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LAWS OF EXCURSIONS ASSOCIATED TO ADDITIVE FUNCTIONALS

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Résumé

Soit (P_t)un semi-groupe droit de Borel, et soit (S_t) l'inverse continu à droite d'une fonctionnelle additive continue (B_t). Soit $\left(Y_t\right)_{t\in R}$ un processus stationnaire à naissance et mort aléatoires, markovien de semi-groupe (P_t) sous la mesure de Kuznetsov Q associée à une mesure excessive. On définit, sous l'hypothèse que la mesure caractéristique $\upsilon_B = Q\int_0^1 I_{\{Y_t\in .\}} B(dt)$ de (B_t) est purement excessive pour le semi-groupe $\left(P_{S_t}\right)$, une fonctionnelle additive pour $\left(Y_t\right)_{t\in R}$ en fonction de (B_t) et on étudie les lois des excursions associées à l'ensemble regénératif constitué des temps de discontinuité de l'inverse continu à droite (U_t) de cette fonctionnelle additive. Plus précisément, si on note par $\left(\Phi_t\right)$ le processus $\left(Y_{U_t}\right)$ et par H la σ -algèbre engendrée par $H_t\left(t\in R\right)$ où H_t est la Q-complétion de H_{+t}^0 [(H_t^0) étant la filtration naturelle de $\left(\Phi_t\right)$], alors si T est un (H_t)-temps d'arrêt tel que $U_{T^-}\neq U_T$ est $\Phi_{T^-}\neq\Phi_T$, la loi conditionnelle de l'excursion chevauchant $\left]U_{T^-},U_T\right[$ par rapport à H dépend uniquement de Φ_T et de Φ_{T^-} . Les lois conditionnelles des couples d'excursions ont été également étudiées. MSC: 60J25; 60J40; 60J55.

Mots clés: *Standard process; Predictable process; Excursion; Additive functional; Conditional law; Exit measure; Kuznetsov process.*

Abstract

Let (P_t) be a right borel semigroup and let (S_t) the right inverse of a continuous additive functional (B_t). Let $\left(Y_t\right)_{t\in R}$ be a right stationary process with random birth and death, Markov with semi group (P_t) under the Kuznetsov measure Q associated to an excessive measure. We define, under the assumption that the characteristic measure $\upsilon_B = Q\int_0^1 I_{\{Y_t\in .\}} B(dt)$ of (B_t) is purely excessive for the semigroup (P_s), an additive functional for $\left(Y_t\right)_{t\in R}$ in terms of (B_t) and we study the laws of excursions associated to the regenerative set which consists in times of discontinuity of the right inverse (U_t) of this additive functional. More precisely, if we note by $\left(\Phi_t\right)$ the process $\left(Y_{U_t}\right)$ and by H the σ -algebra generated by H_t $\left(t\in R\right)$ where H_t is the Q-completion of $H_{t^+}^0$ ($\left(H_t^0\right)$ is the natural filtration of $\left(\Phi_t\right)$), then if T is a $\left(H_t\right)$-stopping time such that $U_{T^-}\neq U_T$ and $\Phi_{T^-}\neq\Phi_T$, the conditional law of the excursion straddling $\left]U_{T^-},U_T\right[$ with respect to H depend only on Φ_T and Φ_{T^-} . Conditional laws of pairs of excursions are also considered.

Keywords: *Standard process; Predictable process; Excursion; Additive functional; Conditional law; Exit measure; Kuznetsov process.*

MSC: 60J25; 60J40; 60J55

H. BOUTABIA

Département de Mathématiques, Faculté des Sciences
Universté Badji-Mokhtar
Annaba 23000, B.P.12.
Algérie.

ملخص

لتكن $\left(P_t\right)$ نصف زمرة من اليمين، وليكن $\left(S_t\right)$ العكوس من اليمين للدوال الجمعية المستمرة $\left(B_t\right)$.
وليكن $\left(Y_t\right)_{t\in R}$ النمط المستقر من اليمين لعشوائية المواليد والوفيات المار كوفية تبعا لنصف الزمرة $\left(P_t\right)$ وفق قياس
Q Kuznetsov المرافق لفوق قياس. نعرف وفق الفرضيات المتعلقة بالقياس المميز $\upsilon_B = Q\int_0^1 I_{\{Y_t\in .\}} B(dt)$ لـ $\left(B_t\right)$ وهي
صافية فوقية لنصف الزمرة الدوال الجمعية ل $\left(Y_t\right)_{t\in R}$ المتعلقة بـ $\left(B_t\right)$ و $\left(U_t\right)$ تدرس القوانين للرحلة العشوائية المرافقة
لمجموعة متولدة، مشكلة من أزمنة التقطع للعكوس من اليميننأين $H_t\left(t\in R\right)$ - جبر المولد عن σ ل.
H بـ و $\left(Y_{U_t}\right)$ للنمط $\left(\Phi_t\right)$ لهذه الدوال الجمعية، بالتحديد إذا رمزنا بـ زمن توقف بحيث $\left(H_t\right)$ هو T فإنه إذا كان
$\left(\Phi_t\right)$ هي السريان الطبيعي لـ $\left(H_t^0\right)$) $H_{t^+}^0$ تكملة لـ - Q هي لأ H فقو $\left]U_{T^-},U_T\right[$. القوانين الشرطية لرحلة
عشوائية قفزية للمجال , $\Phi_{T^-}\neq\Phi_T$ $U_{T^-}\neq U_T$ والقوانين الشرطية لأزواج الرحلات العشوائية تم دراستها كذلك
Φ_T و Φ_{T^-} تعتمد الأ على

In 1985 Kaspi [6] constructs, for a standard process X, an additive functional B associated to a regenerative system M and gives, under the classical duality hypothesis, the probability measures $P^{x,y}$ allowing the law of excursions associated to B with respect to the σ-algebra $K=\sigma\left(Z_t : t\in R_+\right)$, known to start at x end at y $(Z_t = X_{S_t}$ where $S_t = \inf\{u : B_u > t\})$.

It was given in $[2]$, without duality, the measures $P^{x,y}$ in terms of the $\left(F_{D_t}\right)$-predictable exit measures for a regenerative system consisting of the closure of the set of times that the regular points of an arbitrary continuous additive functional are visited. The purpose of this paper is to give the conditional laws of pairs of excursions for a Markov process with random birth and death $(Y_t)_{t\in R}$ having the same semigroup as X. To this respect, we define an " additive functional " for $(Y_t)_{t\in R}$ and we extend this result to $(Y_t)_{t\in R}$. Laws of pairs of excursions for $(Y_t)_{t\in R}$ are discussed.

Preliminaries and notations

Let $\left(\Omega, F, F_t, X_t, \theta_t, P^x\right)$ be the canonical realization for a borel standard semigroup (P_t). We assume that the state space E is lusinian, and we note by E its σ-algebra of borel sets. The cemetery point δ is absorbent and outside of E. Let (B_t) be a continuous additive functional and let R be the perfect exact terminal time $\inf\{u : B_u > 0\}$. We note by $C = \{x : P^x(R=0)=1\}$ the fine support of (B_t).

Let F^* be the universal completion of the σ-algebra $\sigma(X_t : t\in R_+)$. We consider the random homogeneous set $M = \{t + R\circ\theta_t : t\in R_+\}$, and its family of $\left(F_{D_t}\right)$ -predictable exit measures $\left({}^0P^x\right)_{x\in E\cup\{\delta\}}$ (we assume that R is F^*-measurable), where D_t is the random variable $\inf\{s>t : s\in M\}$. Note that if $S_{t^-}\neq S_t$, then $D_{S_{t^-}} = S_t$, hence the excursion associated to t is defined by:

$$e_t(\omega)(s) = k_R\circ\theta_{S_{t^-}}(\omega)(s) = \begin{cases} X_{S_{t^-}+s}(\omega) \text{ if } s < S_t(\omega) - S_{t^-}(\omega) \\ \delta \quad \text{if} \quad s \geq S_t(\omega) - S_{t^-}(\omega) \end{cases}$$

where k_t killing operator at t defined by: $k_t(\omega)(s) = \omega(s)$ if $s<t$ and δ if $s\geq t$. We consider for $(x,y) \in E\times E$ such that $x\neq y$, the measures $P^{x,y}$ on $\left(\Omega, F^*\right)$ "defined by":

$$P^{x,y} = H^x\left(k_R\in ./X_R = y\right) \text{ where } H^x = {}^0P^x\left(.; X_R\neq x\right)$$

Since $\left(\Omega, F^0\right)$ is an U-space, and according to a classical lemma of Doob the measures $P^{x,y}$ can be chosen measurable for the pair (x,y).

We associate to the right inverse $S_t = \{u : B_u > t\}$ of (B_t), the following notations: $Z_t = X_{S_t}$, $M_t = F_{S_t}$ and $\overline{\theta}_t = \theta_{S_t}$. It is well known that the process $Z = \left(\Omega, F, M_t, Z_t, \overline{\theta}_t, P^x\right)$ is strong Markov with semigroup $\left(\overline{P}_t\right) = \left(P_{S_t}\right)$ and takes values on $\left(C, C\cap E^*\right)$ (cf. Jacods $[5]$).

Let $(K_t)_{t\in R_+}$ be the filtration, where K_t is the intersection of the P^π-completions of the $\sigma-$algebra $K^0_{t^+}$ where π is in the set of all the bounded measures on E; $\left(K^0_t\right)$ is the natural filtration of the process (Z_t). It was shown in $[2]$ that if T is a finite $(K_t)-$stopping time such that $S_{T^-}\neq S_T$ $a.s.$, then we have:

$$F_{\left(S_{T^-}\right)^-} = K_{T^-} \tag{1}$$

and that if $S_{T^-}\neq S_T$ and $Z_{T^-}\neq Z_T$ $a.s.$, then

$$P\left(f(e_T)/K\right) = P^{Z_{T^-},Z_T}(f) \tag{2}$$

for all positive and F^*-measurable function f, where $P(A) = \int P^x(A)\mu(dx)$

(μ is an arbitrary law on E).

Note that the formula (2) was proved by Kaspi $[8]$ under the duality hypothesis.

Excursions of Kuznetsov processes

Let W be the set of applications $w : R\mapsto E\cup\{\delta\}$ which satisfies the following properties: there exits an open subset of R on which w is E-valued right -continuous with left limits and out which equals δ. We note by $(Y_t)_{t\in R}$ the coordinate process on W.

Let $\left(G_t^0\right)_{t\in R}$ be the natural filtration of $\left(Y_t\right)_{t\in R}$ and let $G^0 =_{t\in R} VG_t^0$. Then the birth and the death times of $\left(Y_t\right)_{t\in R}$ are respectively:

$$\alpha = \inf\{t \in R : Y_t \in E\} \quad (\inf \varnothing = +\infty),$$
$$\beta = \sup\{t \in R : Y_t \in E\} \quad (\sup \varnothing = -\infty).$$

We define the families of operators on W by:

$\tau_t : W \mapsto \Omega$ such that $\tau_t w(s) = w(s+t)$ for $s \in R_+, t \in R$

$\sigma_t : W \mapsto W$ such that $\sigma_t w(s) = w(s+t)$ for $s, t \in R$

Note that $X_s \circ \tau_t = Y_{t+s}$ on $\{Y_t \in E\}$ and $\sigma_t \circ \sigma_u = \sigma_{t+u}$ for $t, u \in R$, $s \in R_+$. Let η be an excessive measure with respect to (P_t) and let Q be the Kuznetsov measure on W that corresponds to $(\eta, (P_t))$ (cf. $[8]$, $[10]$). We note by G_t and G the Q-completions of G_t^0 and G^0, and we assume that the semigroup (P_t) satisfies "les hypothèses droites de Meyer". It follows by $[10]$ that the process $Y = (W, G, G_t, (Y_t)_{t\in R}, \tau_t, \alpha, \beta, Q)$ is stationary $\left(\text{i.e. } \sigma_t(Q) = Q\right)$ and strong Markov with semigroup (P_t).

For the generalization of formula (2), we consider the additive functionals B and S given in the previous section. We also note by B the random measure on W, carried by $]\alpha, \beta[$ such that:

$$B_s \circ \tau_t = B]t, t+s] \quad \text{on } \{Y_t \in E\} \text{ for all } s > 0 \text{ and } t \in R$$

We assume that the characteristic measure $\upsilon_B = Q\int_0^1 I_{\{Y_t \in .\}} B(dt)$ of B is purely excessive for the semigroup $\left(\overline{P_t}\right)$ (i.e. $\int \overline{P_t} f(x) \upsilon_B(dx) \to 0$ as $t \to \infty$ if $\upsilon_B(f) < \infty$). It was shown in $[7]$ that Q a.e. $B]\alpha, t] < \infty$ for all $t > \alpha$.

Let $(V_t)_{t\in R}$ be the nondecreasing process defined on W by:

$$V_t = \alpha + B]\alpha, t] \text{ on } \{\alpha < t\} \text{ and } V_t = \alpha \text{ on } \{t \le \alpha\},$$

and let $(U_t)_{t\in R}$ be the right-continuous inverse of $(V_t)_{t\in R}$ that is:

$$U_t = \inf\{u > \alpha : V_u > t\}$$

We also note by M the closed random subset of $]\alpha, \beta[$ defined by: $M = \alpha < t < \beta \cup \{t + R \circ \tau_t\}$ which verifies the following property of homogeneity (cf. $[4]$)):

$$(M - t) \cap]0, \infty[= M \circ \tau_t \text{ on } \{Y_t \in E\}$$

Remark:

1) If $\alpha = -\infty$, $\{u > \alpha : V_u > t\} = \%$ and $U_t = +\infty$, then $\alpha > -\infty$ on $\{\alpha < U_t < \beta\}$.

2) $U_t = \alpha$ on $\{t \le \alpha\}$.

For $t \in R$, let

$$\Phi_t = Y_{U_t}, \overline{G}_t = G_{U_t}, \overline{\tau}_t = \tau_{U_t}, H_t^0 = \sigma(\Phi_u : u \le t)$$

and $H^0 = \sigma\left(H_t^0 : t \in R\right)$. We note by H_t (resp. H) the Q-completion of $H_{t^+}^0$ (resp. H^0). Note that for all the following formulas, the $\sigma -$ finiteness of Q is guaranteed by the argument used in $[1]$. It is not hard to show that (Φ_t) has the same properties as (Z_t) and the following result hold.

Proposition:

1) The process (U_t) is right continuous, has left limits, and satisfies $U_t = U_\beta$ for all $t \ge \beta$ Q a.e..

2) (U_t) is $\left(\overline{G}_t\right)$-adapted.

3) For all $t \in R$ and $s > 0$ we have:

a) $U_t = \alpha + S_{t-\alpha} \circ \tau_\alpha$ on $\{-\infty < \alpha < t\}$

b) $V_{t+s} = V_t + B_s \circ \tau_t$ on $\{Y_t \in E\}$ and $U_{t+s} = U_t + S_s \circ \overline{\tau}_t$ on $\{\alpha < U_t < \beta\}$.

4) On $\{U_t \ne U_{t^-}\}$, $]U_{t^-}, U_t[$ is a contiguous interval of M.

If $U_t \ne U_{t^-}$, let E_t be the excursion associated to B defined by:

$$E_t(w)(s) = \begin{cases} Y_{U_{t^-}+s}(w) \text{ if } 0 \le s < U_t(w) - U_{t^-}(w) \\ \delta \text{ if } s \ge U_t(w) - U_{t^-}(w) \end{cases}$$

According to the previous proposition, the process $(V_t)_{t\in R}$ has got the same role as B for the process

$(Y_t)_{t\in R}$. We say that $(V_t)_{t\in R}$ is an " additive functional " for $(Y_t)_{t\in R}$. We have the extension of theorem 2 $[2]$ on W.

Theorem1:

1)*The process* $\Phi = \left(W, \Phi_t, G, \overline{G}_t, \overline{\tau}_t, Q\right)$ *is strong Markov in the sense that for all* $\left(\overline{G}_t\right)-$*stopping time* T *and* $s>0$:

$$Q\left(f\left(\Phi_{T+s}\right) / \overline{G}_T \right) = \overline{P}_s\left(f, \Phi_{\mathbf{T}}\right) \text{ on } \left\{\alpha < U_T < \beta\right\} \quad (3)$$

for all function positive and F *-measurable* f .

2) *Assume that* T_1 *is a finite* $(H_t)-$*stopping time such that* $U_{T_1} \neq U_{T_1^-}$ *and* $\Phi_{T_1} \neq \Phi_{T_1^-}$ Q $a.e.$.*Then we have:*

$$Q\left(F\left[E_{T_1}\right]/H\right) = P^{\Phi_{T_1^-},\Phi_{T_1}}\left(F\right) \text{ on } \left\{\alpha < U_{T_1} < \beta\right\} \quad (4)$$

for all $F \geq 0$, F^**-measurable.*

Proof : If $T \equiv t$ is constant, the formula (3) follows from the Markov property of the process $(Y_t)_{t\in R}$ at time U_t and the fact that $\Phi_{t+s} = Z_s \circ \overline{\tau}_t$ and $\overline{\tau}_{t+s} = \overline{\theta}_s \circ \overline{\tau}_t$ on $\{\alpha < U_t < \beta\}$. This formula is also true for T_n instead of T, where (T_n) is the decreasing dyadic approximation of T , which extends for a general T by the right continuity of the processes (Φ_t), (U_t) and $(\overline{\tau}_t)$. The formula (4) is argued in the same manner as (2) by using the formula (30) of $[1]$.

Conditional laws of pairs of excursions

We consider now the time-reversed process $\left(\check{Y}_t\right)_{t\in R} = \left(Y_{(-t)^-}\right)_{t\in R}$. It is an E-valued right continuous process with left limits on $\left]\check{\alpha},\check{\beta}\right[= \left]-\beta,-\alpha\right[$, and which is equal to δ outside of $\left]\check{\alpha},\check{\beta}\right[$. As in $[1]$ and $[10]$, we assume that $\left(\check{Y}_t\right)$ is also Markov with respect to another standard semigroup $\left(\check{P}_t\right)$ satisfying " les hypothèses droites de Meyer" , which implies the strong Markov property and the existence of exit systems. The measure

$$\hat{\eta}(B) = Q\left(\check{Y}_t \in B; \check{\alpha} < t < \check{\beta}\right)$$

is $\left(\check{P}_t\right)$-excessive and the stationarity of $\left(\check{Y}_t\right)$ is guarantee. Let $\hat{\tau}_t, \check{G}_t, \check{B}, \hat{S}, \check{V}_t, \check{U}_t$ and $\check{E}_t$ be the analogous of $\tau_t, G_t, B, S, V_t, U_t,$ and E_t corresponding to $\left(\check{Y}_t\right)$. As previously we assume that Q $a.e.$ $\check{B}\left]\check{\alpha},t\right] < \infty$ for all $t > \check{\alpha}$. For the process $\left(\Psi_t\right) = \left(\check{Y}_{\check{U}_t}\right)$ and the random subset

$$\check{M} = \left]\check{\alpha} < t < \check{\beta}\right[\cup \left\{t + R\circ\hat{\tau}_t\right\}$$

of $\left]\check{\alpha},\check{\beta}\right[$, we have the analogous of theorem 1. In particular if we design by $\check{H}$ and $\check{H}_t$ the Q completions of $\sigma\left(\Psi_u : u \in R\right)$ and $\sigma\left(\Psi_u : u \leq t\right)_+$ respectively, and by $\check{P}^{x,y}$ the measure defined as $P^{x,y}$ in terms of the exit measures ${}^0\check{P}^x$ of $\check{M}$ for the canonical realization of $\left(\check{P}_t\right)$, we have the following formula:

$$Q\left(F\left(\check{E}_{T_2}\right)/\check{H}\right) = \check{P}^{\Psi_{T_2^-},\Psi_{T_2}}\left(F\right) \text{ on } \left\{\check{\alpha} < \check{U}_{T_2} < \check{\beta}\right\} \quad (5)$$

for all finite $\left(\check{H}_t\right)-$stopping time T_2 such that $\check{U}_{T_2} \neq \check{U}_{T_2^-}$ and $\Psi_{T_2^-} \neq \Psi_{T_2^-}$ Q $a.e.$, and for all positive function F^*-measurable F.

For the following theorem which gives the conditional law of pairs of excursions, we consider the family of probability measures

$$Q^{x,y,z,u} = P^{x,y} \otimes \check{P}^{z,u}.$$

Theorem 2:

Let T_1 (*resp.*T_2) *as in* (4) $\left(\text{resp.}(5)\right)$. *We assume that the following hypotheses are satisfied:*

1) $\sigma\left[U_{T_1^-}\right] \cap \Lambda \subset \check{H}_{T_2^-}$ *2)* $\sigma\left(\check{U}_{T_2^-}\right) \cap \Lambda \subset H_{T_1^-}$,

where $\Lambda = \left\{\alpha < -\check{U}_{T_2^-} \leq U_{T_1^-} < \beta\right\}$. *Then we have the following formula:*

$$Q\left(H\left(E_{T_1},\check{E}_{T_2}\right)/H \cap \check{H}\right) = Q^{\Phi_{T_1^-},\Phi_{T_1},\Psi_{T_2^-},\Psi_{T_2}}\left(H\right) \text{ on } \Lambda \quad (6)$$

for all positive and $F^* \otimes F^*$*-measurable function* H .

Proof:

We have to prove that:

$$Q\left(F\left[E_{T_1}\right]\check{F}\left(\check{E}_{T_2}\right)ZI_\Lambda\right) = Q\left(P^{\Phi_{T_1^-},\Phi_{T_1}}\left(F\right)\check{P}^{\Psi_{T_2^-},\Psi_{T_2}}\left(\check{F}\right)ZI_\Lambda\right) \quad (7)$$

for all $F, \tilde{F}$ positive functions and F^*-measurable, and for all positive random variable Z, $H \cap \tilde{H}$ -measurable. Since $\hat{\tau}_{\tilde{U}_{T_2^-}} = \theta_{U_{T_1^-}+\tilde{U}_{T_2^-}} \circ \hat{\tau}_{-U_{T_1^-}}$ on Λ, and since $\hat{\tau}_{-U_{T_1^-}}$ is $G_{\left(U_{T_1^-}\right)^-}$ $-$measurable and $G^D_{\left(U_{T_1^-}\right)^-} = H_{T_1^-}$, then $\sigma\left(\hat{\tau}_{\tilde{U}_{T_2^-}}\right) \cap \Lambda \subset H_{T_1^-}$ ($G_t^D = G_{D_t} \supset G_t$ where $D_t = \inf\left\{s > t : s \in M\right\}$ on W, for $t \in R$). By using the same argument, we prove that $\sigma\left(\Psi_{T_2^-}, \Psi_{T_2}\right) \cap \Lambda \subset H_{T_1^-}$, $\sigma\left(U_{T_1^-}\right) \cap \Lambda \subset \tilde{H}_{T_2^-}$ and $\sigma\left(\Phi_{T_1^-}, \Phi_{T_1}\right) \cap \Lambda \subset \tilde{H}_{T_2^-}$.

The Markov property at time T_1 and the formula (5) implied that for all positive random variable Z_1, $H_{T_1^-}$ $-$measurable and for all positive and F^0 $-$measurable function φ :

$$Q\left(F\left(E_{T_1}\right)\tilde{F}\left(\tilde{E}_{T_2}\right)Z_1\varphi\left(\bar{\tau}_{T_1}\right)I_\Lambda\right)$$

$$=Q\left(P^{\Phi_{T_1^-},\Phi_{T_1}}(F)\tilde{F}\left(\tilde{E}_{T_2}\right)Z_1\varphi\left(\bar{\tau}_{T_1}\right)I_\Lambda\right)$$

and according to the fact that H is generated by $H_{T_1^-}$ and $\bar{\tau}_{T_1}$, we have:

$$Q\left(F\left(E_{T_1}\right)\tilde{F}\left(\tilde{E}_{T_2}\right)ZI_\Lambda\right)=Q\left(P^{\Phi_{T_1^-},\Phi_{T_1}}(F)\tilde{F}\left(\tilde{E}_{T_2}\right)ZI_\Lambda\right) \quad (8)$$

The formula (7) follows by using formulas (8) and (5).

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