Sciences & Technologie A – **N°37**, Juin 2013, 21-29.

EFFECTS OF INITIAL RADII ON THE PROPAGATION OF NEGATIVE STREAMER IN NITROGEN.

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Reçu le 07/06/2012 – Accepté le 11/02/2013

Abstract

In the prebreakdown stage of electrical breakdown of gases under high pressure and high voltage, the space charge field plays an important role. We present in this paper the results of numerical calculations of negative streamer propagation in nitrogen making use of two dimensional Scharfetter and Gummel of zero order scheme which allow us to solve numerically the transport equations under strongly space charge dominated conditions such as occur at the head of propagating streamers. The algorithm is fully two dimensional (three dimensional with cylindrical symmetry) and is proving stable and capable of dealing with the steep density gradients which appear in our calculations. Poisson's equation is resolved by Biconjugate Gradient Stabilized method. We are interested in using the computer calculations to aid in understanding the notion of streamer. Different sizes of radii of the initial ionized spot are simulated in order to define the role of this parameter on the streamer propagation dynamics.

Keywords : *Streamer discharge simulation, effects of different radii.*

Résumé

Dans la phase de la prédisruption du claquage électrique des gaz à pression élevée et sous haute tension, le champ de charge d'espace joue un rôle important. Nous présentons dans cet article les résultats des calculs numériques de la propagation du streamer négatif dans l'azote. Nous utilisons le schéma de Scharfetter et Gummel d'ordre zéro pour résoudre les équations de transport (en deux dimensions) sous les conditions où une charge d'espace importante a lieu à la tête du streamer qui se propage. L'algorithme est entièrement à deux dimensions (trois dimensions pour une symétrie cylindrique) et se révèle stable et capable de traiter les gradients de densité abrupts qui apparaissent dans nos calculs. L'équation de Poisson est résolue par la méthode Biconjugate Gradient Stabilized. Nous utilisons nos calculs informatiques qui nous aident à comprendre la notion de streamer. Différentes tailles de la dispersion radiale de la gaussienne initiale sont utilisées dans le but de définir le rôle de ce paramètre sur la dynamique de propagation du streamer.

Mots clés: *Simulation de la décharge streamer, effets de la dispersion radiale.*

ملخص.

في المرحلة ما قبل التفريغ للتفريغ الكهربائي للغازات تحت الضغط المرتفع و التوتر العالي حقل شحنة الفضاء يلعب دوراهاما في هذا المنشور نقدم نتائج الحسابات العددية لإنتشارالستريمر السلبي في غاز النتروجن نستخدم االطريقة العددية لشارفتر و قومل من الدرجة صفرلحل معادلات النقل ذات البعدين في الشروط اين شحنة فضاء مهمة تكون موجودة برأس الستريم الذي ينتشر الخوارزمية هي تماما ذات بعدين ثلاثية الابعاد مع التناظر الاسطواني و اثبتت انها مستقرة و قادرة على التعامل مع الكثافات ذات التدرجات الكثيرة الانحدار و التي تظهر في جميع حساباتنا يتم حل معادلة بواسن بواسطة طريقة بيكنجوكيت غرادين ستبلزد نحن مهتمون في استخدام حسابات الكمبيوتر للمساعدة على فهم الستريمر ظاهرة قيم مختلفة تعطى لنصف القطر للكثافة الاصلية و الهدف هو تحديد دور هذا المعامل على دينامكية إنتشار الستريمر.

الكلمات المفتاحية: النمذجة لتفريغ الستريم ; اثار التشتت العرضي

Introduction :

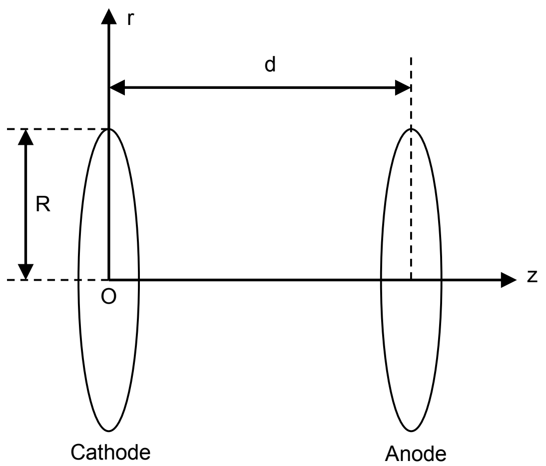
Plasma is a medium in which a number of neutral molecules is ionized creating a sufficient number of free charge carriers to affect electromagnetic properties of the gas. Depending on the degree of equilibration of the plasma constituents, i.e. the neutral atoms or molecules, the excited states, the electrons, the ions, one speaks of thermal or non thermal plasmas. Non thermal or low temperature plasmas are generated by applying a strong electric field on the gas. The energy is then transferred to the electrons rather than the gas, which stays at ambient temperature, whereas the electrons on the contrary can gain more and more energy so that the plasma is not in thermal equilibrium. These non equilibrium plasma discharges appear in various forms depending on the spatio-temporal characteristics of the electric field and on the pressure of the medium. Low temperature plasmas are widely used in industry, e.g. in the processing of semiconductors, flat panel displays and gas cleaning. For a detailed review of the industrial applications of non-thermal plasmas, we can refer to [1], [2].

Streamers are growing filaments of non-equilibrium, moderately or weakly ionized, non-stationary plasma, whose dynamics are controlled by highly localized and nonlinear space charge regions [3], [4]. A streamer can arise from a seed electron that moves in a gas under influence of an electric field. On its path, the electron collides with the neutral gas molecules (or atoms) and if the electric field is strong enough, it eventually gains energy to ionize a molecule, thereby releasing a new electron and a positive ion. The two electrons now gain energy from the field, both colliding with and possibly ionizing other molecules, thereby creating an electron avalanche. The mobility of electrons results in space charge regions, i.e. regions that are electrically non-neutral. Eventually, the amount of space charge can become strong enough to alter the background electric field (geometric field), which in turn influences both the electron velocity and the ionization rate of the gas molecules. A streamer whose dynamics are controlled by non linear mechanisms emerges. Streamers are typically the initial stage of sparks and arc discharges.

A sizable volume of evidence has been accumulated showing the importance of streamers or fast ionizing waves to several aspects of electrical breakdown of gases. Several workers [5], [6] have investigated this phenomenon using high speed photographic technique and have explained the results qualitatively. Unfortunately, the mathematical description of transport under conditions for which space-charge fields play an important role proves very difficult to deal with in general. This phenomenon is an element of the more general subject of space charge dominated transport and a detailed study of the motion of electrons and ions in non uniform fields is necessary to understand the important mechanism involved. There have been attempts to simulate streamer dynamics using kinetics models [7, 8]. However, these models are very time consuming and the vast majority of simulations have been done within the scope of the drift diffusion approximation (hydrodynamic models) and the calculations of streamer propagation have been performed in two dimensions [9-11]. In this paper, we develop a two dimensional code in cylindrical coordinates by using differing radii. We use Scharfetter and Gummel of zero order scheme to solve transport equations and Biconjugate Gradient Stabilized method for solving Poisson's equation [12] .This allows us to follow both the radial and axial development of streamers. We calculate electron density, electric field (radial and axial) along the axis of propagation, net space charge density, source term and the current in the external circuit for a negative streamer (ADS).

2. Mathematical model

In this section, we describe a time dependent hydrodynamic model. This particular model allows us to investigate negative streamers and motion under space-charge dominated condition in general. The streamer is assumed to be cylindrically symmetrical with the axis z (direction of the propagation) and the axis r (radial propagation).



The usual physical model of streamer dynamics is the first order fluid model [9-12] which involves the first two moments (equation of continuity (1) and equation of momentum) of Boltzmann's equation. These two equations are solved for electrons and positive ions (the gas is nitrogen) and are coupled with Poisson's equation (4) for the space charge field calculation. The momentum equation is simplified using the drift diffusion approximation (2):

$$\frac{\partial n_s(\vec{r},t)}{\partial t} + \operatorname{div} \vec{j}_s(\vec{r},t) = S_{ionization}(\vec{r},t) \qquad (1)$$

$$\vec{j}_s(\vec{r},t) = n_s(\vec{r},t)\vec{v}_s(\vec{r},t) - D_s(\vec{r},t)\nabla n_s(\vec{r},t) \qquad (2)$$

Where n_s represents the number density of species s (subscripts "e", " p" respectively refer to electrons and positive ions). In equation (2), the flux $\vec{j}_s$ consists of an advective part with drift velocity $\vec{v}_s$ and a diffusion part with diffusion coefficient D_s. In the right part of the balance equation, the term source $S_{ionization}$ represents the electron impact ionization term and is the dominant source term under high electric field. It is equal to:

$$S_{ionization} = \alpha\, w_e(\vec{r},t)\, n_e(\vec{r},t) \tag{3}$$

α is the Townsend ionization coefficient, n_e the electron density and w_e the magnitude of electron drift velocity.
The photo-ionization source term is omitted and replaced with a fixed pre-ionized neutral background. The pre-ionization needed for stable advancement of negative streamer (ADS) is provided by introducing a uniform neutral background ionization of the gas (10^8 particles par cm^3). Pre-ionization gives an excessive number of electrons around the streamer head and may essentially influence streamer evolution and structure. The electric field must satisfy the Poisson's equation:

$$\Delta\Phi(\vec{r},t) = \nabla^2\Phi(\vec{r},t) = -\frac{|q_e|}{\varepsilon_0}\sum_s n_s(\vec{r},t) \tag{4}$$

Φ is the electrostatic potential, $|q_e|$ the absolute value of electron charge , ε_0 the permittivity of free space and

$$\vec{E} = -\vec{\nabla}\Phi \tag{5}$$

Equations (1), (2) and (4) are closed by using the local electric field approximation which implies that local equilibrium of electrons is achieved instantaneously in time in response to the electric field and assumes that the transport coefficients and source terms are explicit functions of the norm of the local electric field.

The external circuit consists of a resistor R in series with the gap. If $V_g(t)$ represents the voltage on the discharge gap and V the externally applied voltage then the current in the gap is given by [13]:

$$I_g(t) = \frac{q_e}{V_g(t)}\int_V \left(j_p(\vec{r},t) - j_e(\vec{r},t)\right) E_{geometric}(\vec{r})\, d\vec{r} \tag{6}$$

$$V_g(t) = V - R.\, I_g(t) \tag{7}$$

3. Numerical implementation :

We know that a streamer propagates rapidly (its speed is about 1/100 of light speed). During this rapid transient process, electron, ion densities and electric field vary fastly with time and space. For simulating the flux in transport equations, the numerical method must have two main properties:

-First, it has to be sufficiently accurate to describe the fast variation of charged particles;

- Second, it must avoid inducing negative values of densities. Different numerical schemes were proposed in the literature. With a scheme of first- order accuracy (e.g. Upwind scheme), the solution is strictly positive but a numerical diffusion occurs. With a scheme of higher accuracy (e.g. Lax Wendroff scheme), the numerical diffusion is less but the solution is no more positive [14].

Kulikovsky [16] proposed a modified version of the original explicit Scharfetter Gummel method (noted SG0 in our paper) that improves the accuracy of the original scheme [15] significantly without requiring the use of a fine grid. All the details on the scheme can be found in [16].We present the main characteristics of the method. Kulikovsky's basic idea is to put a pair of virtual nodes one on each side of the cell interface where the flux has to be calculated. The distance between these virtual nodes is chosen to be small enough to satisfy the condition for which the Scharfetter Gummel method is accurate. Densities at virtual nodes are obtained using an interpolation of the density between two grid nodes (we choice an exponential interpolation).Then based on the densities at the virtual nodes, the flux at the cell edge can be calculated accurately using the Scharfetter Gummel scheme. It is interesting to note that in the original Scharfetter Gummel method, the electric field is assumed to be constant between the virtual nodes. Kulikovsky [16] has proposed a more accurate approximation of the flux, in which the field is assumed to vary linearly between the virtual nodes. In [16], it was shown that accounting for the linear field profile has a minor influence on the results obtained on a simplified test case. In the modified Scharfetter Gummel scheme, the determination of the location of the virtual nodes is essential and Kulikovsky [16] has proposed a criterion with a constant factor ε which has to be selected. He suggests that a value of ε in the range [0.01, 0.04] gives good results in most cases.

For streamer simulations, the electric field is a key parameter for two reasons:

-First, the transport parameters and source terms have a non linear dependence on it;

-Second, the electric field is directly related to charged species densities. The electric field is derived from the electric potential given by Poisson's equation. A small error in the calculation of the electric potential leads to large fluctuations in the electric field which may lead to considerable errors in the simulation results. Usually, Poisson's equation (4) is solved by using a centered second order scheme. Bi-conjugate gradient stabilized method [17, 18] is used in our paper to solve equation (4). It is an iterative algorithm and we consider that the solution is obtained as soon as the maximum of residual defined as
$Res = \max\left|\nabla^2\Phi^k_{i,j} - \rho^k_{i,j}\right|$ becomes less than a value
$Res_{max} = \delta_\rho \max\left(\left|\rho^k_{i,j}\right|\right) + \rho_0$. The values $\delta_\rho = 10^{-5}$ and $\rho_0 = 1$ are used in the present paper and lead typically to 50-100 iterations. (We remember that subscripts i and j are related to radial and axial directions respectively, k enumerates time levels).

4. Domain of simulation, initial density, transport parameters and boundary conditions :

The computational domain is a cylinder of radius R (figure (1)). This domain is limited by two infinite parallel, planes, circular and metallic electrodes separated by a distance d equal to 0.5 cm. We have taken V_g (t = 0) 26 kV giving a geometric field 52 kVcm^{-1} and corresponding to 47 % overvoltage. The gas nitrogen is at atmospheric pressure (760 Torrs). The initially quasi-neutral plasma spot of Gaussian profiles in the radial and axial directions takes the same form for electrons and positive ions:

$$n(z,r,t=0) = n_0 \exp\left(-\left(\frac{z}{\sigma_z}\right)^2 - \left(\frac{r}{\sigma_r}\right)^2\right) + n_{preionization} \quad (8)$$

$$\begin{cases} n_0 = 10^{14}\,\mathrm{cm}^{-3} \\ n_{\text{préionisation}} = 10^{8}\,\mathrm{cm}^{-3} \\ \sigma_r = 0.021\ \mathrm{cm} \\ \sigma_z = 0.027\ \mathrm{cm} \end{cases}$$

By placing the initial electron and ion densities at the left electrode (z = 0, axis of symmetry), we can simulate the propagation of negative streamer (ADS) moving from the cathode (voltage = 0 kV) to the anode (voltage = 26 kV). The choice of a high value of the Gaussian height n_0 equal to 10^{14} cm^{-3} in the expression (8) provides immediate formation of the streamer in the gap space (with this value, we by-pass the avalanche stage). The following transport parameters are available for nitrogen:

Ionization coefficient	α =5.7 P exp (-260 E/P)	cm^{-1}
Electron mobility	μ_e = 2.9 10^5/ P	cm^2 V^{-1} s^{-1}
Electron axial diffusion coefficient	D_{ea} = 1800	cm^2 s^{-1}
Electron radial diffusion coefficient	D_{er} = 2190	cm^2 s^{-1}
Ion mobility	μ_p = 2.6 10^5/ P	cm^{2}V^{-1}s^{-1}
Ion axial diffusion coefficient	D_{pa}= 10	cm^2 s^{-1}
Ion axial diffusion coefficient	D_{pr}= 10	cm^2 s^{-1}

Table 1. Transport parameters

The primary ionization coefficient α and the electron and ion motilities are given by [19]. The electron diffusion coefficients are given by [20].
The boundary conditions for the potential are chosen to satisfy:

$$\Phi_{(z=0)} = 0 \quad (9)$$

$$\Phi_{(z=d)} = V_g(t=0) \quad (10)$$

$$\Phi_{(r=R)} = \frac{V_g(t=0)}{d} z \quad (11)$$

Conditions (9) and (10) mean that the potential at the anode and the cathode is fixed; condition (11) means that the electric field is undisturbed at the side surface of the computational domain. For potential, the axial symmetry of the problem imposes condition (12) and for particles condition (13):

$$\left.\frac{\partial \Phi}{\partial r}\right|_{(r=0)} = 0 \quad (12)$$

$$\left.\frac{\partial n_s}{\partial r}\right|_{(r=0)} = 0 \quad (13)$$

Boundary conditions at the electrodes are as follows:

$$\left.\frac{\partial n_s}{\partial z}\right|_{(z=0)} = 0 \quad (14)$$

$$\left.\frac{\partial n_s}{\partial z}\right|_{(z=d)} = 0 \quad (15)$$

The conditions (14) and (15) mean that the electrodes don't influence the particles fluxes and provide continuity of current.

6. Results and discussion

To determine the effect of the initial ionization conditions on negative streamer propagation, we carry out several calculations in which differing radii of the initial Gaussian distribution are assumed. The first results are in one dimension.

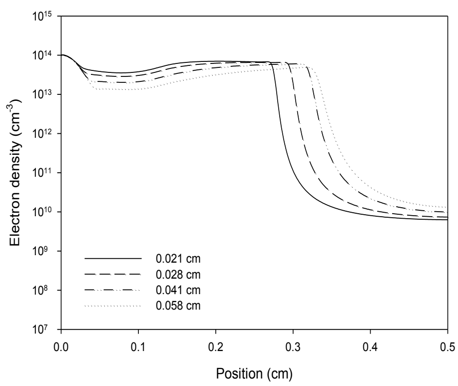


Figure 2. Electron density along the axis of propagation for time = 2 ns

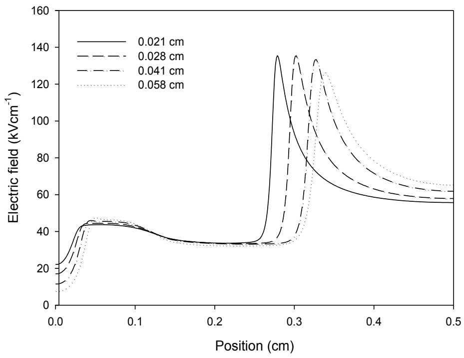


Figure 3. Electric field along the axis of propagation for time = 2 ns

Figures (2) and (3) compare the profiles of the electron density and of the magnitude of the electric field for four differing radii or half width in the radial direction (0.021, 0.028, 0.041 and 0.058 cm) at a fixed time t = 2 ns. All other parameters are fixed (height of the Gaussian or peak density, background ionization, voltage, pressure…). Once the quasi steady-state is reached, the four streamers with different radii propagate with different speeds. The streamer that originates from a wider initial distribution propagates faster and has a peak electric field (126 kV cm^{-1}) which is smaller than the peak electric field of the streamer that originates from a narrower distribution (135 kV cm^{-1}).

The smaller diameter streamer produces the higher streamer density in the channel, the steeper gradient density at the tip and the lower background charge.

We remember that the mechanism of propagation of an anode directed streamer (ADS) is as follows: the electron drift is along the direction of the wave propagation. The electrons move forward to shield the bulk of the plasma left behind; this causes the peak of electric field to move forward and the electron density near the peak grows due to electron impact ionization. The new charge created shields itself; in the bulk of the plasma (see figure (4)), due to shielding, the electron and ion density are nearly the same, but ahead of the streamer, the electron drift velocity caused the electron density to be higher than the ion density.

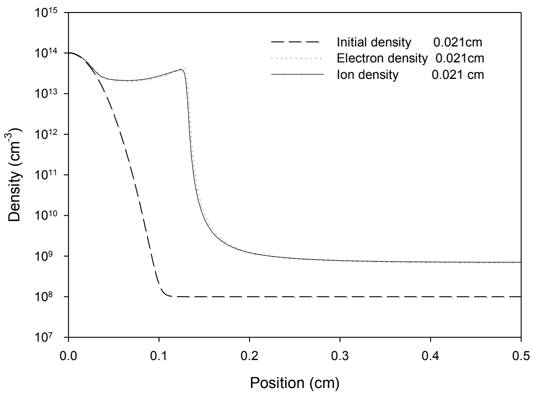


Figure 4. Profile of densities along the axis of propagation for time = 1 ns

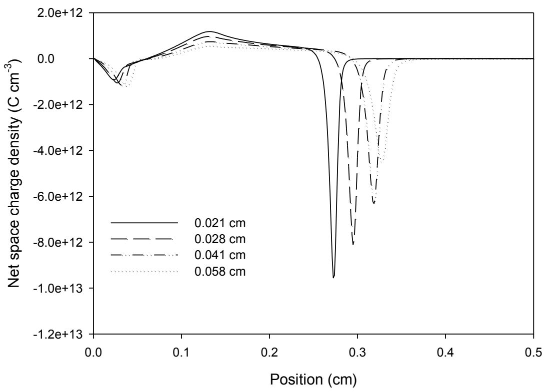


Figure 5. Net space charge density along the axis of propagation for time = 2 ns

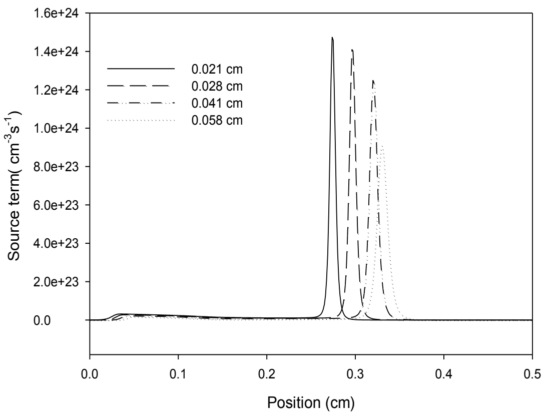


Figure 6. Source term along the axis of propagation for time = 2 ns

The net space charge density for an anode directed streamer is negative and this is shown in figure (5). Since the ionization coefficient α is a very strong function of the electric field, the maximum growth of the electron density takes place when the field is the highest. This remark is also valuable for the net space charge density (figure (5)) and the source term (figure (6)).

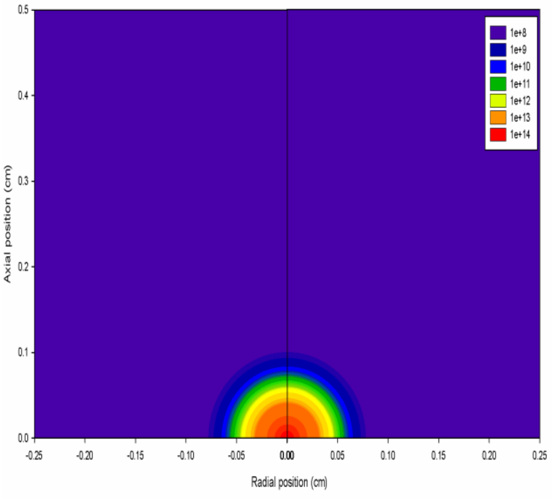


Figure 7. Cross sectional view of the initial electron density (cm^{-3}); radial dispersion 0.021 cm, logarithm scale

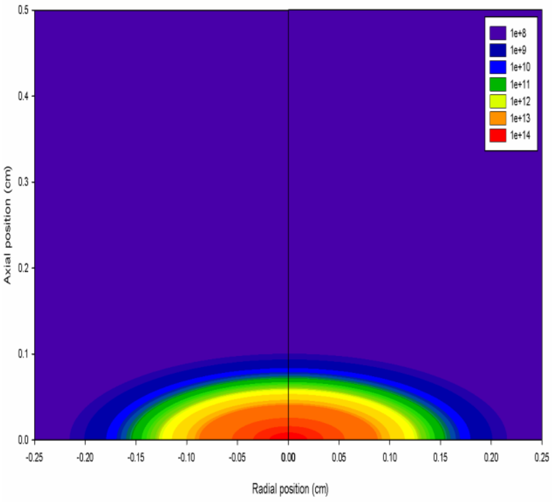


Figure 8. Cross sectional view of the initial electron density (cm^{-3}); radial dispersion 0.058 cm, logarithm scale

We present also a two dimensional picture of streamer propagation. The initial distributions of the electron density are shown in figure (7) where the radial dispersion is 0.021 cm and in figure (8) where the radial dispersion is 0.058 cm. For the two cases, the z-distributions are the same. We see that for the second radius, we have the wider electron density. Figures (9,10) and (11,12) show respectively the cross sectional views of the distribution of electron density and the electric field for two differing radii (0.021 cm and 0.058 cm) at a fixed time t = 2 ns. Figures (9-12) present two main regions of the streamer: a crescent shaped high field region produced by the streamer head and a low field region behind the streamer head where the field is lower than the breakdown field and which corresponds to the high electron density region (channel) caused by streamer propagation.

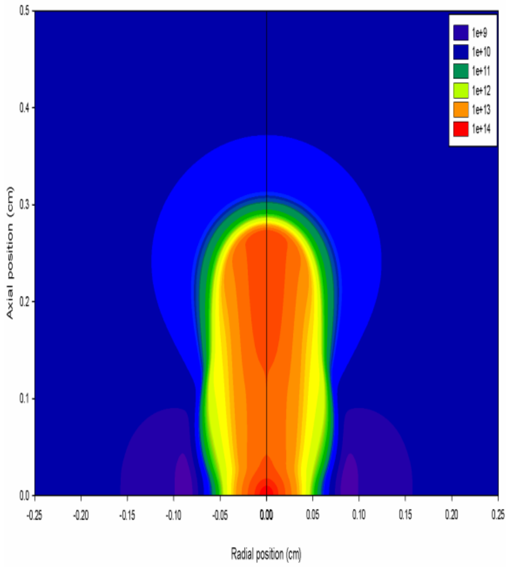


Figure 9. Cross sectional view of the electron density (cm^{-3}); radial dispersion 0.021 cm, logarithm scale

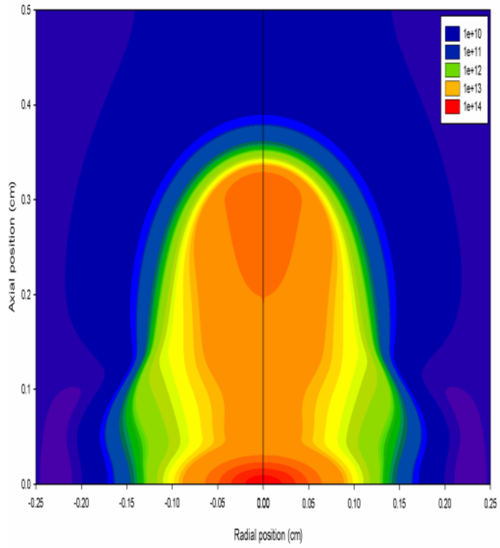


Figure 10. Cross sectional view of the electron density (cm^{-3}); radial dispersion 0.058 cm, logarithm scale

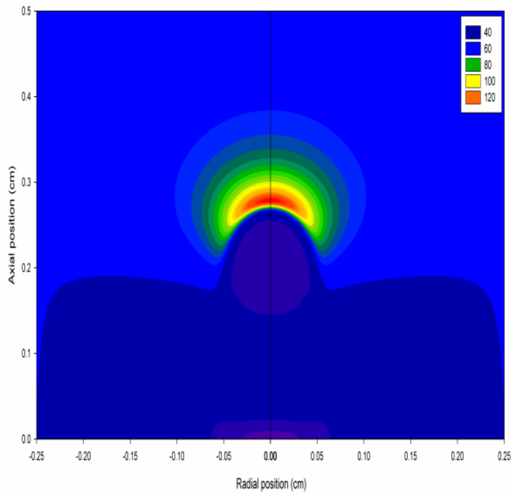


Figure 11. Cross sectional view of the electric field (kV cm^{-1}); radial dispersion 0.021 cm

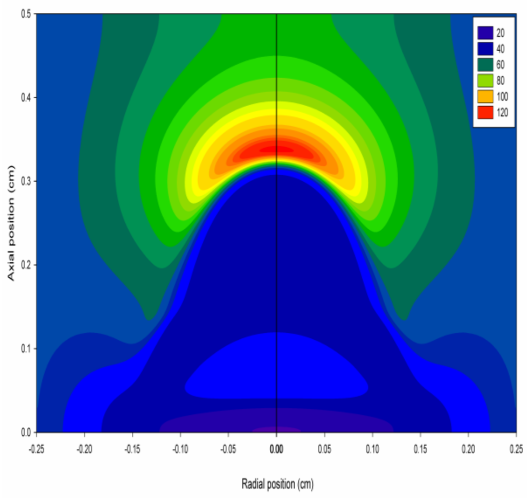


Figure 12. Cross sectional view of the electric field (kV cm^{-1}); radial dispersion 0.058 cm

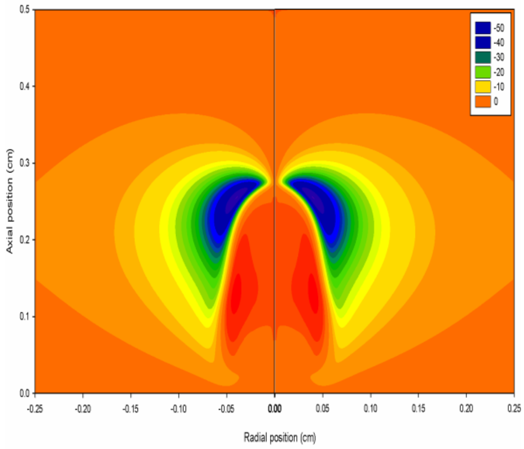


Figure 13. Cross sectional view of the radial field (kV cm^{-1}); radial dispersion 0.021 cm

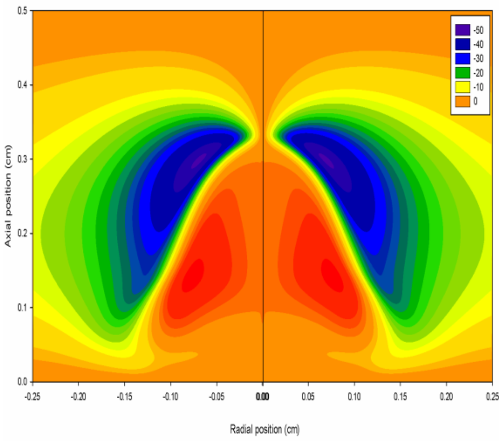


Figure 14. Cross sectional view of the radial field (kV cm^{-1}); radial dispersion 0.058 cm

Figures (9, 10) show that the radial profile of the streamer after its formation is wider than the initial charge in both cases but the streamer that originates from the initial charge of bigger radius continues to propagate with a bigger radius. Although the initial background and the applied field are the same, the space charge field to the initial charge distribution shapes the density ahead of the streamer.

The difference in the radius of two streamers is also apparent from the axial and the radial fields shown respectively in figures (11, 12) and (13, 14). The field is depleted over a bigger radius for the streamer with a wider electron density. We see also that the values for radial field don't change for the two streamers.

Figures (15, 16) and (17, 18) show respectively the cross sectional views of net space charge density and source term for radius 0.021 cm and 0.058 cm. Anode directed streamer for the two cases is created with space charge of the opposite sign at its head; it is surrounded by an electronic and narrow coat. This space charge takes the form of a horse shoe with the maximum value at the tip (head). The source term is also maximum at the head of the streamer. As the two parameters are function of the electric field, they reduce as the electric field becomes smaller for the radius 0.058 cm.

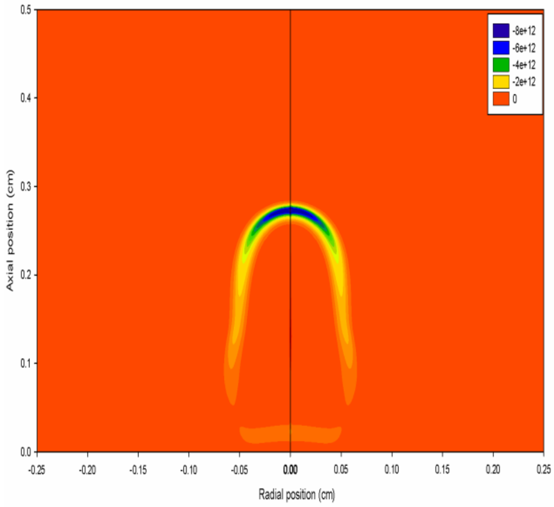


Figure 15.Cross sectional view of net space charge density (C cm^{-3}), radial dispersion 0.021 cm

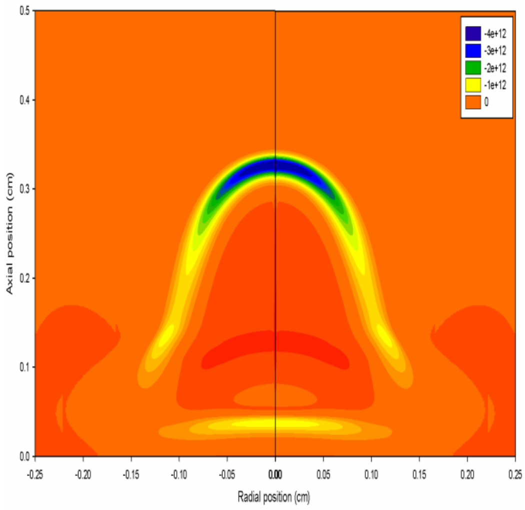


Figure 16.Cross sectional view of net space charge density (C cm^{-3}), radial dispersion 0.058 cm

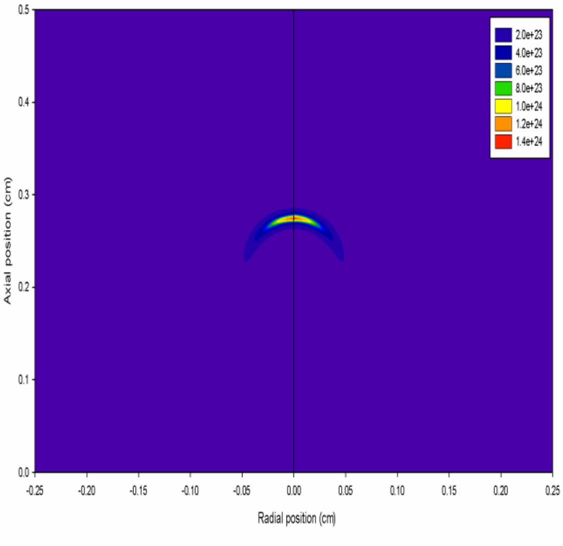


Figure 17.Cross sectional view of source term ($cm^{-3}s^{-1}$), radial dispersion 0.021 cm

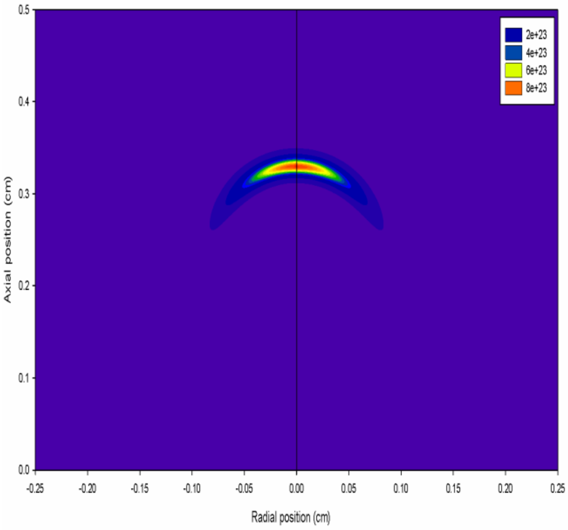


Figure 18.Cross sectional view of source term ($cm^{-3}s^{-1}$), radial dispersion 0.058 cm

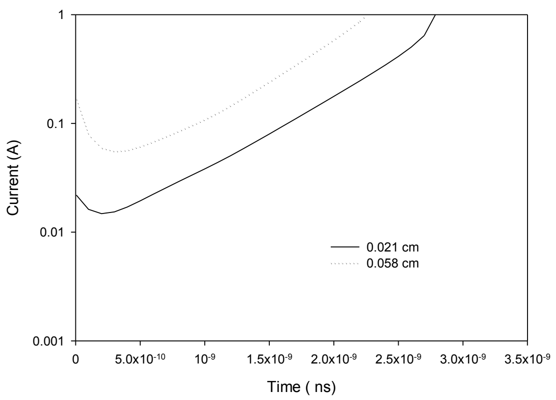


Figure 19. External electrical current as function of time (ADS)

Figure (19) shows the external circuit for the anode directed streamer for the two radii 0.021 and 0.058 cm. The initial current results from the initial ionization and decreases as the initial charge shields itself from the external field. At later times, the current starts to rise exponentially because of the exponential growth of the electron density in the main body of the plasma due to the penetration of electric field. The current for the radius 0.058 cm is more important than for the radius 0.021 cm (the avalanche multiplication of ionization density is more important (see figure (10)).

7. Conclusion

In this paper, we study the effect of differing radii on the propagation of negative streamer. We find that:

-The radial size of the streamer is influenced by the radial size of the initial charge although the steady state streamer radial size is larger than the initial radial size.

-The streamer parameters groups (streamer dimensions, velocity and current) essentially depend on the spot radius and are sensitive to the processes in the streamer channel.

-The shape of the streamer and of the streamer head near the anode are similar for both radii, we conclude that the dynamics of the streamer head are not controlled by initial conditions.

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