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Abstract

We present a space-time transformation to the harmonic oscillator with a time-dependent mass and frequency and we transform the problem to that of constant mass and time-dependent potential of the form . The propagator and the corresponding wave functions are given. A new general model of time-dependent mass is proposed.

Keywords: *Path integral, Time dependent mass, time dependent harmonic oscillator, time dependent transformations.*

Résumé

Nous présentons une transformation spatio-temporelle de l'oscillateur harmonique dont la masse et la fréquence sont des fonctions du temps. On transforme ce problème à celui d'une masse constante et un potentiel en fonction du temps de la formeΩ^2 (t) x^2. Le propagateur et les fonctions d'onde sont donnés. Un nouveau modèle généralisé de la masse en fonction du temps est proposé.

Mots clés *: L'intégrale des chemins, La masse qui dépend du temps, L'oscillateur harmonique qui dépend du temps, La transformations qui dépend du temps*.

ملخص

عرضنا مسالة الهزاز التوافقي المعتمد على الزمن باستعمال تحويلات فضائية زمنية تم تحويل المسالة الي تلك ذات الكتلة الثابتة والكمون التاليx^2 (t) Ω^2. الناشر والدوال الموجية اعطيت, ونموذج جديد تم تقديمه.

الكلمات المفتاحية : تكامل المسالك، الكتلة المعلقة بالزمن، الهزاز التوافقي المعلق بالزمن، التحويلات المتعلقة بالزمن.

I. INTRODUCTION

Recently, a great deal of attention has been devoted to the subject of time dependent Hamiltonians. The importance of this problem in various areas of physics, quantum optics [1], cosmology [2], nanotechnology [3], plasma physics [4] is the main reason for these intensive studies. The problem of the harmonic oscillator with time-dependent mass and frequency is a common problem in this area. Abdalla and Colograve [5] studied this problem with a time dependent mass and constant frequency in order to describe the electromagnetic field intensities in a Fabry-Pérot cavity by applying a time dependent canonical transformation. Kandekar and Lawand [6] have considered the case of exponentially varying mass with variable frequency by means of path integral method. The same problem with a constant frequency has been treated by path integral by many authors for example: Sabir and Rajagopalan [7] treated the cases of the strongly pulsating mass and a model of growing mass, the power-low suppressed harmonic oscillator [8] is also solved. In [9] the problem with an arbitrary time dependent mass and frequency is treated using space-time transformations. The same problem has been solved in[10]. Cheng [11] evaluated the propagator of a forced time dependent harmonic oscillator. Looking through the literature one finds that an explicit expression for the propagator could not be obtained for all time varying mass-functions because the procedure involves the solutions of non-linear differential equations. This is the reason why only few cases of varying mass has been solved. As mention above the strongly pulsating mass [7], the exponentially time-dependent mass [6], the power-low mass [8] and some examples are given in Ref.[12]

In this paper we will give a model of harmonic oscillator with a time-dependent mass and constant frequency. We will transform the problem of the time-dependent mass to that of constant mass and frequency.

II. EXPLICITLY TIME-DEPENDENT TRANSFORMATION:

In order to discuss explicitly time-dependent space-time transformations we start by considering the usual path integral formulation of a particle with a time-dependent mass and time-dependent potential according to:

$$K(x'',t'';x',t') = \int D[x(t)]\exp\frac{i}{h}\int dt\left(\frac{m(t)}{2}\dot{x}^2 - V(x,t)\right)$$
$$= \lim_{N->\infty}\prod_1^N\left(\frac{m_j}{2\pi ih}\right)^{1/2}\prod_1^{N-1}\int dx_j \exp\{\frac{i}{h}\sum_{j=1}^N[\frac{m_j}{2}\Delta x_j{}^2 - V(x_j,t_j)]\} \quad \textbf{(1)}$$

We consider an explicitly time-dependent coordinate transformation $x = h(q,t)$, the propagator will be:

$$K(q'',q';t'';t') = (h'(q'',t'')h'(q',t'))^{-1/2}$$
$$\int D[q(t)]\exp\left(\frac{i}{\hbar}\int(\frac{m(t)}{2}h'^2(q,t)\dot{q}^2 + \frac{m(t)}{2}\dot{h}^2(q,t) - \frac{i}{\hbar}\frac{\dot{g}(q,t)}{g(q,t)} - \frac{\hbar^2}{8m(t)}\frac{h''^2(q,t)}{h'^4(q,t)} - V(q,t)\right)dt \quad \textbf{(2)}$$

where $g(q,t) = \exp\left(\frac{im(t)}{\hbar}\int^q h'(z,t)\dot{h}(z,t)dz\right)$ and $h(q,t)$ is an arbitrary function.

III. EVALUATION OF THE EXACT PROPAGATOR FOR THE HARMONIC OSCILLATOR WITH TIME-DEPENDENT MASS AND FREQUENCY

The general time-dependent Lagrangian for oscillator harmonic is given by:

$$L = \frac{1}{2}[m(t)\dot{x} - m(t)\omega^2(t)x^2] \quad \textbf{(3)}$$

where m(t) and ω (t) are well-behaved functions of time. By using explicitly space-time transformations such that $x = c(t)q$, we can rewrite the propagator corresponding to the Lagrangian Eq.(3) as:

$$K(q'',t'';q',t') = [c(t')c(t'')]^{-\frac{1}{2}}exp\frac{im_0}{2h}[\frac{c\,(\dot{t''})}{c(t'')}q''^2 - \frac{c\,(\dot{t'})}{c(t')}q'^2]\int D[q(t)]exp\int dt(\frac{m_0}{2}\dot{q}^2 - \frac{m_0}{2}\Omega^2(t)q^2) \quad \textbf{(4)}$$

we have put $c(t)^2m(t)=m_0$ and $\Omega^2(t) = \left(\frac{\ddot{c}}{c} - 2\frac{\dot{c}}{c^2}^2 + \omega^2(t)\right)$

The fact that $\Omega^2(t)$ is a general time-dependent function, enabled us to investigate the system separately for three cases where $\Omega^2(t)>0$, $\Omega^2(t)<0$ and $\Omega^2(t)=0$.

<u>**The first case: $\Omega^2(t)>0$**</u>

This is the case of the well-known harmonic oscillator and the propagator of this system [les ref] is given by

$$K(q'',t'';q',t') = \left(\frac{(\dot{\gamma}''\dot{\gamma}')^{\frac{1}{2}}}{2\pi c(t')c(t'')\sin(\gamma''-\gamma')}\right)^{\frac{1}{2}}$$
$$exp\frac{im_0}{2h}\left[\left(\frac{\dot{c''}}{c''} + \frac{\dot{\mu''}}{\mu''}\right)q''^2 - \left(\frac{\dot{c'}}{c'} + \frac{\dot{\mu'}}{\mu'}\right)q'^2\right]$$
$$\exp\{\frac{im_0}{2\hbar\sin(\gamma''-\gamma')}([\dot{\gamma}''^2q''^2 + \dot{\gamma}'^2q'^2]\cos(\gamma''-\gamma') - 2(\dot{\gamma}''\dot{\gamma}')^{\frac{1}{2}}q'q'')\} \quad \textbf{(5)}$$

where $\gamma(t)$ and $\mu(t)$ satisfy the following coupled differential equations

$$\ddot{\mu} - \mu\dot{\gamma}^2 + \Omega^2(t)\mu = 0$$
$$\mu\ddot{\gamma} + 2\dot{\mu}\dot{\gamma} = 0$$

<u>**The second case: $\Omega^2(t)<0$**</u>

The propagator corresponds to this case can be given by:

$$K(q'',t'';q',t') = \left(\frac{(\dot{\gamma}''\dot{\gamma}')^{\frac{1}{2}}}{2\pi c(t')c(t'')\sinh(\gamma''-\gamma')}\right)^{\frac{1}{2}}$$
$$exp\frac{im_0}{2h}\left[\left(\frac{\dot{c''}}{c''} + \frac{\dot{\mu''}}{\mu''}\right)q''^2 - \left(\frac{\dot{c'}}{c'} + \frac{\dot{\mu'}}{\mu'}\right)q'^2\right]$$

$$\exp\left\{\frac{im_0}{2\hbar\, sinh(\gamma''-\gamma')}[\dot{\gamma}''^2 q''^2+\dot{\gamma}'^2 q'^2]\,cosh(\gamma''-\gamma')-2(\dot{\gamma}''\dot{\gamma}')^{\frac{1}{2}}q'q''\right\} \quad \textbf{(6)}$$

The third case: $\Omega^2(t)$=0

This case as known is that of the free particle. The propagator of the time dependent mass and frequency in this specific case can be treated as the propagator of the free particle with a constant mass which could be given by:

$$K(q'',t'';q',t')=\left(\frac{1}{2\pi c(t')c(t'')}\right)^{\frac{1}{2}} exp\frac{im_0}{2h}\left[\frac{\dot{c}''}{c''}q''^2-\frac{\dot{c}'}{c'}q'^2\right]\ \exp\frac{im_0}{2\hbar}[q''^2+q'^2-2q'q''] \quad \textbf{(7)}$$

IV. APPLICATION:

We consider the problem of the harmonic oscillator that have a mass of the form $m(t)=m_0(a\,exp(\lambda t)+\ bexp(-\lambda t))^2$, where m_0 is a real number, a,b are complex numbers and λ is a pure real or pure complex number such that $m(t)$ has a physical meaning. The Lagrangian corresponding to this system is:

$$L(x,\dot{x},t)=\frac{1}{2}m_0(a\,exp(\lambda t)+\ b\exp(-\lambda t))^2\dot{x}^2-\frac{1}{2}m_0(a\,exp(\lambda t)+\ bexp(-\lambda t))^2\omega^2(t)x^2 \quad \textbf{(9)}$$

Then the propagator could be given by

$$k(x'',t'';x',t')=\int D[x(t)]\exp\frac{i}{\hbar}\int L(x,\dot{x},t)dt \quad \textbf{(10)}$$

By using the transformation $x=c(t)q$, where $c(t)=(a\exp(\lambda t)+b\exp(-\lambda t))^{-1}$, the propagator Eq.(10) will transform to the following form :

$$k(x'',t'';x',t')=\left(c(t'')c(t')\right)^{-\frac{1}{2}}\frac{g(q'',t'')}{g(q',t')}k(q'',t'';q',t') \quad \textbf{(11)}$$

where $k(q'',t'';q',t')$ is the propagator corresponding to the Lagrangian $\tilde{L}(q,\dot{q},t)=\frac{m_0}{2}\dot{q}^2-\frac{m_0}{2}(\omega^2-\lambda^2)q^2$ *and* $g(q,t)=\exp\left(\frac{im_0}{2\hbar}\frac{\dot{c}(t)}{c(t)}q^2\right)$. The propagator $k(q'',t'';q',t')$ is the propagator of the simple harmonic oscillator when $\omega^2>\lambda^2$, the inverted oscillator …. when $\omega^2<\lambda^2$, and when $\omega^2=\lambda^2$ it is the free particle propagator , at any rate it could be written as :

$$k(q'',t'';q',t')=\left(\frac{m_0\Omega}{2\pi\hbar\sin(\Omega(t''-t'))}\right)^{\frac{1}{2}}\exp\frac{i}{\hbar}\left(\frac{m_0\Omega}{2\sin(\Omega(t''-t'))}[(q''^2+q'^2)\cos(\Omega(t''-t'))\ -2q''q')]\right) \quad \textbf{(12)}$$

where $=\sqrt{\omega^2-\lambda^2}$. Then, the whole propagator $k(x'',t'';x',t')$ corresponding to the Lagrangian E*q.(9)* will take the form:

$$k(x'',t'';x',t')=\left(\frac{(m_0\Omega)(a\,exp(\lambda t'')+b\,exp(-\lambda t''))(a\,exp(\lambda t')+b\,exp(-\lambda t'))}{2\pi i\hbar\,sin(\Omega(t''-t'))}\right)^{1/2}$$

$$exp\frac{-\lambda im_0}{2\hbar}(\ (a^2\exp(2\lambda t'')-b^2\exp(-2\lambda t''))x''^2+-(a^2\,exp(2\lambda t')-b^2\exp(-2\lambda t'))x'^2)$$

$$\exp\frac{i}{\hbar}\left(\frac{m_0\Omega}{2\pi\hbar\sin(\Omega(t''-t'))}([((a\,exp(\lambda t'')+\quad b\,exp(-\lambda t''))\hat{}x''^2+(a\,exp(\lambda t')+b\,exp(-\lambda t'))^2x'^2)\cos(\Omega(t''-t'))\,])\right)$$

$$\exp\left(\frac{im_0\Omega}{2\,\pi\hbar sin(\Omega(t''-t'))}((2x''x'(a\,exp(\lambda t'')+b\,exp(-\lambda t''))(a\,exp(\lambda t')+b\,exp(-\lambda t'))))\right) \quad \textbf{(13)}$$

V. SCHRODINGER EQUATION AND THE INVARIANT OPERATOR:

For a quantal system characterized by time-dependent Hamiltonian H(t) the Schrödinger equation is:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t)=H(t)\psi(x,t) \quad \textbf{(14)}$$

where $\psi(x,t)=\sum_{n=0}^{\infty}C_n\psi_n(x,t)$, C_n are constants and each $\psi_n(x,t)$ (the normalized solution of the time-dependent Schrodinger) satisfies the time-dependent Schrodinger's equation. For the bounded systems the propagator can be written as

$$k(x'',t'';x',t')=\sum_n\psi_n{}^*(x',t')\psi_n(x'',t'') \quad \textbf{(15)}$$

By using Mehler's formula to the propagator Eq.(13)

$$exp[-(x^2+y^2)]\sum_{n=0}^{+\infty}\frac{z^n}{2^n n!}H_n(x)H_n(y)=\frac{\exp\left[-\frac{(x^2+y^2-2xyz)}{(1-z^2)}\right]}{\sqrt{1-z^2}} \quad \textbf{(16)}$$

where $H_n(x)$ is nth Hermite polynomial, $x=\sqrt{m(t'')\dot{\mu}(t'')/\hbar}x''$, $y=\sqrt{m(t')\dot{\mu}(t')/\hbar}x'$ and $x=\exp(-i\Omega(t''-t'))$ then we obtain the normalized solution of the time-dependent Schrodinger as:

$$\psi_n(x,t)=\left[\frac{1}{2^n n!}\left(\frac{m(t)\Omega}{\pi\hbar}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}exp[-i\left(n+\frac{1}{2}\right)\sqrt{\omega^2-\lambda^2}\,t]$$

$$exp\left[\frac{1}{2\hbar}\left(i\frac{\dot{m}(t)}{2}+\Omega m(t)\right)x^2\right]H_n\left(\left(\frac{\Omega m(t)}{\hbar}\right)^{\frac{1}{2}}x\right) \quad \textbf{(17)}$$

Lewis and Riesenfield [ ] have shown that the general solution of the Eq.(14) could be written as:

$$\psi(x,t)=\sum_n C_n\,exp(i\theta_n)\,\Phi_n=\sum_n C_n\psi_n \quad \textbf{(18)}$$

where $\Phi_n(x,t)$ are the eigenfunctions of the invariant operator I(t) corresponding to the system, and we have :

$$I(t)\Phi_n=\gamma_n\Phi_n \quad \textbf{(19)}$$

γ_n are the eigenvalues of the invariant operator which are time –independent .The hermitian invariant operator for our system can be obtained (Colegrave and Abdella (ref)) as :

$$I = \Omega\, m(t)x^2 + \frac{1}{m(t)\Omega}\left(\frac{\dot{m}(t)}{2}x + p\right)^2 \quad \textbf{(20)}$$

where $m(t) = (a exp(\lambda t) + b\, exp(-\lambda t))^2$ and $\Omega = \sqrt{\omega^2 - \lambda^2}$, then one could find that the solutions of the Eq.(19) could be given by:

$$\Phi_n(x,t) = \left[\frac{1}{2^n n!}\left(\frac{m(t)\Omega}{\pi\hbar}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} exp\left[\frac{1}{2\hbar}\left(i\frac{\dot{m}(t)}{2} + \Omega m(t)\right)x^2\right] H_n\left(\left(\frac{\Omega m(t)}{\hbar}\right)^{\frac{1}{2}} x\right) \quad \textbf{(21)}$$

VI. PARTICULAR CASES

A. The exponentially changing mass: $m(t) = m_0 exp(2\lambda t)$:

By putting $b = 0$, and $a = 1$ then $m(t) = m_0 exp(2\lambda t)$ the propagator Eq (13) in this case could simplify to the form:

$$k(x'',t'';x',t') = \left(\frac{m_0\Omega exp(\lambda(t''+t'))}{2\pi i\hbar \sin(\Omega(t''-t'))}\right)^{\frac{1}{2}} \exp\left(\frac{\lambda m_0}{2i\hbar}(\exp(2\lambda t'')x''^2 - \exp(2\lambda t')x'^2)\right) \exp\left(\frac{i}{\hbar}\left(\frac{m_0\Omega}{2\sin(\Omega(t''-t'))}\left([\exp(2\lambda t'')x''^2 + \exp(2\lambda t')x'^2]\cos(\Omega(t''-t')) - 2\exp(\lambda(t''+t'))x''x'\right)\right)\right) \quad \textbf{(22)}$$

B. The strongly pulsating mass $m(t) = m_0\cos^2(\sigma t + \delta)$:

By putting $a = b = \frac{1}{2}\exp(i\delta)$, $\lambda = i\sigma$ where σ is a real number, the mass will take the from $m(t) = m_0\cos^2(\sigma t + \delta)$. The propagator Eq.(13) in this case could simplify to take the form:

$$k(x'',t'';x',t') = \left(\frac{\cos(\sigma t''+\delta)\cos(\sigma t'+\delta)(m_0\Omega)}{2\pi i\hbar\, sin(\Omega(t''-t'))}\right)^{\frac{1}{2}} \exp\frac{i\sigma m_0}{4\hbar}(\,sin(2\sigma t''+2\delta)x''^2 - sin(2\sigma t'+2\delta)x'^2) \exp\frac{i}{\hbar}\left(\frac{m_0\Omega}{2\sin(\Omega(t''-t'))}([(\cos^2(\sigma t''+\delta)\,x''^2 + \cos^2(\sigma t'+\delta)\,x'^2)\cos(\Omega(t''-t') - 2cos(\sigma t''+\delta)cos(\sigma t'+\delta)x''x'])\right) \quad \textbf{(23)}$$

which is the same as given in [6] by putting $\delta = 0$.

c. The mass $m(t) = m_0\cosh^2(\lambda t + \vartheta)$:

Here we have to put $a = b = \frac{1}{2}\exp(\vartheta)$. the propagator Eq.(13) will be:

$$k(x'',t'';x',t') = \left(\frac{\cosh(\lambda t''+\vartheta)\cosh(\lambda t''+\vartheta)(m_0\Omega)}{2\pi i\hbar\, sin(\Omega(t''-t'))}\right)^{1/2} \exp\frac{-i\lambda m_0}{2\hbar}(\,\sinh(2\lambda t''+2\vartheta)x''^2 - sinh(2\lambda t'+2\vartheta)x'^2) \exp\left(\frac{m_0\Omega}{2\,sin(\Omega(t''-t'))}([(cosh^2(\lambda t''+\vartheta)\,q''^2 + cosh^2(\lambda t'+\vartheta)\,q'^2)\,cos(\Omega(t''-t') - 2\,cosh^2(\lambda t''+\vartheta)\,cosh^2(\lambda t'+\vartheta)\,q''q'])\right) \quad \textbf{(24)}$$

which is the same result as given in [13]

VII. CALDIROLA-KANAI OSCILLATOR WITH AN INVERSE QUADRATIC POTENTIAL:

Consider the Caldirola-Kanai oscillator with an inverse quadratic potential

$$L(\dot{x},x,t) = m_0(a exp(\lambda t) + \text{b}\exp(-\lambda t))^2\left[\frac{1}{2}M_0\dot{x}^2 - \frac{1}{2}m_0{\omega_0}^2x^2\right] - \frac{k(a exp(\lambda t)+b\exp(-\lambda t))^{-2}}{m_0x^2} \quad \textbf{(25)}$$

where $k > -\frac{h^2}{8m_0}$ to avoid the fall in the center . By using the transformations given above to the lagrangian in Eq(25), we obtain the converted Lagrangian:

$$L(\dot{q},q,t) = \tilde{L}(\dot{q},q,t) + m_0\frac{d}{dt}\left(\frac{\dot{c}(t)}{2c(t)}q^2\right) \quad \textbf{(26)}$$

where

$$\tilde{L}(\dot{q},q,t) = \left[\frac{1}{2}m_0\dot{q}^2 - \frac{1}{2}m_0({\omega_0}^2 - \lambda^2)\,q^2\right] - \frac{k}{m_0q^2} \quad \textbf{(27)}$$

the propagator after the space-time transformations, will take the form as in Eq( ) .

$$K(x'',t'';x',t') = (c(t'')c(t'))^{-\frac{1}{2}}\frac{g(t'')}{g(t')}k(q'',t'';q',t') \quad \textbf{(28)}$$

with:

$$K(q'',q';t'',t') = \int D[q(t)]exp\frac{i}{h}\int dt\{\frac{1}{2}m_0\dot{q}^2 - \frac{1}{2}m_0({\omega_0}^2 - \lambda^2)\,q^2 - \frac{k}{m_0q^2}\} \quad \textbf{(29)}$$

which is the propagator of the stationary harmonic oscillator plus an inverse potential . One could find in the literature that :

$$K(q'',q';t'',t') = \left(\frac{(q'q'')^{\frac{1}{2}}m_0\Omega}{i\hbar\, sin(\Omega(t''-t'))}\right)\exp\left(\frac{im_0\Omega}{2\hbar}(q''^2 + q'^2)\,cos(\Omega(t''-t'))\right) I_z\left(\frac{m_0\Omega}{i\hbar\, sin(\Omega(t''-t'))}q''q'\right) \quad \textbf{(30)}$$

where $z = (1/4 + 2k/\hbar^2)^{1/2}$ I_z is the modified Bessel function and $g(t) = \exp\left(\frac{im_0}{2\hbar}\frac{\dot{c}(t)}{c(t)}q^2\right)$,then the whole propagator related to the system above Eq (25) could be given as:

$$K(x'',x';t'',t') = \frac{m_0\Omega(x''x')^{\frac{1}{2}}}{i\hbar\, sin(\Omega(t''-t'))} exp\, \frac{im_0}{2\hbar}\left(\frac{\dot{c}(t'')}{c(t'')^3}\, x''^2 - \frac{\dot{c}(t')}{c(t')^3} x'^2\right)\, \exp\left(\frac{im_0\Omega}{2\hbar}\left(\frac{x''^2}{c(t'')^2} + \frac{x'^2}{c(t')^2}\right) cos(\Omega(t''-t'))\right) I_z\left(\frac{m_0\Omega}{i\hbar\, sin(\Omega(t''-t'))}\, \frac{x''}{c(t'')}\frac{x'}{c(t')}\right)$$

$$= \frac{m_0\Omega}{i\hbar\, sin(\Omega(t''-t'))}\left((a\, exp(\lambda t'') + b\, exp(-\lambda t''))(a\, exp(\lambda t') + b\, exp(-\lambda t'))\right)^{1/2}$$

$$\exp\left(\frac{im_0\Omega}{2\hbar}((a\, exp(\lambda t'') + b\, exp(-\lambda t''))x''^2 + (a\, exp(\lambda t') + b\, exp(-\lambda t'))x'^2)\, cos(\Omega(t''-t'))\right)$$

$$\exp\frac{-i\lambda m_0}{2\hbar}((a^2\exp(2\lambda t'') - b^2\exp(-2\lambda t''))\, x''^2 - (a^2\, exp(2\lambda t') - b^2\, exp(-2\lambda t'))x'^2)\, I_z\left(\frac{m_0\Omega}{i\hbar\, sin(\Omega(t''-t'))}(a\, exp(\lambda t'') + b\, exp(-\lambda t''))(a\, exp(\lambda t') + b\, exp(-\lambda t'))x''x'\right) \quad \textbf{(31)}$$

CONCLUSION

The study of quintals harmonic oscillators with time-dependent mass has assumed because it is very important in different areas of physics like plasma physics, cosmology, quantum optics etc. Looking through the literature one can notice in this context that the path integral method has been used to solve some problems with specific time-dependent mass like exponentially varying mass, strongly pulsating mass, growing mass etc. In this paper we have used a space-time transformations to solve a new generalized model. We gave the related propagator to the harmonic oscillator. The normalized eigen-function and the invariant operator are found for the harmonic oscillator.

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