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AGGREGATION IN FUZZY FAULT TREE QUANTIFICATION: COMPARISON OF MEANS AND EXPERTONS TECHNIQUES

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Abstract

This paper presents a comparison of the two techniques: arithmetic means and expertons, used for aggregation of experts' judgments relative to basic events of fault trees. Valuations as confidence intervals included in [0, 1] have been considered. First, bounds are numbers to one decimal; next, numbers belonging to [0, 1]. In this last case, R+_expertons concept is used, with a counter-expertise form proposed. The means technique is well known in practice, but as fault tree is a logical diagram built by "AND" and "OR" gates, i.e. nonlinear operators, its use leads to wrong results and expertons technique should be used.

***Keywords**: Fuzzy Fault Tree Quantification, Aggregation, Arithmetic Mean, Experton.*

Résumé

Le présent article présente une comparaison de deux techniques : moyennes arithmétiques et expertons, utilisées pour l'agrégation des jugements d'experts relatifs aux événements de base des arbres de défaillance. Des valuations en intervalles de confiance inclus dans [0, 1] ont été considérées. Dans un premier temps, les bornes sont des nombres à une décimale, puis dans un autre, des nombres appartenant à [0, 1]. Dans ce dernier cas, le concept des R+_expertons est utilisé avec proposition d'une certaine forme de contre-expertise. La technique des moyennes est bien connue en pratique, mais comme l'arbre de défaillance est un diagramme logique construit avec des portes "ET" et "OU", i.e., des opérateurs non linéaires, son utilisation conduit à des résultats erronés et delà, la technique des expertons devrait être utilisée.

***Mots clés**: Quantification Floue d'un Arbre de Défaillance, Agrégation, Moyenne Arithmétique, Experton.*

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ملخص

هذا المقال يعرض مقارنة بين تقنيتين: المعدلات الوسطية و الاكسبرتونات، استعملتا لدمج أحكام خبراء متعلقة بالأحداث العنصرية لأشجار الخلل. تقديرات في شكل مجالات ثقة محتواة في [0 ، 1] استعملت. أولا، بحدود من أعداد برقم واحد وراء الفاصلة؛ ثانيا، من أعداد تنتمي للمجال [0 ، 1]. في هذه الحالة الأخيرة، مفهوم ح+ـاكسبرتونات استعمل، مع اقتراح شكل من أشكال الخبرة المعاكسة. إن تقنية المعدلات معروفة جدا عمليا، غير أنه بما أن شجرة الخلل هي بيان مكون من أبواب **'و'** و **'أو'**، أي معاملين غير خطيين، فان استعمالها يؤدي إلى نتائج غير صحيحة، ومن ثم وجب استعمال تقنية الاكسبرتونات.

***الكلمات المفتاحية**: التكميم الغامض لشجرة الخلل، الدمج، المعدل الحسابي، الاكسبرتون.*

In conventional Fault Tree (FT) analysis, calculation of the Top Event (TE) probability is carried out from the probabilities of Base Events (BE), which are treated as exact values. However, for many systems, it is often difficult to evaluate these probabilities from past components failures, either because of the lack of sufficient statistical data due to the fact that failures are rare events, or because of the change in systems environment. Furthermore, in the design phase new components whose probability of failure is needed often does not exist and must be estimated [1] [2]. In such situations, the use of experts' judgments becomes more and more an acceptable part in the risk assessment process and more effort is deployed to arrive to the best possible result. In particular, the way in which these judgments can be combined has been widely treated. As a result of this work there is no unique mode of aggregation that would be satisfactory in all instances [12].

The weighted average method, belonging to the compromise mode, is often encountered in theory decision and risk assessment to aggregate criteria. This is even the only attitude considered as rational by a number of researchers [3] [11]. It considers the experts' opinions in a more statistical way, i.e., as random variables, and is based on convex weighted sums (i.e., mixtures). The simplest form, corresponding to equal weights, is the ordinary Arithmetic Mean (AM), which will be dealt with in this paper. Although AM allows very simple calculations, its use with non-linear functions cannot be justified and would lead to wrong results [4] [14]. In particular, since FT is a logical diagram with "AND" and "OR" gates, i.e., its probability function is not linear, it should be necessary to employ another alternative to AM. Expertons technique due to Kaufmann and GIL Aluja [5, 6] seems more rigorous than AM when dealing with non-linear operators as the case of FT. Indeed, contrary to AM, expertons have the advantage to keep different judgments about probabilities of BE without deformation, and calculation of the top event experton is carried out level by level according to a hendecadarian scale. Thus, the entropy is made to fall later by calculation of mathematical expectation of this final experton, which will be considered in decision-making.

The main purpose of this paper is to present the results of a comparison of AM and expertons techniques which are used separately to aggregate valuations as Confidence Intervals (CI) originating from experts. The first section addresses the representation of expert's knowledge using CI, a particular case of fuzzy numbers. Then we consider theoretical formulations of combining theses intervals by AM and expertons. Intervals bounds considered are either numbers to one decimal or small numbers belonging to [0, 1]. In this last case corresponding to real values, the R^+_experton concept is used with a counter expertise form we have proposed. Lastly, a section addresses numerical results obtained and some comments.

1. REPRESENTATION OF EXPERTS' JUDGMENTS

Fuzzy sets theory has been developed to deal with fuzzy phenomena [15]. In particular, it offers an adequate framework that explicitly takes into account the lack of precision of the expert's knowledge. Thus, a vague response of an expert like "the probability of failure is between 0.45 and 0.55, and is perhaps around 0.5" can be well represented by the possibility concept, i.e., a fuzzy set defined in probability space [8, 13]. Explicitly, the possible values as described by the membership function of the fuzzy set are ordered according to their compatibility with the true value (s) of the probability. The simplest form of this representation is the CI defined as an interval that contains all the possible values of the probability with a level of confidence that the true value of this probability may exist within the interval [7] (see figure 1). The intervals with great possibility correspond to levels near to unity. It has been shown that the reliability or failure function based on the concept of possibilistic failure rates can be well manipulated by CI corresponding to any degree of possibility [2].

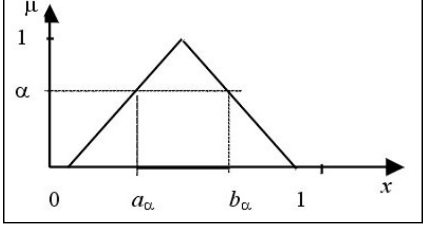

Figure 1 : α-level confidence interval.

The symbolic representation of a CI at α-level can be written as:

$$I_\alpha = [a_\alpha, b_\alpha] \quad (1)$$

This way of expressing knowledge is simpler and more natural than giving a point-value. Of course, allowing for imprecision reduces the uncertainty of the assessment, but imprecise statement are always safer than precise ones [4]. For a CI provided by an expert in order to locate the "probability" of a BE, we do not take into account the level of confidence of the CI. Thus, the representation we adopt is:

$$\tilde{p}_i = [p_*, p^*]^{(i)} \quad (2)$$

where p_* et p^* are valuations (i.e., subjective probabilities).

2. CALCULATION OF THE CI OF THE TE

To calculate the TE probability, the method of the Simple Structure Function (SSF) can be used [9]. Assuming that we have determined all the minimal cuts C_j (j=1,m) of a FT, and let x_i be the state variable associated to the BE e_i (i=1, n), the structure function of the FT is:

$$\varphi(x) = 1 - \prod_{j=1}^{m}(1 - C_j) \quad (3)$$

where: $C_j = C_j \prod_{e_i \in c_j} x_i$

Taking into account the idempotence property: $x_i.x_i = x_i$, (3) can be reduced to a simple form $\varphi_S(x)$ whose major advantage is to allow a direct calculation of the probability of the TE using a simple replacement of each state variable x_i by the corresponding probability p_i. We find:

$$P_S(p) = P_S(p_1, p_2, ..., p_n) \equiv \varphi_S(p_1, p_2, ..., p_n) = \varphi_S(p) \quad (4)$$

An important property of (4) is the increasing monotony expressed as:

$$((p) \succ (q)) \Rightarrow (P_S(p) \geq P_S(q)) \quad (5)$$

where: $(p) = (p_1, p_2, ..., p_n)$ and $(q) = (q_1, q_2, ..., q_n)$; $\succ$: domination sign, $\forall\ i, p_i \geq q_i$.

If we consider now for each BE e_i (i=1, n) valuation as CI $\tilde{p}_i = [p_{*i}, p^*_i]$, $p_{*i} \leq p^*_i$, the monotony property is written as:

$$((p_{*1}, p_{*2}, ..., p_{*n}) \prec (p^*_1, p^*_2, ..., p^*_n)) \Rightarrow (P_{*S} \leq P^*_S) \quad (6)$$

Expression (6) justifies clearly the feasibility of the calculation of the CI of the TE from the CIs of BE. The calculation of the CI of the TE using $\varphi_S(x)$ is carried out as follows: first, we replace the state variables of $\varphi_S(x)$ by lower bounds of fundamental CI; next, by upper bounds of these intervals. We obtain:

$$\tilde{P}_S(p) = [P_{*S}(p), P^*_S(p)] = [\ _S(p_*),\ _S(p^*)] \quad (7)$$

3. AGGREGATION TECHNIQUES

3.1. Arithmetic Means technique

Assuming that for a given BE e_i (i=1,n), each expert provides a CI $\tilde{p}_k^{(i)} = [p_{*k}, p^*_k]^{(i)}$, $p_{*k} \leq p^*_k$; k=1, r, with r is the number of experts. The AM of these r intervals is:

$$\tilde{m}^{(i)} = \frac{1}{r}\sum_{k=1}^{r} \tilde{p}_k^{(i)} \quad (8)$$

Explicitly:

$\widetilde{m}^{(i)} = [m_*, m^*]^{(i)} = \frac{1}{r}([p_{*1}, p^*_1]^{(i)}$ (+) $[p_{*2}, p^*_2]^{(i)}$ (+) ... (+)

$$[p_{*r}, p^*_r]^{(i)}) = \left[\frac{1}{r}\sum_{k=1}^{r}\widetilde{p}_{*k}, \frac{1}{r}\sum_{k=1}^{r}\widetilde{p}^*_k\right]^{(i)}$$

So, the CI of the TE is given by:

$$\widetilde{P}_S(p) = [P_{*S}(p), P^*_S(p)] = [\ _S(m_*),\ _S(m^*)] \quad (9)$$

3.2. Expertons Technique

3.2.1. CI having bounds belonging to J={0, 0.1, 0.2, .., 0.9, 1}

To help experts in their estimation, we suggest to them a semantic scale with 11 levels as shown in table 1.

Valuation	Semantic correspondence
1	Failing
0.9	Practically failing
0.8	Nearly failing
0.7	Quite failing
0.6	Rather failing than reliable
0.5	Neither failing nor reliable
0.4	Rather reliable than failing
0.3	Quite reliable
0.2	Nearly reliable
0.1	Practically reliable
0	Reliable

Table 1: Experts' valuations in J.

The expert can provide either a number belonging to the set J or an interval with bounds belonging to the same set. If we consider, for a BE e_i ($i=1,n$), r experts each of them provides a CI $\widetilde{p}_k^{(i)} = [p_{*k}, p^*_k]^{(i)}$, $p_{*k} \le p^*_k$ (k=1, r), the following stages allow to build the corresponding experton:

- Statistic: it is carried out on the upper bounds p_{*k} and on the lower bounds. Thus, for each scale level we obtain the interval: $\widetilde{S}_j = [S_{*j}, S^*_j]^{(i)}; j \in J = \{0, 0.1, 0.2, ..., 0.9, 1\}; \forall j \in J: S_{*j} \le S^*_j$.
- Normalisation: admitting that the statistic constitutes a low probability, we normalize frequencies dividing by the number of experts. We obtain:

$$\widetilde{N}_j^{(i)} = \frac{\widetilde{S}_j^{(i)}}{r} = \left[\frac{S_{*j}}{r}, \frac{S^*_j}{r}\right]^{(i)} = [N_{*j}, N^*_j]^{(i)}; \forall j \in J: N_{*j} \le N^*_j$$

- Complementary accumulation: starting from the level j=1, we find the cumulative complementary function or "the experton". For a level v, we have:

$$\widetilde{F}(v)^{(i)} = [F_*(v), F^*(v)]^{(i)} = 1 - \sum_{j=0}^{v-1}\widetilde{N}_j^{(i)} = \sum_{j=1}^{v}\widetilde{N}_j^{(i)} = \sum_{j=1}^{v}[N_{*j}, N^*_j]^{(i)} \quad (10)$$

Note that the interval $\widetilde{F}_*(v)$ is not a probability because it does not satisfy all the axioms of the probability theory, i.e., Borel-Kolmogorov axioms, there are only the bounds $F_*(v)$ and $F^*(v)$ that give by accumulation the unity [6]. The experton conception enables it to maintain the CI disorder. Indeed, the representation of their bounds with a cumulate function materializes clearly their weights in "probabilistic" point of view. All expertons have the following monotony property:

$$\forall(v,v') \in [0,1]^2, (v < v') \Rightarrow ([F_*(v'), F^*(v')] \le [F_*(v), F^*(v)]) \quad (11)$$

That is to say:

$$(v < v') \Rightarrow (F_*(v') \le F_*(v), F^*(v') \le F^*(v))$$

3.2.2. CI having bounds belonging to [0, 1]

In fault trees analysis, we often encounter BE with low probabilities. Therefore, computing an experton as described previously from such values appears impractical, because we will be obliged to consider a semantic scale with several levels which do not become easily identifiable. For this reason, we have used R^+_experton concept to aggregate experts' judgments in such case. To build this experton from CI $\widetilde{p}_k^{(i)} = [p_{*k}, p^*_k]^{(i)}$ (k=1, r) included in [0, 1], the following steps are to be considered:

- Compute reference interval:
  $\widetilde{A}^{(i)} = [A_*, A^*]^{(i)} = [\min(p_{*k}{}^{(i)}, k=1,r), \max(p^*_k{}^{(i)}, k=1,r)]$
- Compute assessment intervals: all the intervals $\widetilde{p}_k^{(i)} = [p_{*k}, p^*_k]^{(i)}$ (k=1, r) are linearly positioned within the interval $\widetilde{A}^{(i)} = [A_*, A^*]^{(i)}$ according to the hendecadrian scale shown in the table 2.

Level	Semantic correspondence
0	For A_*
0.1	Practically A_*
0.2	Nearly A_*
0.3	Near to A_*
0.4	Nearer to A_* than to A^*
0.5	As close to A_* than to A^*
0.6	Nearer to A^* than to A_*
0.7	Near to A^*
0.8	Nearly A^*
0.9	Practically A^*
1	For A^*

Table 2: Counter-expertise levels.

Thus, it will be associated to each interval $[p_{*k}, p^*_k]^{(i)}$ an assessment interval $\widetilde{\alpha}_k^{(i)} = [\alpha_{*k}, \alpha^*_k]^{(i)}$, where α_{*k} and α^*_k belong to {0, 0.1, 0.2, …, 0.9, 1}; $\alpha_{*k} \le \alpha^*_k\ \forall\ k$. Figure 2 illustrates the algorithm for assessment intervals.

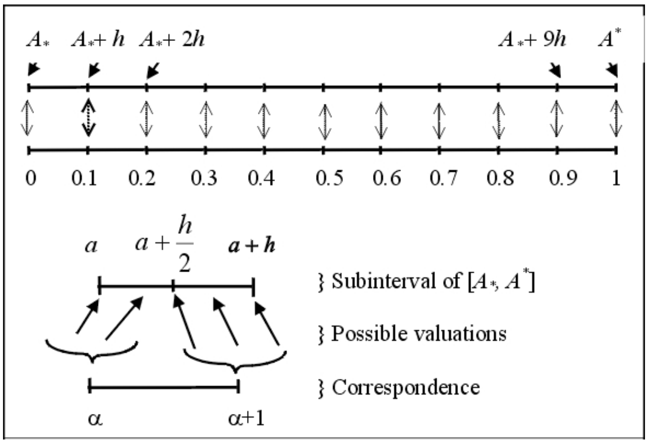

Figure 2: Illustration of assessment intervals algorithm.

- Compute the experton $\dot{\tilde{\alpha}}_k^{(i)}$ following steps described in 4.2.1.
- Compute the R+_experton using the following linear equation:

$$\dot{\tilde{p}}^{(i)} = A_*(+)\left((A^* - A_*)(\cdot)\dot{\tilde{\alpha}}^{(i)}\right) \quad (12)$$

Where the operation (+) and (.) are carried out level by level.

3.2.3. Experton of the TE

All algebraic operations applied to CIs can be used with expertons as long as the monotony property will be preserved, as this is the case with the operators $(\hat{+})$ and (.) from which the probability of the TE is computed, Considering the SSF of a FT, the experton of the TE can be obtained with the replacement of the state variables by the corresponding BE expertons. Then, each bound is calculated from expertons' level by level. We arrived at:

$$\dot{\tilde{P}}_S(\dot{\tilde{p}}^{(i)}, i/x_i \in \varphi_S(x)) = \begin{array}{|cc|} P_{*S}^{\,0} & P^*_S{}^{0} \\ P_{*S}^{\,0.1} & P^*_S{}^{0.1} \\ P_{*S}^{\,0.2} & P^*_S{}^{0.2} \\ P_{*S}^{\,0.3} & P^*_S{}^{0.3} \\ P_{*S}^{\,0.4} & P^*_S{}^{0.4} \\ P_{*S}^{\,0.5} & P^*_S{}^{0.5} \\ P_{*S}^{\,0.6} & P^*_S{}^{0.6} \\ P_{S*}^{\,0.7} & P_S^{*\,0.7} \\ P_{S*}^{\,0.8} & P_S^{*\,0.8} \\ P_{S*}^{\,0.9} & P_S^{*\,0.9} \\ P_{S*}^{\,1} & P_S^{*\,1} \end{array} \quad (13)$$

Information which will be taken into account at the moment of the decision-making is given by the mathematical expectation of (13) which is obtained by adding each of the two columns without taking into account level α=0 and then dividing the result by 10:

$$\tilde{E} = [E_*, E^*] = \frac{1}{10}\left(\tilde{F}(1) + \tilde{F}(0.9) + \ldots + \tilde{F}(0.2) + \tilde{F}(0.1)\right) \quad (14)$$

On the other hand, at this stage that experts' judgments entropy is made to fall.

4. RESULTS AND DISCUSSION

The present section is devoted to a comparison of the two techniques examined previously, using a suitable numerical treatment. We have considered 3 FT taken from the literature (figures 1.1, 1.2 and 1.3 in appendix 1) and treated numbers belonging to the set J={0, 0.1,..., 0.9, 1} and more generally numbers belonging to [0, 1] ($J \subset$ [0, 1]). The choice of the number of experts consulted could be justified by practical considerations which include the limited number of experts in industrial systems concerned with reliability analysis, especially when FT contains BE referring to different kinds of failure, i.e., component failure, human error and environment damage, and refusal attitude of certain experts to give quantitative data, even with intervals. For the BE of the first FT, it is supposed that the maximum number of experts is 5. For those of the second and the third one, the number varies from 5 to 10 and from 3 to 10, respectively.

Although expertise becomes one of the main sources of reliability data, numbers provided by experts to describe their knowledge, rely on subjective human reasoning methods, which introduce biases into these numbers. Besides, research in experimental psychology have shown that extracting subjective probabilities from experts cannot be taken as a simple question/answer operation, but as an engagement between analyst and experts, in which rigorous methodology must be followed [10]. To take into account the fact that an expert can fail, we have jugged useful to consider experts' deficiencies with regard to four instances as shown in figure 3.

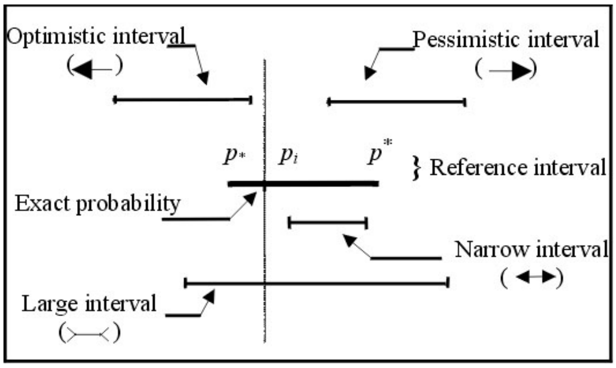

Figure 3: Experts' deficiencies.

Thus, in reference to data D1, where it is assumed that all experts agree (i.e., perfect coherence), we have considered other data characterized by a divergence varying in an increasing way and, in addition to AM, both expertons relative to the set J and R+_expertons are used in order to make rigorous comparison. For the experts' valuations relative to the first FT, see table 1.1 in appendix 1. Results obtained with different techniques are shown in figures 4, 5 and 6, and the differences existing between them are more clarified using the relative Hamming distance defined for two CI as:

$$\delta\left([P_{*y}, P^*{}_y],[P_{*z}, P^*{}_z]\right) = \frac{|P_{*y} - P_{*z}| + |P^*{}_y - P^*{}_z|}{2} \quad (15)$$

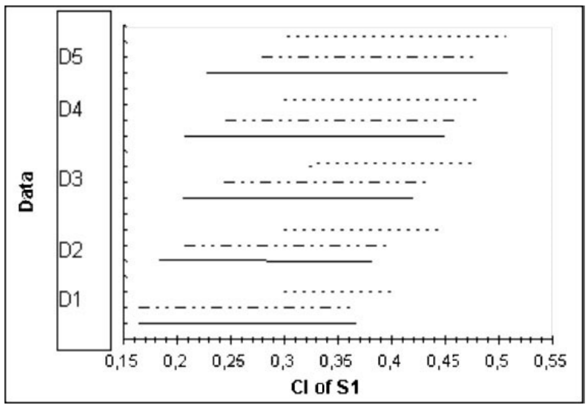

Figure 4: CI of the TE of the first FT.

.... Exp. in J , —..—... R+_Exp. , ——— AMs

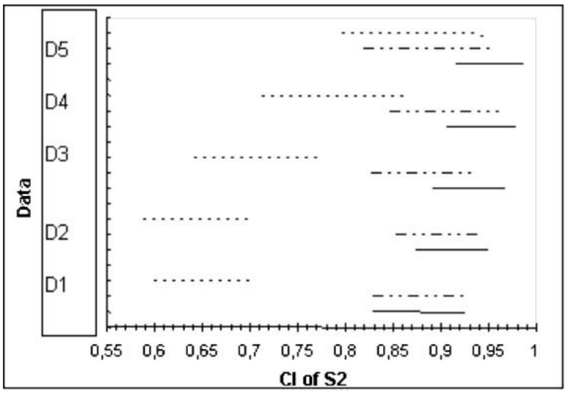

Figure 5: CI of the TE of the second FT.

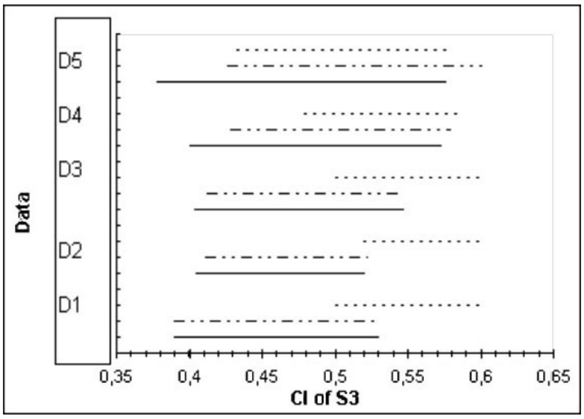

Figure 6: CI of the TE of the third FT.

Let CI_{AM}, CI_{R+_exp} and $CI_{exp\ in\ J,}$ be the CI of the TE calculated from AM, R^+_expertons and expertons in J, respectively. In reference to figures 4, 5 and 6, we can notice that for D1, i.e., reference CIs, both CI_{AM} and CI_{R+_exp} are identical. Indeed, we can see that in R^+_expertons of BE the CI given by experts are kept for the ten levels of the hendecadarian scale from the level α=0.1. For the same data, CI_{AM} and CI_{R+_exp} differ from $CI_{exp\ in\ J}$. This difference is more important in the case of the second FT (except for D5, we have: $CI_{AM} \cap CI_{exp\ in\ J} = \Phi$). Regarding the data D2, D3, D4 and D5, corresponding to an experts' judgments divergence which reflect the most frequent instance, we remark that $CI_{exp\ in\ J}$ bounds move away from those of CI_{AM} and CI_{R+_exp}. However, $CI_{exp\ in\ J}$ bounds are closer to those of CI_{R+_exp} than those of CI_{AM}, as confirmed also by figures 7, 8 and 9.

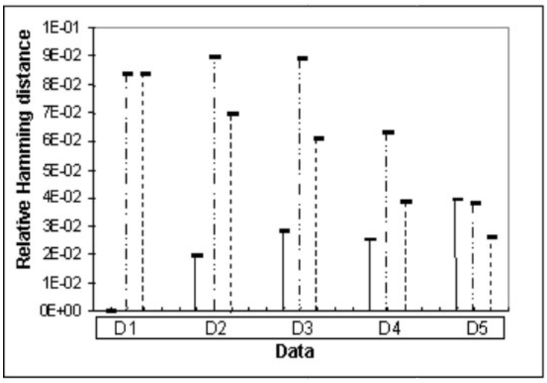

Figure 7: Difference between CI: case of the first FT.

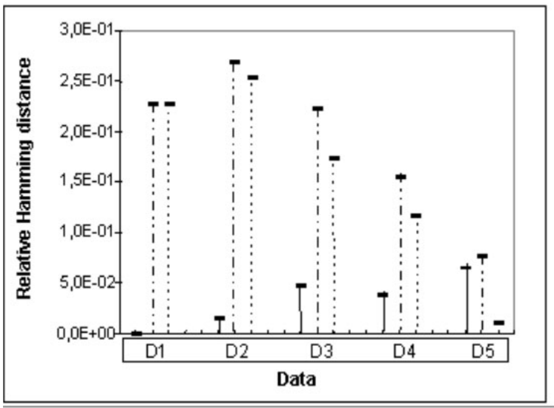

Figure 8: Difference between CI: case of the second FT.

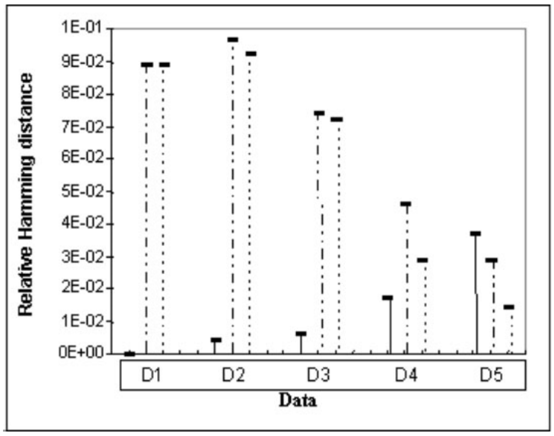

Figure 9: Difference between CI: case of the third FT.
—— Ams-R+_Exp, –··– Ams-Exp. in J,·········R+_Exp.-Exp. in J.

This result is due to the fact that both expertons in J and R^+_expertons relative to BE have common thread that is the possibility to maintain judgments disorder through the hendecadarian scale, considered of course in two different contexts. On the other hand, with AM this disorder is made to fall very early.

Let us consider now valuations in [0, 1]. The table 2.2 in appendix 2 shows data relative to the first FT. Both CI_{AM} and CI_{R+_exp} of S1, S2 and S3 are given in figure 10, 11 and 12.

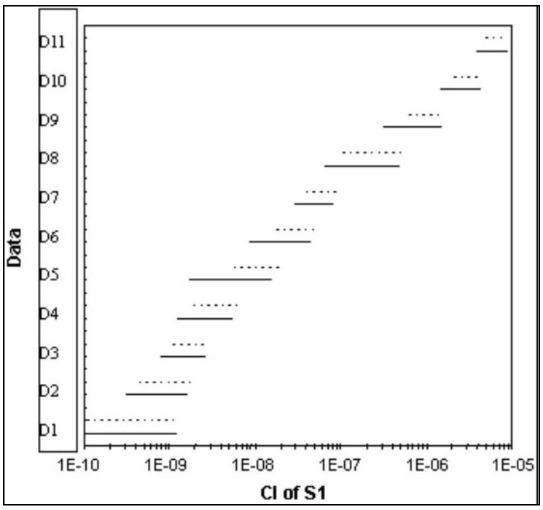

Figure 10: CI of the TE of the first FT.

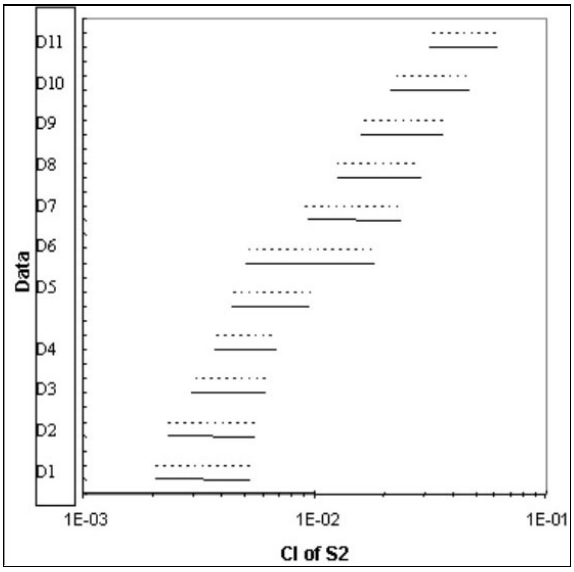

Figure 11: CI of the TE of the second FT.

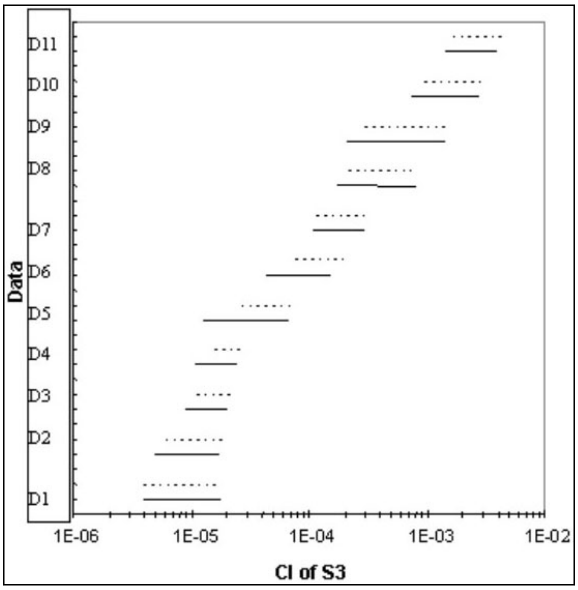

Figure 12: CI of the TE of the third FT.
.... R+_Exp., _____ AMs

Analysing results, we can remark that with data D1 (i.e., judgments perfectly coherent), CI_{AM} and CI_{R+_exp} are identical as in the case with valuations to one decimal. With the other data, a particular difference between these two kinds of intervals can be noticed in the case of the first and the third FTs, while it is markedly weak with the second FT. Nevertheless, it can be seen that CI_{R+_exp} width is less or most equal to the one of CI_{AM} and, therefore, CI_{R+_exp} can be considered as less uncertain than CI_{AM}.

CONCLUSION

In this paper we have examined the aggregation of uncertain information supplied by several experts to quantify fault trees. Two techniques: arithmetic means and expertons were the subject of a rigorous comparison, considering an adequate numerical treatment. A first important conclusion is that the use of arithmetic means for the aggregation of valuations relative to each basic event is clearly challenged. In fact, the differences between the confidence intervals of top events, calculated with the two techniques taken separately, are sometimes remarkable, especially when referring to expertons issued from valuations to one decimal. As a second conclusion, comparing with arithmetic means, expertons have a potential capacity for reducing uncertainty. Indeed, all the top-events intervals calculated with expertons are smaller or equal to those obtained with arithmetic means.

It is clear that there is no special interest for using expertons technique when dealing with linear functions because, in this case, arithmetic means give the same result with a simple calculation. But when dealing with non-linear function, as in the case of the function probability of the top event, arithmetic means will be invalid and expertons should be used. In addition, it appears important to emphasize on the information contained in experton: different valuations are represented in experton with their real weight. With means, this information is lost by removing dispersion. Finally, we must notice that further interpretation of the results obtained needs a lot more exploitation of theoretical aspects of expertons and non-linear operators.

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APPENDIX 1

FAULT TREES USED FOR AGGREGATION PROBLEM

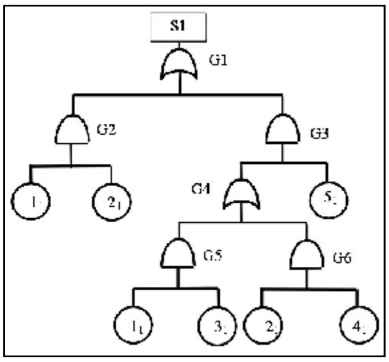

Figure 1.1: First FT.

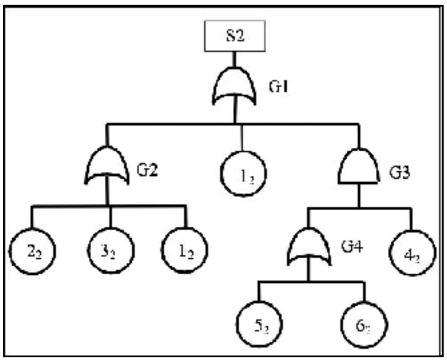

Figure 1.2: Second FT.

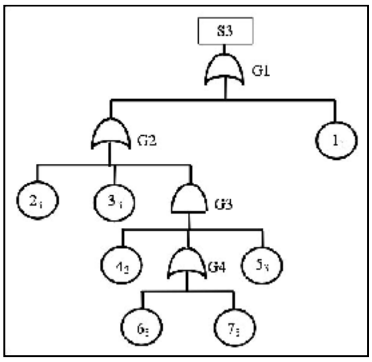

Figure 1.3: Third FT.

APPENDIX 2

EXPERTS' VALUATIONS RELATIVE TO THE FIRST FT

2.1. Valuations in J={0, 0.1, …, 0.9, 1}

BE	1_1		2_1		3_1		4_1		5_1	
Experts number	5		4		3		4		5	
Reference CI (D1)	0.4	0.5	0.2	0.4	0.3	0.4	0.6	0.8	0.5	0.6
D2										
Expert 1 →	0.5	0.6	0.3	0.5	0.4	0.5	0.7	0.9	0.6	0.7
Expert 2 ←	0.3	0.4	0,1	0,2	0.2	0.3	0.5	0.6	0.4	0.5
Expert 3 ↔	0.5	0.5	0.3	0.4	0.3	0.3	0.7	0.8	0.5	0.5
Expert 4 >-<	0.3	0.6	0.2	0.5			0.5	0.9	0.3	0.6
Expert 5 →	0.6	0.7							0.6	0.7
Data D3										
Expert 1	0.6	0.7	0.4	0.6	0.5	0.6	0.8	1	0.7	0.8
Expert 2	0.2	0.3	0	0.1	0.1	0.2	0.4	0.5	0.5	0.6
Expert 3	0.5	0.5	0.4	0.4	0.3	0.3	0.8	0.8	0.5	0.5
Expert 4	0.2	0.5	0.2	0.6			0.4	0.9	0.2	0.7
Expert 5	0.7	0.8							0.7	0.8
Data D4										
Expert 1	0.7	0.8	0.5	0.7	0.6	0.7	0.9	1	0.8	0.9
Expert 2	0.1	0.2	0	0.1	0	0.1	0.3	0.4	0.4	0.5
Expert 3	0.5	0.5	0.4	0.4	0.3	0.3	0.8	0.8	0.5	0.5
Expert 4	0.1	0.5	0.1	0.6			0.3	0.9	0.2	0.8
Expert 5	0.8	0.9							0.8	0.9
Data D5										
Expert 1	0.8	0.9	0.6	0.8	0.7	0.8	0.9	1	0.9	1
Expert 2	0	0.1	0	0.1	0	0.1	0.2	0.3	0.3	0.4
Expert 3	0.5	0.5	0.4	0.4	0.3	0.3	0.8	0.8	0.5	0.5
Expert 4	0	0.5	0.1	0.7			0.3	1	0.2	0.9
Expert 5	0.9	1							0.9	1

Table 2.1: Data in J, relative to the first FT

2.2. Valuations in [0, 1]

BE	1_1	2_1	3_1	4_1	5_1
Number of experts	5	4	3	4	5
Reference CI (D1)	10^{-5} 3.10^{-5}	10^{-5} 3.10^{-5}	8.10^{-6} 2.10^{-5}	10^{-3} 4.10^{-3}	9.10^{-4} 3.10^{-3}
			D2		
Expert 1 →	2.10^{-5} 5.10^{-5}	3.10^{-5} 6.10^{-5}	10^{-5} 4.10^{-5}	3.10^{-3} 6.10^{-3}	2.10^{-3} 4.10^{-3}
Expert 2 ←	8.10^{-6} 10^{-5}	6.10^{-6} 10^{-5}	5.10^{-6} 10^{-5}	7.10^{-4} 10^{-3}	8.10^{-4} 2.10^{-3}
Expert 3 ↔	2.10^{-5} 3.10^{-5}	2.10^{-5} 3.10^{-5}	10^{-5} 2.10^{-5}	2.10^{-3} 4.10^{-3}	9.10^{-4} 2.10^{-3}
Expert 4 ⋈	9.10^{-6} 4.10^{-5}	8.10^{-6} 4.10^{-5}		8.10^{-4} 5.10^{-3}	8.10^{-4} 4.10^{-3}
Expert 5 →	3.10^{-5} 4.10^{-5}				10^{-3} 4.10^{-3}
			D3		
Expert 1	4.10^{-5} 7.10^{-5}	6.10^{-5} 9.10^{-5}	3.10^{-5} 6.10^{-5}	5.10^{-3} 8.10^{-3}	5.10^{-3} 6.10^{-3}
Expert 2	6.10^{-6} 8.10^{-6}	3.10^{-6} 6.10^{-6}	2.10^{-6} 6.10^{-6}	5.10^{-4} 7.10^{-4}	6.10^{-4} 8.10^{-4}
Expert 3	3.10^{-5} 3.10^{-5}	3.10^{-5} 3.10^{-5}	2.10^{-5} 2.10^{-5}	3.10^{-3} 4.10^{-3}	9.10^{-4} 10^{-3}
Expert 4	8.10^{-6} 5.10^{-5}	7.10^{-6} 5.10^{-5}		7.10^{-4} 6.10^{-3}	7.10^{-4} 5.10^{-3}
Expert 5	5.10^{-5} 6.10^{-5}				2.10^{-3} 5.10^{-3}
			D4		
Expert 1	7.10^{-5} 10^{-4}	8.10^{-5} 2.10^{-4}	5.10^{-5} 9.10^{-5}	7.10^{-3} 9.10^{-3}	7.10^{-3} 8.10^{-3}
Expert 2	3.10^{-6} 5.10^{-6}	10^{-6} 3.10^{-6}	10^{-6} 3.10^{-6}	3.10^{-4} 5.10^{-4}	4.10^{-4} 6.10^{-4}
Expert 3	3.10^{-5} 3.10^{-5}	3.10^{-5} 3.10^{-5}	2.10^{-5} 2.10^{-5}	4.10^{-3} 4.10^{-3}	9.10^{-4} 9.10^{-4}
Expert 4	7.10^{-6} 6.10^{-5}	6.10^{-6} 6.10^{-5}		6.10^{-4} 7.10^{-3}	6.10^{-4} 6.10^{-3}
Expert 5	6.10^{-5} 7.10^{-5}				4.10^{-3} 6.10^{-3}
			D5		
Expert 1	9.10^{-5} 3.10^{-4}	10^{-4} 4.10^{-4}	8.10^{-5} 2.10^{-4}	8.10^{-3} 10^{-2}	9.10^{-3} 10^{-2}
Expert 2	2.10^{-6} 3.10^{-6}	7.10^{-7} 10^{-6}	7.10^{-7} 10^{-6}	2.10^{-4} 4.10^{-4}	2.10^{-4} 4.10^{-4}
Expert 3	3.10^{-5} 3.10^{-5}	3.10^{-5} 3.10^{-5}	2.10^{-5} 2.10^{-5}	4.10^{-3} 4.10^{-3}	9.10^{-4} 9.10^{-4}
Expert 4	6.10^{-6} 7.10^{-5}	5.10^{-6} 7.10^{-5}		5.10^{-4} 8.10^{-3}	5.10^{-4} 7.10^{-3}
Expert 5	8.10^{-5} 10^{-4}				5.10^{-3} 7.10^{-3}
			D6		
Expert 1	2.10^{-4} 4.10^{-4}	4.10^{-4} 8.10^{-4}	2.10^{-4} 5.10^{-4}	10^{-2} 2.10^{-2}	10^{-2} 2.10^{-2}
Expert 2	10^{-6} 2.10^{-6}	4.10^{-7} 6.10^{-7}	4.10^{-7} 8.10^{-7}	10^{-4} 2.10^{-4}	10^{-4} 2.10^{-4}
Expert 3	3.10^{-5} 3.10^{-5}	3.10^{-5} 3.10^{-5}	2.10^{-5} 2.10^{-5}	4.10^{-3} 4.10^{-3}	9.10^{-4} 9.10^{-4}
Expert 4	5.10^{-6} 8.10^{-5}	4.10^{-6} 8.10^{-5}		4.10^{-4} 9.10^{-3}	4.10^{-4} 8.10^{-3}
Expert 5	9.10^{-5} 2.10^{-4}				7.10^{-3} 9.10^{-3}
			D7		
Expert 1	4.10^{-4} 6.10^{-4}	8.10^{-4} 10^{-3}	5.10^{-4} 8.10^{-4}	$1,5.10^{-2}$ $2,5.10^{-2}$	2.10^{-2} 3.10^{-2}
Expert 2	8.10^{-7} 10^{-6}	10^{-7} 3.10^{-7}	10^{-7} 4.10^{-7}	9.10^{-5} 10^{-4}	8.10^{-5} 9.10^{-5}
Expert 3	3.10^{-5} 3.10^{-5}	3.10^{-5} 3.10^{-5}	2.10^{-5} 2.10^{-5}	4.10^{-3} 4.10^{-3}	9.10^{-4} 9.10^{-4}
Expert 4	4.10^{-6} 9.10^{-5}	3.10^{-6} 9.10^{-5}		3.10^{-4} 10^{-2}	3.10^{-4} 9.10^{-3}
Expert 5	10^{-4} 3.10^{-4}				8.10^{-3} 10^{-2}
			D8		
Expert 1	7.10^{-4} 9.10^{-4}	10^{-3} 4.10^{-3}	8.10^{-4} 10^{-3}	2.10^{-2} 3.10^{-2}	$2,5.10^{-2}$ $3,5.10^{-2}$
Expert 2	5.10^{-7} 7.10^{-7}	8.10^{-8} 10^{-7}	8.10^{-8} 10^{-7}	8.10^{-5} 9.10^{-5}	7.10^{-5} 8.10^{-5}
Expert 3	3.10^{-5} 3.10^{-5}	3.10^{-5} 3.10^{-5}	2.10^{-5} 2.10^{-5}	4.10^{-3} 4.10^{-3}	9.10^{-4} 9.10^{-4}
Expert 4	3.10^{-6} 10^{-4}	2.10^{-6} 10^{-4}		2.10^{-4} 2.10^{-2}	2.10^{-4} 10^{-2}
Expert 5	3.10^{-4} 5.10^{-4}				10^{-2} 2.10^{-2}
			D9		
Expert 1	10^{-3} 2.10^{-3}	3.10^{-3} 6.10^{-3}	10^{-3} 3.10^{-3}	3.10^{-2} 4.10^{-2}	3.10^{-2} $4,5.10^{-2}$
Expert 2	2.10^{-7} 4.10^{-7}	6.10^{-8} 8.10^{-8}	5.10^{-8} 7.10^{-8}	6.10^{-5} 8.10^{-5}	6.10^{-5} 7.10^{-5}
Expert 3	3.10^{-5} 3.10^{-5}	3.10^{-5} 3.10^{-5}	2.10^{-5} 2.10^{-5}	4.10^{-3} 4.10^{-3}	9.10^{-4} 9.10^{-4}
Expert 4	2.10^{-6} 2.10^{-4}	10^{-6} 2.10^{-4}		10^{-4} 3.10^{-2}	10^{-4} 2.10^{-2}
Expert 5	6.10^{-4} 9.10^{-4}				2.10^{-2} 3.10^{-2}
			D10		
Expert 1	4.10^{-3} 6.10^{-3}	5.10^{-3} 8.10^{-3}	3.10^{-3} 5.10^{-3}	5.10^{-2} 6.10^{-2}	4.10^{-2} 5.10^{-2}
Expert 2	10^{-7} 2.10^{-7}	5.10^{-8} 6.10^{-8}	3.10^{-8} 5.10^{-8}	5.10^{-5} 6.10^{-5}	4.10^{-5} 6.10^{-5}
Expert 3	3.10^{-5} 3.10^{-5}	3.10^{-5} 3.10^{-5}	2.10^{-5} 2.10^{-5}	4.10^{-3} 4.10^{-3}	9.10^{-4} 9.10^{-4}
Expert 4	10^{-6} 3.10^{-4}	9.10^{-7} 3.10^{-4}		9.10^{-5} 4.10^{-2}	9.10^{-5} 3.10^{-2}
Expert 5	8.10^{-4} 10^{-3}				3.10^{-2} 4.10^{-2}
			D11		
Expert 1	7.10^{-3} 9.10^{-3}	8.10^{-3} 10^{-2}	6.10^{-3} 8.10^{-3}	6.10^{-2} 7.10^{-2}	5.10^{-2} $5,5.10^{-2}$
Expert 2	7.10^{-8} 10^{-7}	3.10^{-8} 5.10^{-8}	10^{-8} 2.10^{-8}	4.10^{-5} 5.10^{-5}	3.10^{-5} 4.10^{-5}
Expert 3	3.10^{-5} 3.10^{-5}	3.10^{-5} 3.10^{-5}	2.10^{-5} 2.10^{-5}	4.10^{-3} 4.10^{-3}	9.10^{-4} 9.10^{-4}
Expert 4	9.10^{-7} 4.10^{-4}	8.10^{-7} 4.10^{-4}		8.10^{-5} 5.10^{-2}	8.10^{-5} 4.10^{-2}
Expert 5	10^{-3} 3.10^{-3}				4.10^{-2} 5.10^{-2}

Table 2.2: Data in [0, 1], relative to the first FT.