

A LINEAR PROGRAMMING ALGORITHM FOR LEAST-COST SCHEDULING

Abstract

In this research, some concepts of linear programming and critical path method are reviewed to describe recent modeling structures that have been of great value in analyzing extended planning horizon project time-cost trade-offs problems.

A simplified representation of a small project and a linear programming model is formulated to represent this system. Procedures to solve these various problems formulations were cited and the final solution is obtained using LINDO program. The model developed represents many restrictions and management considerations of the project. It could be used by construction managers in a planning stage to explore numerous possible opportunities to the contractor and predict the effect of a decision on the construction to facilitate a preferred operating policy given different management objectives.

An implementation using this method is shown to outperform several other techniques and a large class of test problems. Linear programming show that the algorithm is very promising in practice on a wide variety of time-cost trade-offs problems. This method is simple, applicable to a large network, and generates a shorter computational time at low cost, along with an increase in robustness.

Key words: *Linear programming, cost optimization, least cost-scheduling, time-cost trade-off, project compression, expediting.*

Résumé

Dans ce travail de recherche, quelques concepts de programmation linéaire et de méthodes de parcours critiques sont examinés en vue de décrire la structure de modèles récents qui se sont avérés d'une grande valeur pour l'optimisation de la durée et du coût des projets de construction.

Ce travail est illustré par la représentation simplifiée d'un projet et un modèle de programmation linéaire est formulé pour représenter ce système. Les méthodes de résolution de ces différentes formulations des problèmes sont citées et une solution est obtenue en utilisant le programme LINDO. Le modèle développé signale un certain nombre de restrictions et des considérations de gestion du projet. Il peut-être utilisé durant la phase de planification afin d'explorer les nombreuses opportunités qui s'offrent au constructeur dans ses prises de décisions finales.

On montre que la mise en œuvre de cette approche d'étude, appliquée à un large éventail de problèmes faisant intervenir les paramètres durée et coût, rend plus performant l'utilisation d'autres techniques.

Mots clés: *Programmation linéaire, optimisation du coût et de la durée du projet.*

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ملخص

أن الهدف من هذا البحث هو استخدام نموذج البرمجة الخطية كنظام لتحليل التكاليف الإنشائية وذلك باختصار العمر الزمني للمشروع والحصول على أقل كلفة ممكنة. تعتمد هذه الطريقة على خوارزمية فاعلة لحل مجموعة من معادلات البرمجة الخطية لتحقيق درجة أعلى من الضبط وإمكانيات الرقابة على التكاليف. والفكرة الرئيسية للبحث هي استخدام المعلومات المتعلقة بالكلفة الإنشائية والفترة الزمنية اللازمة لكل نشاط بهدف إيجاد أفضل حل لدالة الهدف بشرط التوافق مع القيود. تم بعد ذلك تغذية تلك المعلومات إلى برنامج LINDO لحساب المعاملات وتحديد الحل الأمثل.

لقد تم اختيار هذا النموذج لما له من ميزات عديدة تتفوق على الطرق الأخرى والمستخدمة في تخطيط وبرمجة المشاريع مثل طريقة المسار الحرج وطريقة تقويم ومراجعة البرامج. وقد أثبتت الخوارزمية المعروضة في هذا البحث فعاليتها في العديد من الأمثلة المتعلقة باختصار العمر الزمني للمشروع.

الكلمات المفتاحية: *خوارزمية البرمجة الخطية، أقل كلفة ممكنة، اختصار العمر الزمني للمشروع.*

Construction management involves the coordination of group of activities where in the manager plans, organizes, staffs, directs and controls construction projects to achieve an object, including a specification of their interrelationships and considering the required resources in an acceptable time span.

Most construction managers are continually facing a situation in which they must take a decision whether to complete the project sooner than originally specified in the contract because of the clients request and /or to optimize the cost of expediting. The planned duration is decreased by crashing all critical activities either by authorizing overtime work or applying additional resources.

The objective of critical Path Method (CPM) and Program Evaluation and Review Techniques (PERT) is to establish a feasible and desirable relationship between the time and cost of the project by reducing the target time and taking into account the cost of expediting. A number scheduling methods were developed for planning and scheduling of construction projects using graphical methods such as line of balance and vertical production method. These techniques are neither suitable for the scheduling of

linear projects nor adequate for addressing typical challenge related to time-cost trade-off [1].

fundamental failures in these models. Field and academic research have failed to question the feasibility of the network technique for construction. It is suggested that these methods are neither true models nor best approximate model of the construction process because control of construction. It is suggested that these methods are neither a true models nor best approximate model of the construction process because control of construction resources is more desirable than minimum calendar duration of the whole project. Therefore, minimizing cost is an objective to be considered as much as minimizing overall duration [2].

Optimal schedule cost can be determined by trial and error for small project, but in a realistic project consisting of many activities, such trial-and error becomes extremely tedious and impossible. A very limited number of computer programs are available but far from perfect. Such programs have a limited capacity to accept time-cost data and at a very high price. Other limitations of these programs are that, the only data the computer can handle is the time-cost slope for individual activities. Another serious shortcoming has been the computational time when changes of network logic are involved. Finally, the excessive or inappropriate use of computers specially in a moderately sized network is another major factor of such failures. Because of these major failings, such programs have led to dissatisfaction and found little acceptance in the construction industry.

Increased sophistication in optimization techniques have lead to examine the possibility of incorporating a time-cost trade-off within an optimization framework. When changes in the network logic are involved, this method has advantage that decisions required of the decision maker are simple, and can handle a large data or alternatives. Thus optimization technique has been developed to aid in the quick determination of the minimum cost for every possible value of project duration. Clearly, the use of optimization techniques incorporated with time-cost trade-off becomes an economic necessity and the objective of this research.

LITERATURE REVIEW

Selinger [3] developed a dynamic programming model of a linear project. His work ignored to incorporate the cost as decision variable in the optimization process. As extension of the Selinger's work, Russel and Caselton [4] formalized a N-stage dynamic programming solution into two state variable to determine the minimum project duration. In the optimization process, the developed model ignored the activities costs as a decision variable.

Reda [5] developed a linear programming to identify minimum cost maintaining constant production rates and repetitive projects. This method can only be used for nontypical linear project and not applicable to construction projects. Most of the developed models assume the activities are accomplished serially. In reality, most construction activities are accomplished concurrently while others accomplished serially.

Failure of many contractors to fully use CPM or PERT exposes

Elmaghraby [6] considered completion schedules on an arbitrary set of milestone events by developing an efficient algorithm to determine the project schedule, which minimizes the sum of the total cost plus penalties for late completion. Another extension was by Moore [7] by using goal programming to consider multiple objectives, such as completion times, resources leveling and operation within a limited budget.

Senouci [1] presented a dynamic programming formulation for the scheduling of non sequential or nonserial activities to determine the project time-cost profile which determines possible project duration and their minimum project total cost. The formulation considers the effects of interruptions, minimum project direct cost, and minimum project duration.

COMPUTATIONAL PROCEDURE

Concept of project expediting has a considerable economic importance especially in large and costly projects. Cost slope is given by:

$$C_{ij} = \frac{C_{d_{ij}} - C_{D_{ij}}}{t_{D_{ij}} - t_{d_{ij}}} = \frac{\Delta C}{\Delta t}$$

- Where: C_{ij} = Cost slope
- $C_{d_{ij}}$ = Crashed cost
- $C_{D_{ij}}$ = Normal cost
- $t_{D_{ij}}$ = Normal time
- $t_{d_{ij}}$ = Crashed time
- ΔC = increase in cost
- Δt = increase in time

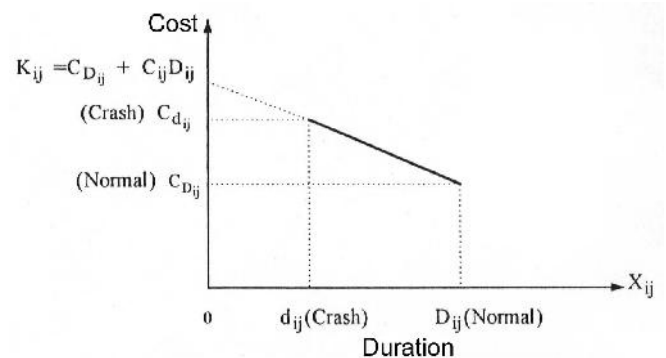


Figure 1: Time-cost trade-off curve nomenclature.

The basic formulation of the time cost trade-off problem has a curve of the form shown in figure 1. The advantage of this notation used in this figure is that it replaces the normal and crash costs by a single slope, C_{ij} . Once the cost details of activities are available, the formulation of linear programming is possible. Letting X_{ij} denote the scheduled

duration of the activity i-j, the total cost as a function of these variables is as follows:

$$\sum_i \sum_j (K_{ij} - C_{ij}X_{ij}) = K - \sum_i \sum_j C_{ij}X_{ij}$$

Where $\sum_i \sum_j$ denote the summation over all activities in the project network and K_{ij} are fixed constants whose sum is denoted by K.

At this point, it must be recognized that when shortening the critical path leads to reduce floats of other activities. Continued shortening of the critical path will lead to the formation of new critical paths and new critical activities. When multiple critical paths are involved, all such paths must be shortened if time advancement is to be achieved. The effect of each shortening action must be checked to ascertain if it has produced new critical activities. This accomplished by network recomputation after each step in the time reduction process. Such recomputation becomes extremely tedious.

To have a sound system of cost and time control of a construction project, mathematical programming is becoming increasingly important. Linear programming uses a mathematical model to describe the problem of concern. It deals with the optimization of a linear objective function subject to a set of constraint conditions in the form of linear inequalities and/or equations. Thus linear programming involves the planning of activities in order to obtain an optimal result, i.e.; a result which reaches the specified goal best among all feasible alternatives. A characteristic of many projects is that all work must be performed in some well-defined order. This formulation concerns the scheduling of the activities, which combine to make a project. The analysis requires a graphical illustration of the starting and ending times and costs for each activity of the project are known. The linear programming formulation provides a means of selecting the least costly schedule for desired completion time.

Linear programming analysis may be utilized to maximize a linear function subject to a finite number of linear constraints. In constructing the model, the objective function is to minimize the overall cost in order to reduce the completion time. The mathematical form of the objective function is given by:

$$\text{Minimize } f(X_1, X_2, \dots, X_n) = c_1X_1 + \dots + c_nX_n$$

where:

c_1, c_2, \dots, c_n are real cost coefficients.

X_1, X_2, \dots, X_n are decision variables to be determined.

The mathematical form consists of a set of "m" constraints of inequality form:

$$a_{11}X_1 + a_{12}X_2 + \dots + a_{1n}X_n \geq b_1$$

$$a_{21}X_1 + a_{22}X_2 + \dots + a_{2n}X_n \geq b_2$$

$$a_{m1}X_1 + a_{m2}X_2 + \dots + a_{mn}X_n \geq b_m$$

Where a_j, b_j, c_j are real constants and $X_j \geq 0$.

Combining the objective, objective function and the constraints, the linear programming model can be written in a concise notation form as follows:

$$\text{Minimize } \sum_{j=1}^n c_j X_j$$

subject to

$$\sum_{j=1}^n a_{ij} X_j \leq b_i \text{ for all "i"}$$

$$X_j \geq 0 \text{ for all "j"}$$

By solving the linear programming problem, the crash schedule and the corresponding crash cost can be found.

Illustrative example

It is useful at this point to illustrate the procedure with a small example in which there are six activities as shown in figure 2. Each activity is accomplished by a single crew starting from the first activity to the last. Table 1 summarizes the input data for the example.

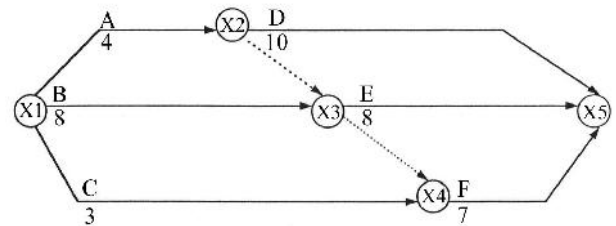


Figure 2: Project network.

Activity	Duration		Cost	
	Normal	Crash	Normal	Crash
A	4	2	400	500
B	8	5	800	980
C	3	2	600	700
D	10	6	500	600
E	8	6	800	950
F	7	4	700	1000

Table 1: Activity cost and duration data for the project.

The proposed method can handle activity time-cost functions. The numerical example was performed on personal computer using LINDO [9]. It was easy to tackle this problem by visual inspection and simple judgement. This simplicity will assist the understanding of the procedure. The computation procedures of the project starting with the first activity to the last one has the form:

Minimize $50Y_A + 60Y_B + 100Y_C + 25Y_D + 75Y_E + 100Y_F$
 Subject to the following constraints:

$$X_2 + Y_A \geq 4$$

$$X_3 + Y_B \geq 8$$

$$\begin{aligned}
 X_4 + Y_C &\geq 3 \\
 X_5 - X_2 + Y_D &\geq 10 \\
 X_3 - X_2 + 0 &\geq 0 \\
 X_5 - X_3 + Y_E &\geq 8 \\
 X_5 - X_4 + Y_F &\geq 7 \\
 X_4 - X_3 + 0 &\geq 0 \\
 Y_A &< 2 \\
 Y_B &< 3 \\
 Y_C &< 1 \\
 Y_D &< 4 \\
 Y_E &< 2 \\
 Y_F &< 3 \\
 X_5 &< 12
 \end{aligned}$$

and all variables are nonnegative. The objective function minimizes the sum of the total construction cost that occurs in all links of the system during all of the periods. Acting within the constraints and related costs, it is required to determine the crashing time for each activity, which will make the cost function a minimum.

ANALYSIS AND RESULTS

The linear programming model will not only take into account the activities on the critical path, but will also consider the noncritical activities, which in their turn become critical as the project time decreases.

Solution of the model is presented in table 2, which indicates that the project can be completed within 11 week deadline by spending an extra 505 JD.

Subtracting the crash-time amounts from the normal completion, the result indicates that activity B should be completed in 5 weeks, D in 7 weeks, E in 6 weeks and F in 6 weeks. The nodes of the network, or milestone of the project, are indicated by X2 through X5.

The linear programming solution also provides a valuable sensitivity analysis. As shown in the computer output in table 2, the "reduced cost" column indicates the cost increase in activities not crashed by this solution and achieve the same project deadline. Further interesting results appear in the computer output is the amount of available reduction used and is presented in the "slack or surplus" column. The net cost of changing the solution is presented in the third column "dual prices" which is solved per period in each constraint. As can be seen in row 16, there will be a cost increase of 200 JD to reduce the overall project time X5 by one week from the 11-week deadline. Finally, the sensitivity analysis indicates the range of the coefficient values over which the optimal solution remains unchanged.

The sensitivity analysis can also be helpful for assessing the effects of uncertainties in some of the data items. The further uncertainties in the data may be examined directly by running the program with upper and lower limits on the data to check the sensitivity of the results in different circumstances. The linear programming analysis carried out to determine the optimal policy of investing in extra resources in order to meet the deadline is obtained. It is important for project manager to recognize the flexibility of

the system that can be used to explore numerous possible opportunities to the contractor.

Many iterative methods for solving this problem have been proposed; they are usually descent methods that do not guarantee a minimum cost. Excessive number of iterations or functions evaluations causes the failures. For comparison of commercially available CPM/PERT computer program see the work of Smith [8].

This approach allows the user to easily manipulate different project networks of various difficulties representing real world applications, and to study the effectiveness of the model in the case of large projects. The implementation of the developed model is tested on a large number of linear optimization problems and shown to have more efficient and reliable results, generates a considerable computational savings, along with an increase in robustness.

Table 2: LINDO output file

LP OPTIMUM FOUND AT STEP		16
OBJECTIVE FUNCTION VALUE		
1)	505.000	
VARIABL	VALUE	REDUCED COST
E		
YA	.000000	25.000000
YB	3.000000	.000000
YC	0.000000	100.000000
YD	3.000000	.000000
YE	2.000000	.000000
YF	1.000000	.000000
X2	4.000000	.000000
X3	5.000000	.000000
X4	5.000000	.000000
X5	11.000000	.000000
ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.00000	-25.00000
3)	.00000	-175.00000
4)	2.00000	.00000
5)	.00000	-25.00000
6)	1.00000	.00000
7)	.00000	-75.00000
8)	.00000	-100.00000
9)	.00000	-100.00000
10)	2.00000	.00000
11)	.00000	115.00000
12)	1.00000	.00000
13)	1.00000	.00000
14)	.00000	.00000
15)	2.00000	.00000
16)	.00000	200.00000

N° ITERATIONS = 16

DO RANGE (SENSITIVITY) ANALYSIS? Y

RANGES IN WHICH THE BASIC IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF.	ALLOWABLE INCREASE	ALLOWABLE DECREASE
YA	50.0000	INFINITY	25.000000
YB	60.0000	115.0000	INFINITY
YC	100.0000	INFINITY	100.000000
YD	25.0000	25.0000	25.000000
YE	75.0000	INFINITY	75.000000
YF	100.0000	INFINITY	100.000000
X2	.0000	25.0000	25.000000
X3	.0000	INFINITY	115.000000
X4	.0000	INFINITY	100.000000
X5	.0000	200.0000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	4.000000	1.000000	3.000000
3	8.000000	.000000	1.000000
4	3.000000	2.000000	INFINITY
5	10.000000	1.000000	3.000000
6	.000000	1.000000	INFINITY
X3	.000000	INFINITY	115.000000
X4	.000000	INFINITY	100.000000
X5	.000000	200.0000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	4.000000	1.000000	3.000000
3	8.000000	.000000	1.000000
4	3.000000	2.000000	INFINITY
5	10.000000	1.000000	3.000000
6	.000000	1.000000	INFINITY
7	8.000000	.000000	2.000000
8	7.000000	2.000000	1.000000
9	.000000	2.000000	1.000000
10	2.000000	INFINITY	2.000000
11	3.000000	1.000000	.000000
12	1.000000	INFINITY	1.000000
13	4.000000	INFINITY	1.000000
14	2.000000	INFINITY	.000000
15	3.000000	INFINITY	2.000000
16	11.000000	1.000000	.000000

CONCLUSION

In this work, we propose an algorithmic model based on linear programming incorporated with a minimal time-cost trade-off in a construction project. The format of the model lends itself to a wide range of variables and considerations.

The present modeling strategy has shown the resources of this interactive approach including a bulk of data to completely analyze the project is easily possible. It allows a great number of parameters to simulate project conditions and contractor's preference and provides potentially useful tool for decision making on project scheduling.

Various computer programs reveal that these packages cannot satisfy the users in a large number of activities. Use of linear programming models outperforms the default algorithm implemented in these packages to minimize total construction cost. It appears that the programming in project time-cost trade-off concept has more to offer than the graphical methods and scheduling software packages. A real understanding of this concept leads to improved solutions to large scale systems facing construction cost control systems and network compression.

The cost of the network activities has been optimized for various overall duration. The optimum trade-off of time against cost has been made. This approach is an acceptable tool of management and proving to be not only superior method for planning, scheduling and controlling project progress, but also is very real and valuable assets to contractors in convincing the owner of their potentials and abilities. With the introduction of better and more rigorous methods of planning work, together with cost analysis, the construction control will become more systematic. Mathematical models are used more and more for executive planning functions. In all of these, decisions must be made to carry out the operation in the best way possible in light of the restraints that are bound to exist.

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