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MASS TRANSFER ANALYSIS IN AN INTERMITTENTLY POROUS CHANNEL

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Abstract

The present work analyses mass transfer in a channel with successive porous matrices. The fluid subject to a concentration gradient flows alternatively through plain and porous regions. The fluid considered is air with different concentrations of solvents corresponding to water vapour, carbon dioxide and ether. Effects of Darcy number indicating the permeability of the porous medium on velocity and concentration profiles in the channel are investigated. The obtained results show that as Darcy number decreases, mass transfer coefficient decreases implying higher performance of filtration. Thus the filtration is more efficient when the permeability is the lowest. In addition, for fixed Darcy and Reynolds numbers, water vapour is better retained within the porous matrix than the two other substances considered with a higher Schmidt number.

Finally, a comparison between a partially and a fully porous channel indicates that both cases yield the same increase in mass transfer coefficient but with a significantly reduced pressure drop in the partially porous configuration. This appears to be promising for application to filtration systems with minimum pressure drop.

***Keywords**: Porous medium - Mass Transfer - Forced Convection - Partially Porous Channel – Filtration.*

Résumé

L'étude entreprise est une contribution à la quantification du transfert de masse s'effectuant au sein d'un écoulement, en convection forcée, dans un canal partiellement poreux. Le thème est inspiré par la nécessité, dans certains procédés industriels, d'une optimisation et d'une maîtrise des transferts massiques. Une investigation sur la séparation des constituants d'un fluide composé de deux éléments est donc entreprise avec une approche consistant en l'analyse de trois mélanges de fluides (air+vapeur d'eau, air+dioxyde de carbone et air+ether) lors de la traversée d'une succession de cloisons poreuses dans un canal plan. Les résultats sont traduits en termes de profils des vitesses, donnant une idée de la variation des pertes de charge en fonction de la variation des paramètres physiques en présence. Une quantification des taux de matière filtrée est ainsi effectuée et l'évolution en fonction, notamment, de la perméabilité du substrat poreux et de la composante du mélange est mise en évidence.

***Mots clés**: Milieu poreux – Transfert de masse – Convection forcée - canal partiellement poreux – Filtration.*

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ملخص

العمل الحالي (قيد البحث) ذو صلة وثيقة بتقنية المواد خلال مائع جاري عبر إخضاعها لسلسلة من الأجسام المشبعة ذات النفاذية داخل قناة (مجرى). جداول للخواص الفيزيائية للمائع واللأغشية المسامية ثم اختيارها لثلاثة مديات مختلفة موجودة في الهواء المناسب.

دراسات نوعية أجريت لتحليل تأثيرات رقم دارسي على مساري السرعة والتركيز داخل القناة, تنجز الدراسة الثلاثة تدرجات من الإذابة وفقا لبخار الماء وثاني أكسيد الكربون والايثر في الهواء مع افتراض أن الجدران متساوية التركيز.

النتائج توضح أنه كلما يقل رقم دارسي, التنقية تزداد وبالتالي فان عملية الترشيح (التنقية) تكون أكثر كفاءة وفعالية عندما تكون النفاذية في حدها الأدنى. كما لاحضنا أنه عند تثبيت رقمي دارسي ورينولدز فان بخار الماء ينقى (يتم ترشيحه) افضل من المادتين الأخرتين اللتان تحددان عند رقم (شميدت) أعلى. وأخيرا فان المقارنة بين قناة جزئية النفاذية وأخرى تامة النفاذية تشير إلى أن تضمن الزيادة في نقل الكتلة (المادة) من الممكن الحصول عليها في كلتا الحالتين بينما هناك تقليل واضح في خسائر الضغط عند النفاذية الجزئية.

***الكلمات المفتاحية**: الأوساط المسامية، نقل مرغ للمادة، قناة جزئية النفاذية، عملية التنقية.*

Several technological and industrial processes require control of mass transfer in porous media. These processes are encountered in numerous applications including drying and dehumidification, filtration and separation of components (in the hydrocarbon or water treatment for example), as well as the catalytic converters used for reduction of air pollution by the exhaust products of combustion in automobile industry. The study of mass and heat transport in porous media has been of a growing interest in the last few years. Nevertheless, in the majority of the studies undertaken, the models are often limited to cases of water vapour diffusion, in particular in the problems of drying where the flow is described by the Darcy law.

A literature survey shows that the work of Bories [1] presented an interesting review of the last investigations conducted on heat and moisture transfer considering various theoretical approaches. Le and Ly [2] proposed a model for simultaneous heat and mass transfer of a vapour by convection through an absorbing textile fibre assembly. They presented a model that accounts for the presence of liquid water in the bed. The Darcy equation is used to describe the flow of the steam and a non-local thermal equilibrium is invoked for computation of the changes associated with the condensation front. Pel *et al.* [3] have given the coefficient of mass diffusion of moisture by measuring profiles of concentration during drying and using the principle of the nuclear magnetic resonance which is proven to be a method adapted for this purpose. Manole and Lage [4] examined in their work thermohaline convection in a porous medium for various geological and environmental situations.

In fact, very few studies addressed the problem of mass convection in porous media including other effects than the macroscopic resistance due to Darcy term. For instance, we consider in the present work an analysis of mass convection in a channel with a succession of saturated porous matrices. A formulation including both Darcy and Brinkmann terms is adopted to describe the flow within the porous medium.

1- MATHEMATICAL FORMULATION

The purpose of the study is to model the separation of components of a fluid by analysing the mass transfer during its flow in an intermittently porous channel. A comparison is carried out with the case of a completely porous case. A detailed numerical study including the influence of the properties of the flowing fluid and the characteristics of the solid matrix is underlined for several mixtures of fluids.

The physical model is the same as the one considered by Hadim [5] for heat transfer. In our work, we consider mass transfer in a homogeneous incompressible fluid constituted of two components (aqueous solution and solvent), flowing in a steady state through a two-dimensional parallel plate channel with a succession of porous partitions. The porous medium is saturated by the mixture, and the physical properties of the various components are assumed to be constant. Thus the flow will successively cross fluid regions and porous obstacles more or less permeable.

The flow in the fluid zones is governed by continuity and momentum (Navier-Stokes) equations as well as the conservation of the chemical species (diffusion). The momentum equation is corrected by the introduction of the Darcy-Brinkmann model which accounts for the macroscopic resistance as well as the viscous effects in the porous region. The diffusion equation is corrected by a factor which includes the effect of porosity.

Under the assumptions cited above, the formulation is written as :

- Mass balance equation for the two regions:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

- Momentum equations :

* Fluid regions:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_f\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right] \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_f\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right] \tag{3}$$

* Porous regions:

$$\frac{\rho}{\varepsilon^2}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_e\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]-\frac{\mu_f}{K}u \tag{4}$$

$$\frac{\rho}{\varepsilon^2}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_e\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]-\frac{\mu_f}{K}v \tag{5}$$

- Diffusion equation:

* Fluid regions:

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right) \tag{6}$$

* Porous regions:

$$u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}=\varepsilon D\left(\frac{\partial^2 c}{\partial x^2}+\frac{\partial^2 c}{\partial y^2}\right) \tag{7}$$

where : ρ, ε, μ_f, μ_e, K are respectively density of the fluid, porosity of the medium, viscosity of the fluid, effective viscosity and permeability of the porous medium, D being mass diffusivity in the fluid.

The previous equations are associated with boundary conditions corresponding to the entry and the exit of the channel, as well as with imposed wall values. Then it follows:

At the inlet ($x = 0$): $u=u_0$ (inlet velocity); $v=0$; $c=c_0$.

At the outlet ($x = L$): $\frac{\partial u}{\partial x}=0$; $v=0$; $\frac{\partial c}{\partial x}=0$.

At the walls ($y=0$ and $y=H$): $u=v=0$ (no slip condition);
$c=c_0$ (uniform concentration).

In order to reduce the number of involved parameters, a non dimensional form is employed and the following scaled variables are used:

$$u^+=\frac{u}{u_0}\ ;\quad v^+=\frac{v}{u_0}\ ;\quad x^+=\frac{x}{H}\ ;$$

$$y^+=\frac{y}{H}\ ;\quad P^+=\frac{P}{\rho u_0^2}\quad C=\frac{c-c_p}{c_0-c_p}$$

The previous set of equations then become:

- Continuity equation for the two regions:

$$\frac{\partial u^+}{\partial x^+}+\frac{\partial v^+}{\partial y^+}=0 \tag{8}$$

- Movement equations :

* Fluid regions:

$$\left(u^+\frac{\partial u^+}{\partial x^+}+v^+\frac{\partial u^+}{\partial y^+}\right)=-\frac{\partial p^+}{\partial x^+}+\frac{1}{Re}\left[\frac{\partial^2 u^+}{\partial {x^+}^2}+\frac{\partial^2 u^+}{\partial {y^+}^2}\right] \tag{9}$$

$$\left(u^+\frac{\partial v^+}{\partial x^+}+v^+\frac{\partial v^+}{\partial y^+}\right)=-\frac{\partial p^+}{\partial y^+}+\frac{1}{Re}\left[\frac{\partial^2 v^+}{\partial {x^+}^2}+\frac{\partial^2 v^+}{\partial {y^+}^2}\right] \tag{10}$$

* Porous regions:

$$\frac{1}{\varepsilon^2}\left(u^+\frac{\partial u^+}{\partial x^+}+v^+\frac{\partial u^+}{\partial y^+}\right)=\frac{\partial p^+}{\partial x^+}+\frac{Rv}{Re}\left[\frac{\partial^2 u^+}{\partial {x^+}^2}+\frac{\partial^2 u^+}{\partial {y^+}^2}\right]-\frac{1}{DaRe}u^+ \tag{11}$$

$$\frac{1}{\varepsilon^2}\left(u^+\frac{\partial v^+}{\partial x^+}+v^+\frac{\partial v^+}{\partial y^+}\right)=-\frac{\partial p^+}{\partial y^+}+\frac{Rv}{Re}\left[\frac{\partial^2 v^+}{\partial {x^+}^2}+\frac{\partial^2 v^+}{\partial {y^+}^2}\right]-\frac{1}{DaRe}v^+ \tag{12}$$

- Diffusion equation:

* Fluid region:

$$u^+\frac{\partial C}{\partial x^+}+v^+\frac{\partial C}{\partial y^+}=\frac{1}{ScRe}\left(\frac{\partial^2 C}{\partial {x^+}^2}+\frac{\partial^2 C}{\partial {y^+}^2}\right) \tag{13}$$

* Porous regions:

$$u^+\frac{\partial C}{\partial x^+}+v^+\frac{\partial C}{\partial y^+}=\frac{\varepsilon}{ScRe}\left(\frac{\partial^2 C}{\partial {x^+}^2}+\frac{\partial^2 C}{\partial {y^+}^2}\right) \quad (14)$$

A number of dimensionless parameters is introduced in the above equations and they are expressed by:

- Reynolds number: $Re = \rho u_0 H/\mu_e$
- Darcy number: $Da = K/H^2$
- Viscosity ratio : $Rv = \mu_e/\mu_f$
- Schmidt number: $Sc = \nu_f/D$

The obtained equations are elliptic partial differential equations requiring the use of a numerical method of resolution. The choice was made on the control volume method recommended by Patankar [6].

2- RESULTS AND INTERPRETATION

The first step consisted of validating the results, in particular for the hydrodynamic part. Hence, a comparison is made with the results of Hadim [5]. An excellent agreement is obtained.

2.1- Hydrodynamic results

The results are first presented in terms of variation of the pressure along the channel. One notices in the partially porous case and for various Reynolds numbers considered, that the presence of the matrix generates an increase in pressure drop (Fig.1a). However, the pressure drop is less important than it is in a completely porous channel (Fig.1b). This is explained perfectly by the fact that the porous obstacle contributes to increase the pressure losses.

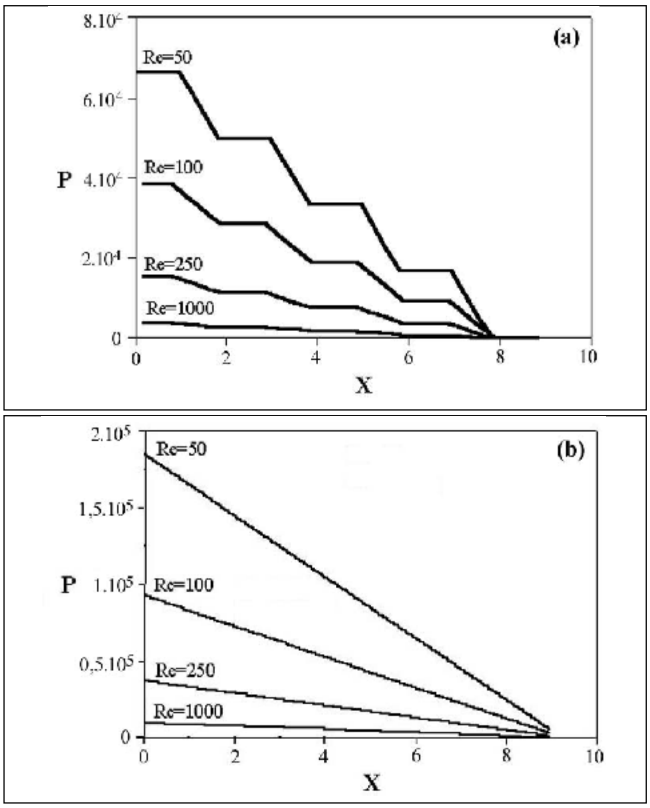

Figure 1: Local pressure for different Reynolds numbers for $Da=10^{-6}$. **(a)** partially porous case ; **(b)** completely porous case.

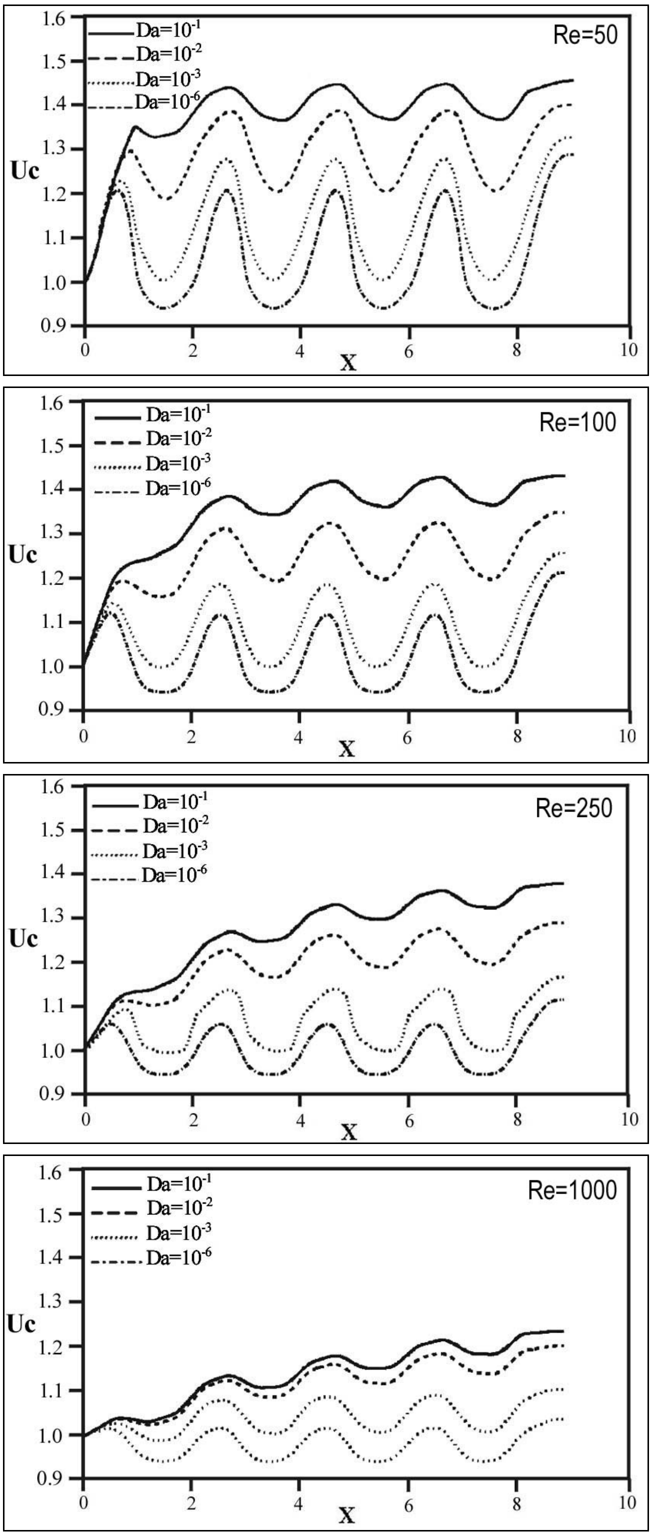

Figure 2 : Effect of Darcy number and Reynolds number values on axial velocity (partially porous case).

In the case of a partially porous channel, the velocity presents a variable intermittent profile between a quasi-parabolic form in the fluid and the flat profile in the porous substrate. This is perfectly illustrated by the curves in figure 2 which represent the axial velocity evolution along the channel. The hollows correspond to the porous zones and the crests to the fluid regions.

One notices on these figures that the presence of the porous matrix generates a deceleration of the flow, mainly for the low values of the permeability ($Da<10^{-3}$). In addition, it is noted that the increase in the value of the Reynolds number causes at the same time a reduction of the axial velocity and a flatness of the amplitude of the oscillation between the fluid and porous zones. Indeed, the increase in the Reynolds number increases the inertia effects in the two regions and masks, consequently, the effects of the viscous forces.

2.2- Mass transfer

To illustrate mass transfer results, we present the evolution, along the channel, of the average concentration of the mixture at different cross sections. Indeed, more this concentration at the exit is weak, more filtration is greater. It is first observed (Fig.3), that for a given solutal mixture, the average concentration is all the more weak since the permeability is low; that means a more important filtration in this case. This result is valid for both the completely porous configuration and the partially porous case.

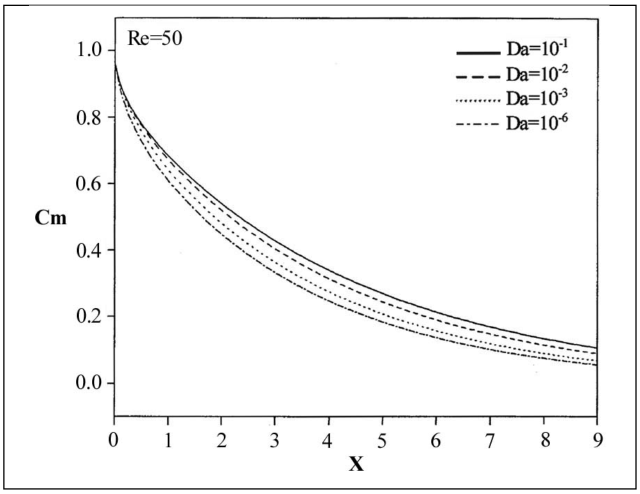

Figure 3: Evolution of the average concentration along the channel for different values of the permeability (Re=50; completely porous channel).

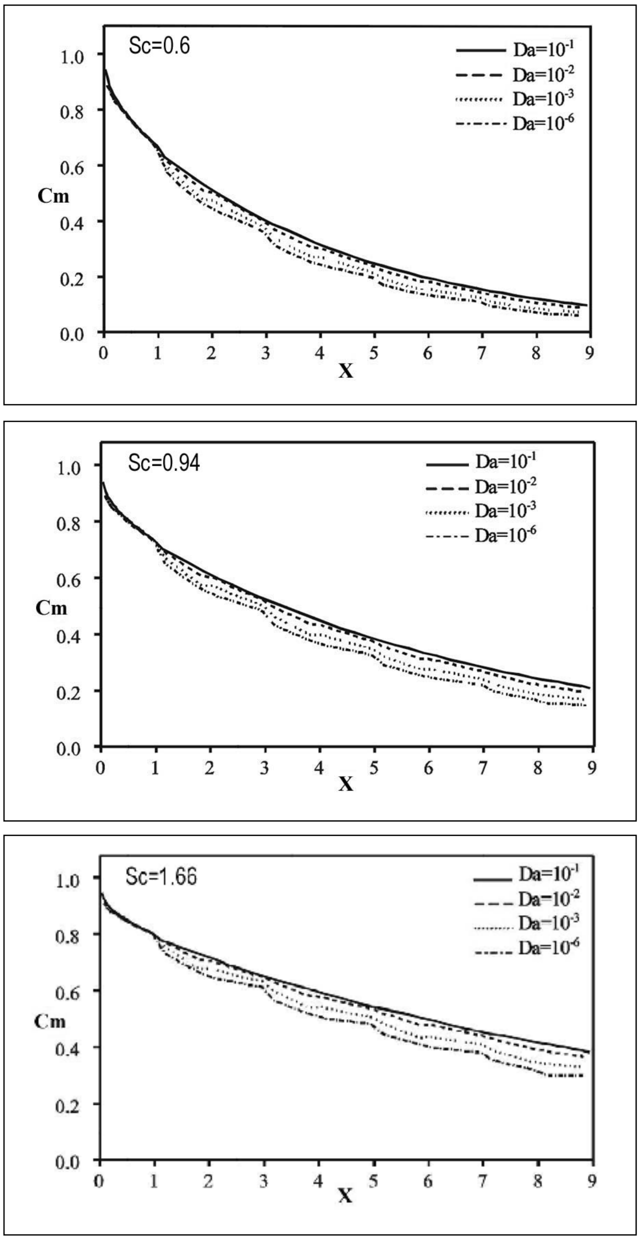

Figure 4: Evolution of the average concentration along the channel for different values of the permeability (Re=50; partially porous channel).

Figure 4 exhibits the evolution of the average concentration in the partially porous configuration for the three cases of solutal gradient considered: Sc=0.6 (steam in the air), Sc=0.94 (carbon dioxide in the air) and Sc=1.66 (ether in the air).

One observes first of all, for the three cases of mixtures, a reduction in the concentration along the channel, which represents a certain filtration of the components contained in the flow. It is noticed in particular that the mixtures with a relatively small Schmidt number are filtered better than the mixtures with a higher Schmidt number. Indeed, the value of this parameter, when it increases, lessens the effects of convection in the mass transfer balance. It follows a reduction in the variations of concentration and, consequently, an attenuation of filtration.

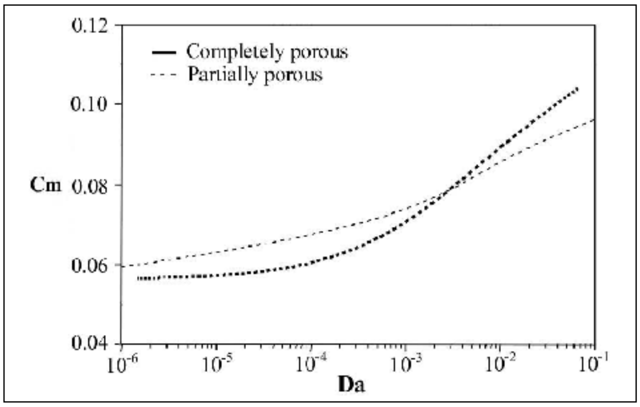

Figure 5: Effect of Darcy number on outlet average concentration (Re=50).

Lastly, figure 5 illustrates the effect of permeability on the outlet value of the average concentration. It shows the existence of a critical value of *Da* below which filtration is better in a partially porous channel and over which the completely porous channel yields a more important filtration. From a practical point of view, this critical value represents the minimum or maximum value of the permeability that should have the porous medium in order to obtain better filtration or enhanced mass transfer respectively without filling completely the channel. For example, it is not necessary to completely fill the channel by the porous substrate (saving in material), if one works with low permeabilities. This was also underlined by Hadim [5] for the case of the heat transfer.

CONCLUSION

A numerical study is carried out to investigate mass transfer and thus filtration of substances contained in fluid mixtures, through porous matrices. The optimal conditions for a better filtration with lower cost are determined. It is shown that the various physical properties of the involved components are of importance and have a great effect on performance of filtration. The permeability of the porous matrix and the diffusive properties of the fluids in presence in the flow are among these parameters.

REFERENCES

[1]- Bories S., "Recent advances in modelisation of coupled heat and mass transfer in capillary porous bodies", *Proceedings of Sixth Intern. Drying Symposium*, Versaille, Sept. (1988), pp. 5-8.

[2]- Le C.V. and Ly N.G., "Heat and mass transfer in the condensing flow of stream through an absorbing fibrous medium", *Int. J. Heat Mass Transfer*, vol. 38, (1995), pp.81-89.

[3]- Pel L., Brocken H. and Kopinga K., "Determination of moisture diffusivity in porous media using moisture concentration profiles", *Int. J. Heat Mass Transfer*, vol. 39, (1996), pp.1273-1280.

[4]- Manole D.M. and Lage J.L., "Convection induced by inclined thermal and solutal gradients, with horizontal mass flow, in a shallow horizontal layer of a porous medium", *Int. J. Heat Mass Transfer*, vol. 37, (1994), pp. 2047-2057.

[5]- Hadim A., "Forced convection in a porous channel with localized heat sources", *J. of Heat Transfer*, vol. 116, may (1994), p. 469.

[6]- Patankar S.V., "Numerical heat transfer and fluid flow", Mc Graw Hill, (1980). □