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EFFECTS OF THE WAVY GEOMETRY ON NATURAL CONVECTION HEAT TRANSFER, BOUNDARY LAYER FLOW ALONG A VERTICAL PLATE WITH VARIABLE WALL TEMPERATURE.

SI ABDALLAH MAYOUF

Physics Department, Faculty of Sciences, University of M'Sila, Algeria

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Abstract :

In this paper, a numerical study of heat transfer by natural convection along a wavy vertical plate with variable wall temperature is presented. The numerical method is applied to obtain the analytic solution. The homotopic transformation is employed to transform the physical domain into a flat plate. The boundary layer equations and the boundary conditions are discretized by the finite difference scheme and solved numerically using the method of Gauss-Seidel algorithm with relaxation coefficient. Effects of the wavy geometry and the constant m ($m \geq 0$) on the velocity profiles, temperature profiles, local Nusselt number and skin friction coefficient are presented and discussed in detail.

Results show that increasing of the constant m leads to increase the heat transfer rate, while the high amplitude wavelength rates reduce in general the heat transfer.

Keywords: Free convection, boundary layer, wavy surface, vertical plate, variable wall temperature.

Résumé

Dans cet article, une étude numérique du transfert de chaleur par convection naturelle le long d'une plaque verticale ondulée avec une température de paroi variable est présenté. La méthode numérique est appliquée pour obtenir la solution analytique. La transformation homotopique est utilisé pour transformer le domaine physique dans une plaque plane. Les équations de la couche limite et les conditions aux limites sont discrétisées par la méthode des différences finies et résolues numériquement en utilisant la méthode de l'algorithme de Gauss-Seidel avec un coefficient de relaxation. Effets de la géométrie ondulée et la constante m ($m \geq 0$) sur les profils de vitesse, les profils de température, le nombre et le frottement de la peau coefficient Nusselt locale sont présentés et discutés en détail.

Les résultats montrent que l'augmentation de la constante m conduit à une augmentation du taux de transfert de chaleur, tandis que les taux de longueurs d'onde d'amplitude élevées réduisent généralement le transfert de chaleur.

Mots clés : convection libre, la couche limite, surface ondulée, plaque verticale, température de la paroi variable.

ملخص.

في هذا المقال، تقدم دراسة عددية لانتقال الحرارة بواسطة الحمل الحراري الطبيعي على طول لوحة متموجة عمودية مع درجة حرارة الجدار المتغيرة. يتم تطبيق طريقة عددية للحصول على حل التحليلي. يعمل التحول بموضع الإصابة لتحويل المجال المادي في لوحة مسطحة. و discretized المعادلات طبقة الحدود وشروط الحدود من قبل النظام الفرق محدود وحلها عدديا باستخدام طريقة خوارزمية جاوس-زايدل مع معامل الاسترخاء. وتعرض آثار الهندسة متموج وم المستمر (م ≤ 0) على ملامح سرعة، وملامح درجة الحرارة، وعدد والاحتكاك الجلد نسلت المحلي معامل ومناقشتها بالتفصيل.

وأظهرت النتائج أن زيادة من م مستمر يؤدي إلى زيادة معدل انتقال الحرارة، في حين تقلل من ارتفاع معدلات الطول الموجي السعة في العام نقل الحرارة.

الكلمات المفتاحية : الحر الحمل الحراري، الطبقة الحدودية، سطح متموج، لوحة عمودية، متغير درجة حرارة الجدار.

Introduction :

Heat and mass transfer driven by natural convection along a surface have received considerable attention in recent years because of their importance in wide range of scientific field such as biology, oceanography, astrophysics, geology, chemical processes and crystal-growth techniques as those reported by Marcoux et al [1999], Mamou [2003] and Markus [2004].

Previous studies of natural convection heat and mass transfer have focused mainly on a flat plate or regular ducts. Adams and Fadden [1966] experimentally studied the free convection with opposing body force. Bottemanne [1971] has considered simultaneous heat and mass transfer by free convection along a vertical flat plate only for steady state theoretical solutions with Pr= 0.71 and Sc=0.63. Gebhart and Pera [1971] made a general formulation of the vertical two-dimensionless boundary layer flows. Moreover, these results were extended to flows about horizontal surfaces by Pera and Gebhart [1972]. Callahan and Marner [1976] studied the free convection with mass transfer on a vertical flat plate with Pr =1 and a realistic range of Schmidt number. Chen and Yuh [1979] investigated the effects of inclination of flat plate on the combined heat and mass transfer in natural convection. Chen and Yuh [1980] also, presented local non similarity solution for natural convection along a vertical cylinder. Chen et al [1980] studied the mixed convection with combined buoyant mechanism along vertical and inclined plates.

The problem of natural convection along a wavy surface was presented by Yao [1983]. The numerical results show that the frequency of the local heat transfer rate is twice that of the wavy surface. The amplitude of the oscillating local Nusselt number gradually decreases downstream where the natural convection boundary layer grows thick. Ching-Yang [2000] analyzed the free thermal and mass transfer near a vertical wavy surface with constant wall temperature and concentration in a porous medium. The results obtained show that increasing the buoyancy ratio tends to increase both the Nusselt and Sherwood numbers. Kefeng and Wen-Qiang [2006] numerically analyzed the magnitude of the buoyancy ratio N on the double-diffusive convection in a vertical cylinder with radial temperature and axial solutal gradients for different values of Gr, Pr and Sc. the thermal laminar natural convection in the boundary layer of a wavy cone trunk has been analyzed numerically by Si-Abdallah et al [2005]. Moreover, Si-Abdallah et al [2006] studied the laminar natural convection heat and mass transfer in the boundary layer along a wavy cylinder. Jer-Huan et al [2003] investigated the effects of the wavy surface on natural convection heat and mass transfer. Anwar et al [2001] analyzed the natural convection flow of a viscous fluid with viscosity inversely proportional to linear function of temperature from a vertical wavy cone. A formulation of combined forced and free convection past horizontal and vertical surfaces has been analyzed by Raju et al [1984]. Lin [1989] studied the free convection on an arbitrarily inclined plate with uniform surface heat flux. The numerical results are very accurate and a simple correlation equation are obtained for arbitrary inclination from the horizontal to the vertical and for $0.001 \leq Pr \leq \infty$. Deswita et al [2009] studied numerically the laminar free convection boundary layer flow on a horizontal plate with variable wall temperature. The numerical results for different values of the Prandtl number Pr and the constant m representing the power index of the wall temperature are obtained. Series solutions for large values of (m >>1) are also obtained and compared with the numerical solutions.

The purpose of the present work is to examine numerically the laminar natural convection heat transfer along a vertical wavy surface with variable wall temperature. The boundary layer equations and the boundary conditions are solved numerically using an implicit finite differences scheme and the Gauss-Seidel algorithm. The numerical results for different values of the constant m and the amplitude wavelength on the heat transfer rate and for the characters of the flow are investigated in detail.

MODEL DESCRIPTION

Consider a vertical wavy plate as shown schematically in Fig.1. The wavy surface of this plate is described by $y = f^*(x) = a^* \sin(2\pi x/L)$ where a^* is the amplitude of the wavy surface and L is the characteristic wavelength of the wavy surface. Moreover, we consider that the surface is maintained at a variable temperature defined by $T_w(x) = T_\infty + x^m$ where x is the coordinate measured along the plate from the leading edge and m is a constant, representing the power index to the wall temperature . The origin of the Cartesian coordinates system (x, y) is placed at the leading edge of the surface. The u and v are the velocity components in the x and y direction respectively.

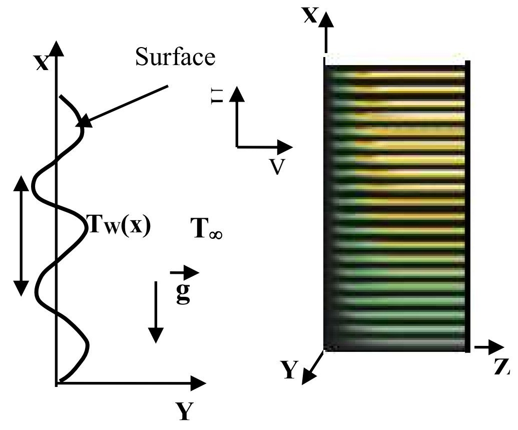

Fig.1 Physical model and coordinates system

The fluid is assumed to have constant physical proprieties except for the density variation in the buoyancy term of the momentum equation.

The governing equations for a steady, laminar and incompressible flow in the boundary layer along a vertical wavy surface with Boussinesq approximation may be written as:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+g\beta\ (T-T_\infty)) \quad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

Energy Equation

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

The appropriate boundary conditions for the problem are:
At the wavy surface, $y=f^*(x)$: $u=0$, $v=0$, $T= T_W(x)$
$y \to\infty$: $u=0$, $T= T_\infty$

Coordinate transformation In order to avoid the non-uniformity of the mesh spacing in the vicinity of the outer surface, the physical domain is transformed into a straight line using the following transformation:

$$x^*=\frac{x}{L};\quad y^*=\frac{y-f^*}{L}Gr^{1/4} \quad (5)$$

Moreover, the dimensionless form of the equations (1)-(4) in the new coordinate system (x^*, y^*), after ignoring terms of small orders in Gr, is:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \quad (6)$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^{*2}}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+f_x\frac{\partial P^*}{\partial y^*}Gr^{1/4}+(1+f_x{}^2)\frac{\partial^2 u^*}{\partial y^{*2}}+\theta \quad (7)$$

$$u^{*2}f_{xx}+\theta f_x=f_x{}^2\frac{\partial p^*}{\partial x^*}-(1+f_x^2)\frac{\partial p^*}{\partial y^*}Gr^{1/4} \quad (8)$$

$$u^*\frac{\partial\theta}{\partial x^*}+v^*\frac{\partial\theta}{\partial y^*}+u^*\theta m(x^*)^{m-1}=\frac{(1+f_x{}^2)}{pr}\frac{\partial^2\theta}{\partial y^{*2}} \quad (9)$$

Where

$$u^*=\frac{L}{\nu Gr^{1/2}}u;\ v^*=\frac{L}{\nu Gr^{1/4}}(v-uf_x);\ p^*=\frac{\rho L^2}{\mu^2 Gr}p,\ Gr=\frac{g\beta_T(T_W-T\infty)}{\nu^2}L^3;$$

$$a=\frac{a^*}{L};\ f(x)=\frac{f^*(x)}{L}$$

$$Pr=\frac{\mu C_P}{k}\quad;\quad\theta=\frac{T-T_\infty}{T_w-T_\infty} \quad (10)$$

We note here that f_x and f_{xx} indicate the first and second derivation of the function f(x) with respect to x.

For the current problem, the pressure gradient along the x direction is zero. Therefore, the term $(\partial p^*/\partial y^*)$ can be eliminated from Eqs. (7) and (8) resulting the following equations:

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=(1+f_x{}^2)\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{1}{1+f_x{}^2}(\theta-u^{*2}f_xf_{xx}) \quad (11)$$

In order to remove the singularity at the leading edge (Yao 1983) we use the following transformation:

$$X = x^* ; \quad Y = \frac{y^*}{(4x^*)^{1/4}} \quad ; \quad U = \frac{u^*}{(4x^*)^{1/2}} \quad ; \quad V = (4x^*)^{1/4} v^* \tag{12}$$

$$(4X)\frac{\partial U}{\partial X} + 2U - Y\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial Y} = 0 \tag{13}$$

$$(4X)U\frac{\partial U}{\partial X} + (V - YU)\frac{\partial U}{\partial Y} + (2 + \frac{4Xf_x f_{xx}}{1+f_x^2})U^2 = (1+f_x^2)\frac{\partial^2 U}{\partial Y^2} + \frac{\theta}{1+f_x^2} \tag{14}$$

$$(4X)U\frac{\partial \theta}{\partial X} + (V - YU)\frac{\partial \theta}{\partial Y} + 4U\theta\,(mX^m) = \frac{(1+f_x^2)}{pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{15}$$

The associate boundary conditions are:

On the wavy surface: $Y = 0$; $U = V = 0$; $\theta = 1$
$Y \to \infty$: the flow is quiescent; $U=0$; $\theta = 0$ (16)

After obtaining the velocity and the temperature along the wavy surface, the computations of the Nusselt number and the local friction coefficient are of practical interest and they are defined in the (X, Y) coordinates system as:

$$Nu_x = -(1+f_X^2)^{1/2}\left(\frac{Gr}{4X}\right)^{1/4}\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} \tag{17}$$

$$Cf_x = (1+f_X^2)^{1/2}\left(\frac{Gr}{4X}\right)^{1/4}\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{18}$$

Numerical Method In the present study, a marching finite differences scheme was used to discretize the coupled equations (13)-(15) for U, V and θ . Moreover, grid independency checks were made. Some of the calculations were tested using 250×250 nodes in the X and Y directions respectively but no significant improvement over 120×100 grid points was found. The algebraic systems of equations were solved using Gauss-Seidel algorithm with a relaxation coefficient equal to 0.7 for the variables U and to 0.5 for θ. During the program test, the convergence criterion used was $|(\Phi^{k+1} - \Phi^k)/\sum\Phi^{k+1}| \le 10^{-5}$, where Φ^k and Φ^{k+1} are the values of the k^{th} and $(k+1)^{th}$ iterations of U and θ . Furthermore, the numerical scheme used in this work is checked.

It is noted that, we have compared our computational results for the local Nusselt number at different X locations of the surface, for different values of the constant m, for Pr=1 and for a vertical flat plate (a=0) with those of Raju et al [1984] as well as by Lin et al [1989].

RESULTS AND DISCUSSION

The controlling parameters of the fluid flow and heat transfer rates for this problem are, Prandetl number Pr, the amplitude wavelength 'a' for the wavy surface described by f(x)= asin(2□x/L) and the constant m representing the power index to the wall temperature defined by Tw(x)= T∞+xm .
The numerical results of velocity profiles, temperature profiles and the heat transfer rate are presented. Hence, we present the results for some values of the constant m and the amplitude wavelength a.

The velocity profiles at a given X (X=0.5) and for Pr=1, m=0 (constant wall temperature) and for different values of the amplitude wavelength are presented in Figs. 2.a. this Figure shows that the values of the velocity profiles decrease as the amplitude wavelength rates increase, while the hydrodynamic boundary layers thickness increase. It is obvious from Fig. 2.b that the boundary layers are thicker near the nodes than those near the holes of the surface.

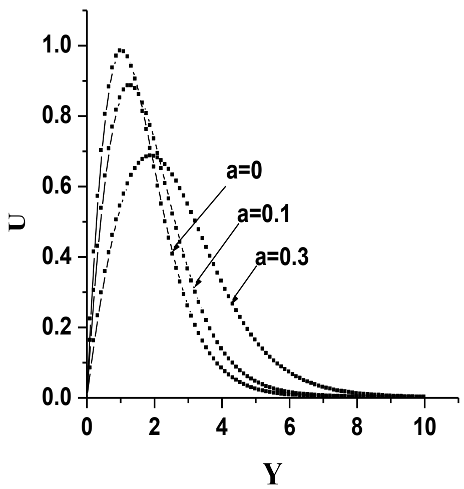

Fig.2.a Velocity profiles at X=0.5 for different values of the amplitude wavelength and for m=0

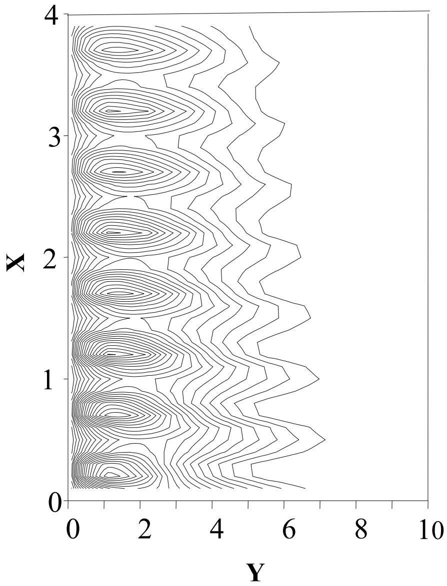

Fig .2.b The velocity contours for a=0.2

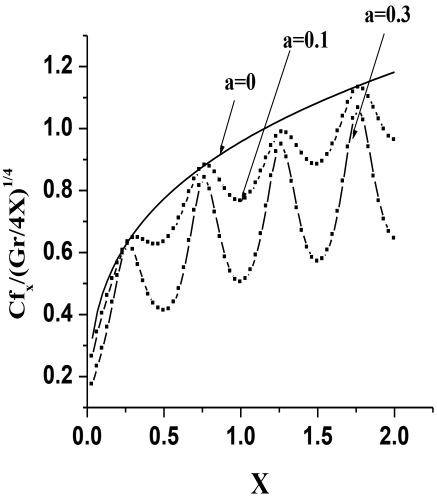

Fig.5 Skin-friction coefficient for different values of the amplitude wavelength and for m=0

Moreover, the temperature profiles at a given X (X=0.5), the local Nusselt number and the skin-friction coefficient for Pr=1, m=0 (constant wall temperature) and for different values of the amplitude wavelength are presented in Figs. 3-5. It is observed from theses figures that higher amplitude wavelength leads to increase the thermal boundary layer thickness, while the both local Nusselt number and skin friction coefficient decrease in general along the surface. It is also worth noting that the maximum values of Nux and Cfx occur on the nodes of the wavy surface while the minimum values occur on the holes.

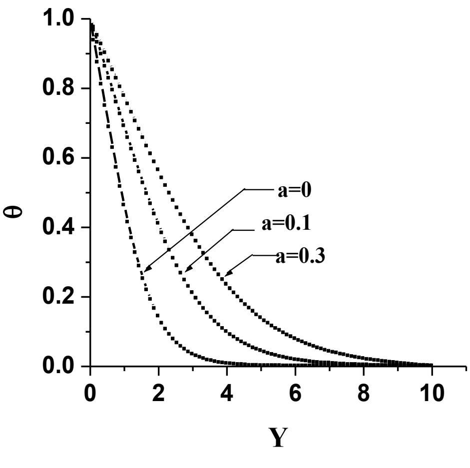

Fig.3 Temperature profiles at X=0.5 for different values of the amplitude wavelength and for m=0

The effect of the variable surface temperature on the velocity profiles, temperature profiles, the local Nusselt number and skin-friction coefficient for a=0.2 is presented in Figs. 6-9. It is seen that increasing of the constant m leads to reduce the velocity profiles, the thermal boundary layer thickness as well as the skin friction coefficient, where the heat transfer rates increase and this can be explained that the heat convection is important than the heat conduction.

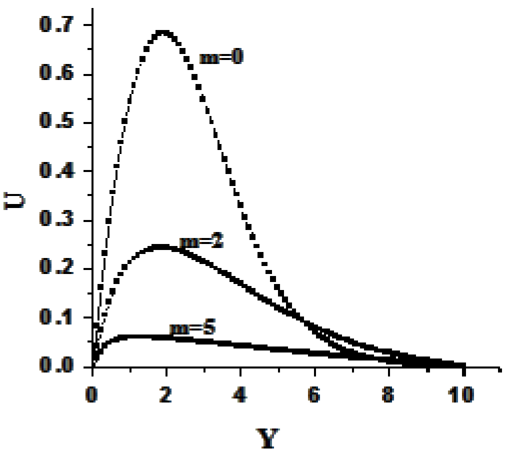

Fig.6 Velocity profiles at X=0.5 for different values of the constant m and for a=0.2

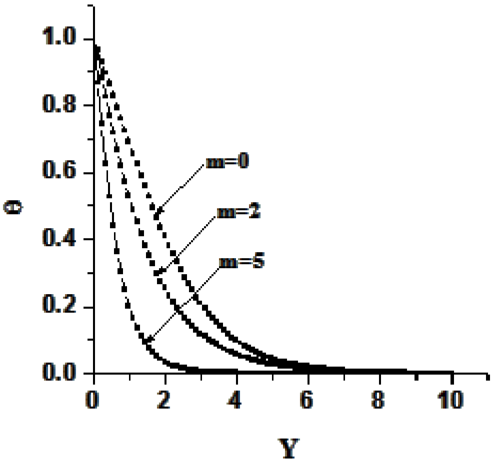

Fig.7 Velocity profiles at X=0.5 for different values of the constant m and for a=0.2

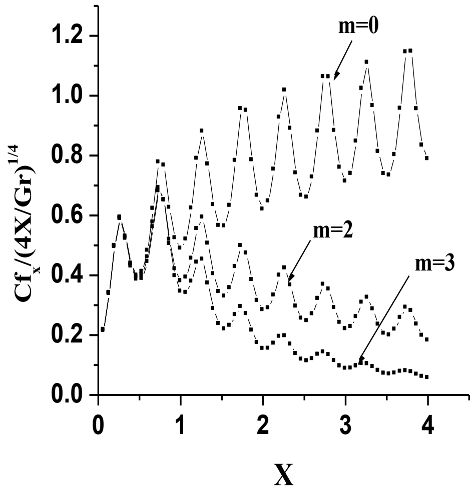

Fig.9 Variation of the local Nusselt number for different values of the constant m and for a=0.2

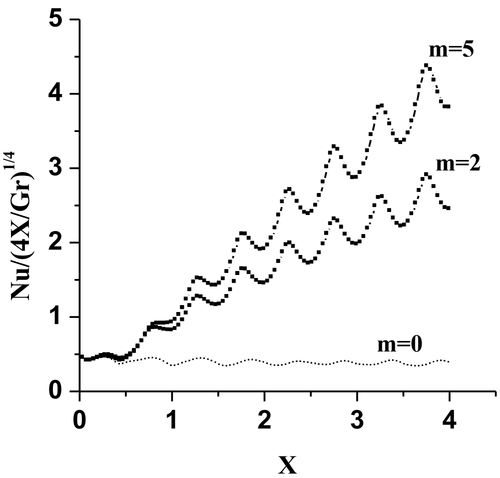

Fig.8 Local Nusselt number for different values of the constant m and for a=0.2

Fig. 10 and 11 represent the isotherms for constant and variable surface temperature respectively. These figures show clearly a sinusoidal behavior but in the case, where the temperature of the wall is variable, the thermal boundary layers thickness decrease as X increases and this can be explained from Fig. 12, that the recirculation of the fluid in the holes of the surface is reduced as the temperature of the wall increases and this conducts to accelerate the flow along the surface.

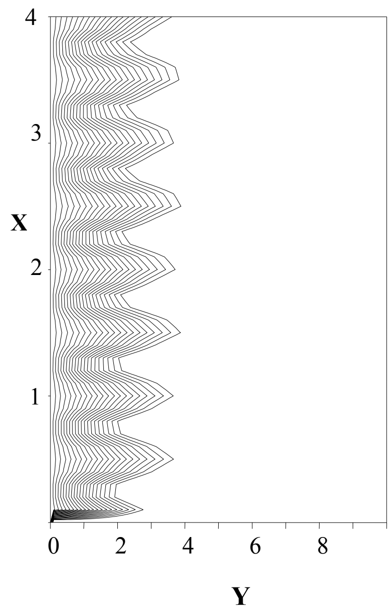

Fig.10 Isotherms for a=0.2 , m=0

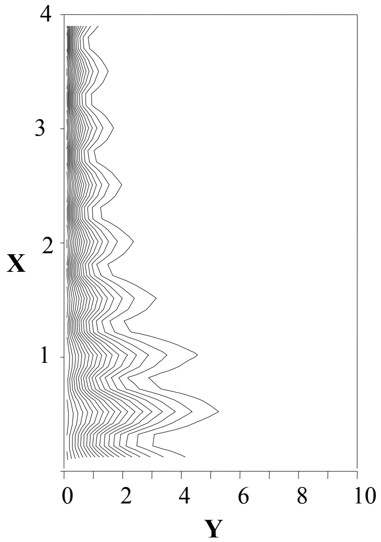

Fig. 11 Isotherms for a=0.2, m=3

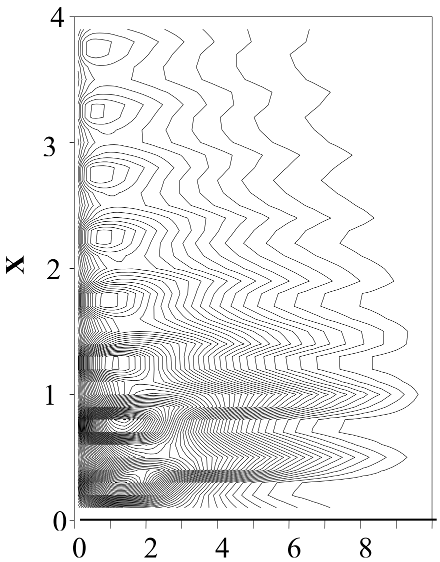

Fig .12 Velocity contours for m= 3, a=0.2

CONCLUSION

The free convection in the boundary layer of a vertical wavy surface maintained at a variable temperature has been studied numerically. The boundary-layer equations were discretized with a finite differences scheme and solved using Gauss-Seidel iterative method.
The effects of amplitude wavelength ratio and the constant m corresponding to the power index of the wall temperature on momentum and heat transfer have been studied in detail.

From this present work, it has been found that the wavy surface disturbs the flow whatever the temperature of the surface const (m=0) or variable (m>0). The higher amplitude wavelength ratio decreases the heat transfer rate. Moreover, increasing of the constant m for a wavy surface conducts to increase the heat transfer rate.

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