}, -\frac{1}{r} \frac{\partial \psi}{\partial r} \right), \quad \bar{\nabla} \wedge \bar{V} = \left(-\frac{1}{r} \frac{\partial \Gamma}{\partial z}, \frac{1}{r} \nabla_*^2 \psi, \frac{1}{r} \frac{\partial \Gamma}{\partial r} \right),$$

$$\text{where } \nabla_*^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The transport equations, in terms of tangential vorticity component and circulation are, respectively:

$$\frac{\partial \xi}{\partial t} + u \frac{\partial \xi}{\partial r} + w \frac{\partial \xi}{\partial z} - \frac{u\xi}{r} - \frac{2\Gamma}{r^3} \frac{\partial \Gamma}{\partial z} = \frac{1}{\text{Re}} \left(\nabla^2 \xi - \frac{\xi}{r^2} \right),$$

$$\frac{\partial \Gamma}{\partial t} + u \frac{\partial \Gamma}{\partial r} + w \frac{\partial \Gamma}{\partial z} = \frac{1}{\text{Re}} \left(\nabla^2 \Gamma - \frac{2}{r} \frac{\partial \Gamma}{\partial r} \right)$$

The Poisson equation, is written in the form:

$$\nabla^2 \psi - \frac{2}{r} \frac{\partial \psi}{\partial r} = r \xi,$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

To solve numerically the above system, boundary and initial conditions are required. These are based on the no slip on the solid walls and symmetry assumptions on the axis.

-Boundary conditions:

$$\text{at the bottom disk: } \psi = 0, \Gamma = r^2$$

$$\text{at the top flat disk: } \psi = 0, \Gamma = S.r^2$$

$$\text{at the sidewall: } \psi = 0, \Gamma = 0$$

$$\text{The tangential vorticity on all solid walls: } \xi = \frac{1}{r} \frac{\partial^2 \psi}{\partial l^2}; \quad l$$

being the direction normal to the boundary.

The symmetry condition at the axis is:

$$\psi = 0, \quad \Gamma = 0, \quad \xi = 0$$

-If a top conical lid, of dimensionless height h_c (height/base radius) and radial extent r_c , is employed, then the conditions on its surface are : $\psi = 0, \quad \Gamma = S_c.r_c^2$ Where $S_c = \Omega_c / \Omega_b$; Ω_c being its constant angular speed.

-When a central thin rod, of radius r_d , is introduced, the symmetry condition is replaced by the no slip condition. The rod can rotate with a rotation ratio $S_r = \Omega_r / \Omega_b$; Ω_r being its constant angular speed.

-Initial conditions:

Let $t < 0$ denote the time when fluid and cavity are at rest. Then, at $t = 0$, the top and/or bottom end walls are impulsively rotated with uniform, but different, angular velocities, while the sidewall remains stationary. These are expressed as follows:

$$t \leq 0 \quad : \quad \psi = 0, \quad \Gamma = 0, \quad \xi = 0;$$

$$(r, z) \in D^* =]0, r_s[\times]0, z_c[.$$

Here, z_c denotes the axial position of the top lid and r_s the radial location of the sidewall.

$t \geq 0$: conditions at the walls, given above, still apply.

To solve the parabolic transport equations, subject to the prescribed conditions, we have adopted a three level time-marching finite difference scheme, akin to that employed successfully, in a related work, and described in detail by Gerrard et al.[4] and by Bellamy-Knights and Saci[6].

The numerical scheme is second order in time and space, and a uniform mesh grid is adopted. For small time, an adequate description of the timewise development of the flow field requires, in general, a time increment $\delta t \approx 0.001$, with a mesh length $\delta r = \delta z = 0.0125$. Since the steady state, in the limit of large time, is our main objective, the time step is relaxed up to $\delta t \approx 0.05$ as the solution proceeds. The flow is governed by non linear coupled differential equations, and stability requirements were based essentially on numerical experiments. Appropriate time and space mesh grids were employed and the time dependent solution is calculated until an essentially ultimate steady state is approached. Poisson's elliptic equation is iterated at each time level and, subsequently, the azimuthal vorticity component is updated. The accuracy of the scheme is first assessed by comparing the present findings, in the case of a cylindrical enclosure, with previous qualitative and quantitative results reported in the literature by Lopez[12] and H.S.Husain [10]. In particular, the velocity fields reported by Lopez [12], using a different approach, have been reproduced and compared favourably. As an additional check, very good agreement is found when comparing the ultimate solution obtained for large times with the solution obtained by solving the steady equations, within the range of the prescribed control parameters.

II. Main results

Convergent steady solutions were obtained in the following range of parameters:

$$250 \leq R_e \leq 2000, 0 \leq S \leq 1, 0.5 \leq \Lambda_r \leq 2, 1 \leq \Lambda_h \leq 2.5.$$

The Reynolds number is fixed below the critical value beyond which flow becomes unstable, as indicated by Escudier's diagram [3] in the case $S=0$.

3.1 Basic flow in a truncated conical cavity:

The basic flow topology, induced by the end disks of a truncated conical cavity, is clearly exhibited in fig.1, for the couple of particular parameters $R_e = 250, -0.5 \leq S \leq 0.5, \Lambda_r = 2, \Lambda_h = 1$. The meridian streamlines indicate that co-rotation displays a two-cell structure, with a dividing stagnation line ($\psi=0$), in the meridian plane, and a core region in a quasi-solid body rotation (fig.1(C₂)). In contrast, counter-rotation induces a three cell-structure which reduces to two cells with increasing rotation ratio. Counter-rotation is also characterised by an azimuthal layer $\Gamma=0$ (fig.1 (c₁)), which tend to coincide with the stagnation line ($\psi=0$) as the rotation ratio increases. The competition of the outward circulations induced by the end disks, give rise to a stagnation point on the sidewall, at which fluid is deviated into the interior, forming a shear layer ($\psi=0$). The leading stagnation point location is, virtually independent of the direction of rotation.

3.2 Steady Flow with vortex breakdown:

3.2.1 Model flow driven by the bottom disk of a cylinder ($S = 0, \Lambda_r = 1$) :

The steady model flow driven by the bottom end disk of a cylindrical enclosure, of aspect ratio $\Lambda_h = 2.5$ is described when $Re=2000$ (fig2.a). In addition to the azimuthal motion, which the fluid acquires, initially, at the rotating disk, there develops a secondary circulation with a concentrated central vortex core which breaks to give rise to two distinct on-axis bubbles, reminiscent to a B type vortex breakdown as defined by Leibovitch[2]. These are depicted in fig.2a, where meridian streamlines are drawn; non- uniformly spaced so as to emphasize the relatively weak, but relevant, reverse flow regions. The breakdown process, may be associated to a centrifugally unstable redistribution of the angular momentum within the central vortex core flow[12].

3.2.2 Influence of the sidewall inclination ($0.8 \leq \Lambda_r \leq 1$).

This part explores the effect of sloping the stationary sidewall of the rotor-stator cylindrical configuration discussed in the above section, which, for a given set of parameters exhibits two distinct on-axis bubbles. Aiming to alter the axial swirl upstream the breakdown region, the radial aspect ratio Λ_r of the resulting truncated conical cavity is varied in the range $0.8 \leq \Lambda_r \leq 1$ (which corresponds to an inclination angle $0^\circ \leq \theta \leq 4.57^\circ$), (fig.2). We recall that $\Lambda_r = 1$ ($\theta=0^\circ$) corresponds to the cylindrical casing. The resulting effect is best viewed and described with reference to (fig.2), which illustrate the meridian streamlines corresponding to the model flow driven by the bottom end disk. It is clearly observed that a relatively small perturbation causes large and relevant changes to the vortex structure. In fact, for $\Lambda_r = 0.9$ ($\theta \sim 2.29^\circ$), fig.2b indicates a substantial size reduction of the bubbles, followed by an axially downward shift; and the threshold value $\Lambda_r = 0.8$ ($\theta \sim 4.57^\circ$), (fig.4c), causes the elimination of both vortex breakdown bubbles.

3.2.3 Influence of a top conical lid:

The rotor- stator cavity described above, has been modified by introducing a top conical lid of height h_c instead of the flat disk. The cone may rotate with a rotation ratio S_c in the range

$-0.5 \leq S_c \leq 0.5$. The influence of the conical geometry is clearly exhibited in fig.3, which illustrates, in the meridian plane, streamlines corresponding to the steady flow induced by the independent rotation of the end walls; the side wall being stationary. For the same parameters Re and Λ_h , computations show that, as h_c increases, the stationary conical lid ($S_c=0$) causes an axial downward shift to the upper bubble which eventually coalesces with the lower one for $h_c = 0.3$ (fig.3b). This latter has not been shifted, as the distance of its trailing edge stagnation point to the bottom disk remained constant. To explore the effect of modifying the kinematics conditions upstream the vortex bubbles, fig.3b,c clearly indicates that a relatively weak

counter-rotation of the top conical lid is sufficient to suppress, successively, the bubbles; while maintaining steady flow. In the process, an axial shifting is noticed, the upper bubble is first eliminated at a counter-rotation rate of 2%, then follows the suppression of the second vortex as the cone counter-rotation attains, approximately, 4% that of the bottom disk. By contrast, meridian streamlines, not depicted here, revealed that co-rotation tend to enhance the bubble size; giving rise to a more elongated vortex breakdown bubbles.

3.2.4 Effect of a near-axis swirl

Previous experimental works [8,10] related to vortex breakdown control in confined flows have introduced a near-axis swirl by means of a rotating central rod mounted at the cylinder axis. Motivated by their findings, the present numerical investigation re-examined and confirmed the effectiveness of this approach to control on-axis vortex bubbles, which occur in the concentrated vortex core of the flow driven by only one end wall.

The influence of a differentially co-rotated thin rod ($S_r = \Omega_r / \Omega_b > 0$), on on-axis vortex breakdown, is best viewed with reference to fig.4. Results indicate that the presence of a stationary central thin rod has virtually no qualitative effect on the vortex structure, but its co-differential rotation (fig.4b), yields, effectively, to the suppression of the bubbles. This result is consistent with H.S.Husain's et al. experimental observations [10], carried out using a cylindrical enclosure of aspect ratio $\Lambda_H=3.25$, but contradicts Mullin's et al. conclusions [8], which reported that the rod ($0 < r_d \leq 0.1$) had no qualitative effect on the vortex structure. Moreover, the present investigation also revealed that counter-differential rotation of the rod induces a centrifugally instable flow and breakdown enhancement, as clearly shown in fig.4c, for a rotation ratio $S_r = -2$.

CONCLUDING REMARKS

Confined vortex breakdown, induced by the independent rotation of the end walls of a cylindrical enclosure, have been numerically studied, and methods of controlling their occurrence and evolution were explored. For on-axis bubbles with axial stagnation points, generated by the rotation of the bottom end disk, a weak counter-rotation of a top conical lid is found sufficient to suppress the bubbles, while co-rotation is observed to enhance their size. Computations have also revealed that, sloping slightly the stationary sidewall, constitutes an effective means of eliminating the vortex structure. Finally, the effectiveness of adding a near-axis swirl, driven by a differentially rotated thin rod, is analysed and found to have a substantial influence on the occurrence and evolution of the vortex bubbles.

REFERENCES:

- [1]-Vogel H.U, "experimentelle Ergebnisse über laminar Strömung in einen zylindrischen Gehäuse mit darin rotierender Scheibe", *Phys.Fluids* 6,(1968), p.2702 .
- [2]-Leibovich S. "The structure of vortex breakdown", *Ann. Rev. Fluid Mech.*,10 (1978),pp. 221-246
- [3]- Escudier M.P., "Observations of the flow produced in a cylindrical container by a rotating endwall". *Exp Fluids* 2, (1984), pp.189-196.
- [4]- Savvides C.N. & Gerard J.H., "Numerical analysis of flow through a corrugated tube with application to arterial prostheses", *J.Fluid Mech* vol.138, (1984), pp. 129-160.
- [5]- Lopez J..M., "Axisymmetric vortex breakdown part 1. Confined swirling flow", *J. Fluid Mech*, 221 (1990), pp.533-552.
- [6]- Bellamy-Knights P.G., SACI R, "Flow between two stationary disks and a rotating shroud". *Comput.Fluids* 20, (1991), pp.77-87.
- [7]- Tsiberblit N and Kit E, "On the onset of unsteadiness in confined steady flows" *Fluid Dyn. Res* 23, (1999), pp.125-152.
- [8]- Mullin T, Kobine J.J, Tavener S.J, and Cliffe K.A, "On the creation of stagnation points near straight and sloped walls", *Phys. Fluids* 12, N°2, (2000), pp.425-431.
- [9]- Brons M, Voigt L.K, Sorensen J.N., "Topology of vortex breakdown bubbles in a cylinder with a rotating bottom and a free surface", *J.Fluid.Mech*, 428(2001), pp.133 -148.
- [10]- Husain H.S., Shtern V., and Hussain F, "Control of vortex breakdown by adding a near-axis swirl", *Phys. Fluids*, Vol.15, N°2 (2003), pp.271-279.
- [11]- Herrada M and Shtern V, "Control of vortex breakdown by temperature gradients" *Phys. Fluids. Rev. E* 68, 041202 (2003).
- [12]- Lopez J.M. & Marques F, "Mode competition between rotating waves in a swirling flow with reflexion symmetry", *J.Fluid.Mech*, vol.507 (2004), pp.265-288.
- [13]- Manunga L, Hourigan K and Thompson M.C, "Confined flow vortex breakdown control using a small rotating disk", *Phys.Fluids*, vol.16, N°12 (2004), pp. 4750-4753.
- [14]- Piva M, Meiburg E, "Steady axisymétrique flow in an open cylindrical container with a partially rotating bottom wall", *physics of fluids*, vol 17, N°063603, (2005) pp.1-12.