

## THE PERFORMANCE OF DISCRETE WAVELET WITH CONVOLUTIONAL CODE IN LEO SATELLITE CONSTELLATION

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### Abstract

In this paper, we propose a new approach to solve the problem of the spectral occupation for the transmissions with high flow in satellites telecommunications systems, this occupation spectral is caused by the cohabitation with other emissions or services sharing the same band frequency, this approach is the association of discrete wavelet with convolutional code. The performance of this method are illustrated in a additive white Gaussian noise channel (AWGN).

**Keywords:** Convolutional coder, discrete wavelet transform, Wavelet thresholding.

### Résumé

Dans cet article, on propose une nouvelle approche pour résoudre le problème de l'occupation spectrale pour les transmissions à haut débit dans les satellites de télécommunication. Cette occupation spectrale est causée par la cohabitation avec d'autres émissions ou services partageant la même bande de fréquence. Cette approche est basée sur l'association de la transformée discrète en ondelette avec le codage convolutionnel. Les performances de cette méthode sont illustrées pour un canal à bruit blanc gaussien additif (AWGN).

**Mots clés:** Codage convolutionnel, transformée discrète en ondelette, seuillage par ondelette.

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The transmissions with high flow by satellite, such as one meets them in Observation of the Earth or Telecommunication for example [1-3], are more and more often confronted with the problems of cohabitation with other emissions or services dividing the same band frequency, because of their important spectral occupation related to the rate/rhythm of the transmitted data [4-6].

In addition, the permanent search for a greater effectiveness of use of energy on board encourages to introduce powerful systems of coding to protect information against the errors from transmission while trying to tend towards the "mythical limit of Shannon"; unfortunately, the profit obtained on the energy balance results in an increase in the band occupied by the signals transmits [6], for coding involves a more or less important rise in the digital rhythm of the connection according to the type of code used [5].

The reduction of the spectral occupation, with informational flow given, can be carried out by means of prefiltering in emission and/or of adapted modulations allowing to approach limiting the band known as "band of Nyquist".

The traditional prefiltering optimum generally recommended in emission, are characterized by transfer functions of Root of Elevated Cosine ; the signals obtained after such a filtering however present a certain number of disadvantages which are:

\* the occupied band higher than the ultimate theoretical band of Nyquist (1,3-1,5 times the rhythm of Rs symbol),

\* they are sensitive to non linearity of the chain of emission because of the amplitude modulation which characterizes them (modulation introduced by filtering emission). For the bands Ku and Ka, the travelling wave tubes constitute the most powerful solution in term of output to carry out the power gain of these signals; they however have strong not linearities of phase and amplitude which involve considerable losses of transmission.

### ملخص

من خلال هذا البحث، نقوم بتقديم حل لمشكل الاحتلال الطيفي للاتصالات العالية التدفق في أقمار الاتصال، هذا الاحتلال الطيفي ناجم عن تعدد الارسلات والخدمات التي تتقاسم نفس مجال التوتر. هذا الحل يتمثل في الجمع بين تقنية التشفير الالتفاتي وتحويل اندلات. أثبتت هذه الطريقة فعاليتها في قناة قوسيان للإرسال.

**الكلمات المفتاحية:** التشفير الالتفاتي، تحويل اندلات، تحديد اندلات.

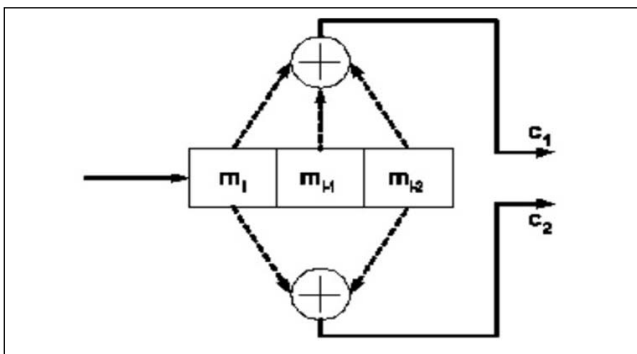
To mitigate this problem, one proposes the use a convolutional coding on the levels of the transmitter, and the discrete wavelet associate with convolutional decoder uses the Viterbi algorithm on the levels of receiver. The use of discrete wavelet comes as of the these performance from denoising in the milieus very disturbed [1], thus to decrease the redundancies cause by the second coding.

This paper is organized as follows; in the first part one gives a description of coding convolutional thus decoding of viterbi, in the second part one presents the transformed discrete one into wavelet and differ them method of thresholding, lastly, the simulation results are presented in the third part and we completes by a conclusion.

### 1- CONVOLUTIONAL CODES (CC)

CC is an error detection and correction method [7]. The concept behind CC is simple. Because every data bit transmitted across a noisy channel has potential for error, a system can use CC to introduce redundancies in the data [8-9]. In other words, instead of transmitting only the data, the transmission system sends CC across the noisy channel. On the receive end, the system recovers the CC with errors and corrects these errors in the decoding process [10].

A convolutional code is a sequence of encoded symbols, which is generated by passing the information sequentially through a binary shift register [7,11]. On the transmit side, the convolutional encoder takes data stream at rate of  $R_s$  as input. Then, it encodes the data to output a code with coding rate of  $R_t$ . Finally, the resulting code is modulated and transmitted across a noisy channel. The code rate can be defined as  $R_s/R_t$ . On the receive side, the convolutional decoder receive data from the demodulator at transmit coding rate of  $R_t$ , and perform the reverse process to get the original data at the rate of  $R_s$  [7]. Figure 1 shows an example of a three-stage shift register that implements a (2,1) binary convolutional encoder with code rate of 1/2 and constraint length of 3.



**Figure 1:** An(n, k) = (2, 1) binary Convolutional Encoder [3].

By interleaving the upper and lower summers, the encoder generates the output. For example [7], an input pulse vector  $v = (1\ 0\ 0\ \dots)$  generates upper and lower branch responses:

$$w^{(1)} = (1\ 0\ 1\ 0\ 0\ \dots) \text{ and } w^{(2)} = (1\ 1\ 1\ 0\ 0\ \dots)$$

The final output sequence  $w$  is

$$w = (w_1^{(1)}, w_1^{(2)}, w_2^{(1)}, w_2^{(2)}, w_3^{(1)}, w_3^{(2)}, \dots)$$

In the current example, the final output to be transmitted

is  $w = (1, 1, 0, 1, 1, 1, 0, 0, \dots)$ . On the receive end, the convolutional decoder uses the Viterbi algorithm. The Viterbi algorithm as a dynamic programming algorithm for finding the shortest path through a trellis, and the algorithm can be broken down into the following three steps [15].

1- Weigh the trellis; that is, calculate the branch metrics.

2- Recursively compute the shortest paths to time  $n$ , in terms of the shortest paths to time  $n-1$ . In this step, decisions are used to recursively update the survivor path of the signal. This is known as add-compare-select (ACS) recursion.

3- Recursively find the shortest path leading to each trellis state using the decisions from Step 2. The shortest path is called the survivor path for that state and the process is referred to as survivor path decode. Finally, if all survivor paths are traced back in time, they merge into a unique path, which is the most likely signal path that we are trying to find.

### 2- THE DISCRETE WAVELET TRANSFORM

The Principe of the discrete wavelet transform is to separate the signal in two components, one representing the general pace of the signal, the other representing its details [12]. The general pace of a function is represented by its low frequencies, the details by its high frequencies [13].

To separate both, we thus need a pair of filters: a low-pass filter to obtain the general pace (also called approximation or average), and a high-pass filter to estimate its details, i.e. the elements which vary quickly. Not to lose information, these two complementary filters must of course being: the frequencies cut by one must be preserved by the other [14].

We can define the supplementary orthogonal  $W_j$  of

$V_j$  in  $V_{j-1}$ :

$$V_{j-1} = V_j \oplus W_j \quad (1)$$

Then, there is a function  $\Psi$  there such as the family.

$\Psi_{j,n}(t) = 2^{-j/2} \Psi(2^{-j}t - n)$ ,  $n$  describing  $Z$ , that is to say an orthonorm base  $W_j$ . The families of the  $\Psi_{j,n}$ ,  $j$  and

describing  $Z$ , is a orthonorm base of  $L^2$  and

$$L^2(R) = \oplus_{j \in Z} W_j = V_j \oplus_{k \leq j} W_k \quad (2)$$

$\Psi$  is an orthogonal wavelet associated with the approximation multiresolution. A signal  $f$  of  $L^2$  breaks up in

$$f(t) = \sum_{j,n \in Z} \langle f, \Psi_{k,n} \rangle \Psi_{k,n}(t) \quad (3)$$

$$= \sum_{n \in Z} \langle f, \varphi \rangle \Psi_{k,n} + \sum_{k \leq j, n \in Z} \langle f, \Psi_{k,n} \rangle \Psi_{k,n}(t) \quad (4)$$

where  $f$  is related to orthogonal scale of the multiresolution [12,14].

A theorem of Mallat and Meyer permits to construct an wavelet from the function of ladder [16].

Above one deducts the equation of ladder that count of coefficients  $a_1[n]$  and  $d_1[n]$  of a signal in  $V_j$  and  $W_j$  from the coefficients  $a_0[n]$  in  $V_{j-1}$  makes himself then by application of the  $h$  filters and  $g$  by an under-sampling:

$$a_1[n] = a_0 * h_1[2n] \text{ and } d_1[n] = a_0 * g_1[2n]. \quad (5)$$

while putting :

$$h_1[n] = h[-n] \quad \text{and} \quad g_1[n] = g[-n]. \quad (6)$$

Coefficients  $h$  of  $g$  and are given by equations of scale:

$$\phi(t) = \sqrt{2} \sum_{-\infty}^{+\infty} h[n] \phi(2t-n) \quad (7)$$

$$\Psi(t) = \sqrt{2} \sum_{-\infty}^{+\infty} g[n] \Psi(2t-n)$$

very well, in domain frequencies:

$$\hat{\phi}(2w) = \frac{1}{\sqrt{2}} \sum_{-\infty}^{+\infty} \hat{h}[w] \hat{\phi}(w) \quad (8)$$

$$\hat{\Psi}(2w) = \frac{1}{\sqrt{2}} \sum_{-\infty}^{+\infty} \hat{g}[w] \hat{\Psi}(w)$$

Inversely, the reconstruction of  $a_0[n]$  from  $a_1[n]$  and of  $d_1[n]$  makes himself while inserting a zero then while doing between every sample the sum of convolution with duals filters  $h_2$  and  $g_2$  partners to equations of ladders duals:

$$a_0[n] = z(a_1) * h_2[n] + z(d_1) * g_2[n] \quad (9)$$

where the  $z$  operator represents the insertion of zero [17].

Coefficients of  $h_2$  and  $g_2$  are given by equations of ladder:

$$\phi(t) = \sqrt{2} \sum_{-\infty}^{+\infty} h_2[n] \phi(2t-n)$$

$$\Psi(t) = \sqrt{2} \sum_{-\infty}^{+\infty} g_2[n] \Psi(2t-n)$$

very well, in domain frequencies:

$$\hat{\phi}(2w) = \frac{1}{\sqrt{2}} \sum_{-\infty}^{+\infty} \hat{h}_2[w] \hat{\phi}(w) \quad (10)$$

$$\hat{\Psi}(2w) = \frac{1}{\sqrt{2}} \sum_{-\infty}^{+\infty} \hat{g}_2[w] \hat{\Psi}(w) \quad (11)$$

It is by this algorithm that one evaluates the values of the functions of scale and the wavelet. Indeed, the coefficients of a wavelet or a function of scale are all null except 1 in its own resolution. The algorithm of rebuilding gives coefficients in the finest resolutions. With high resolution, one compares the coefficients to samples [18,19].

#### ▪ Thresholding

Wavelet thresholding is the basis of wavelet based noise reduction. For a function  $f$  with gaussian noise  $y_i = f(t_i) + \sigma \varepsilon_{i(i \in N)}$  this means that the function  $f$  is restored [19,21].

Two types of thresholding were proposed by Donoho and Johnstone [20].

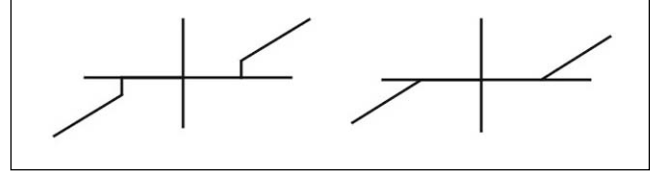
\* Hard thresholding is a simple "keep or kill" selection. All wavelet coefficients below a threshold  $\lambda$  are zeroed.

$$d_{j,k} \rightarrow \begin{cases} 0 & \text{if } |d_{j,k}| < \lambda \\ 1 & \text{if } |d_{j,k}| > \lambda \end{cases} \quad (12)$$

\* Soft thresholding shrinks the coefficients towards zero:

$$d_{j,k} \rightarrow \begin{cases} 0 & \text{if } |d_{j,k}| < \lambda \\ d_{j,k} - \text{signe}(d_{j,k}) \cdot \lambda & \text{if } |d_{j,k}| > \lambda \end{cases} \quad (13)$$

The two shrinkage methods are displayed in Figure 2. The most important step is now a proper choice of the threshold  $\lambda$ . Donoho and Johnstone showed that a universal threshold  $\lambda = \sigma \sqrt{\frac{2 \log N}{N}}$  where  $N$  is the sample size and  $\sigma$  the scale of the noise on a standard deviation scale [22].



**Figure 2:** Soft and hard thresholding.

The overall procedure of noise suppression consists of a wavelet transform (WT) which yields the wavelet coefficients  $c_{j,k}$  thresholding of these coefficients followed by an inverse wavelet transform (IWT) which restores the original spectrum.

### 3- SIMULATION RESULTS

In this section, we present the resultat in AWGN channel, the modulation used BPSK, a convolutional encoder used (2,1,2) when the number of input is 1, the number of output is 2. And in decoder, when the number of input is 2, the number of output is 1.

A convolutional decoder use Viterbi algorithm with soft decision, the length of constraint is 3, the wavelet used is the wavelet of Haar [22].

The Haar function is a simple step function a

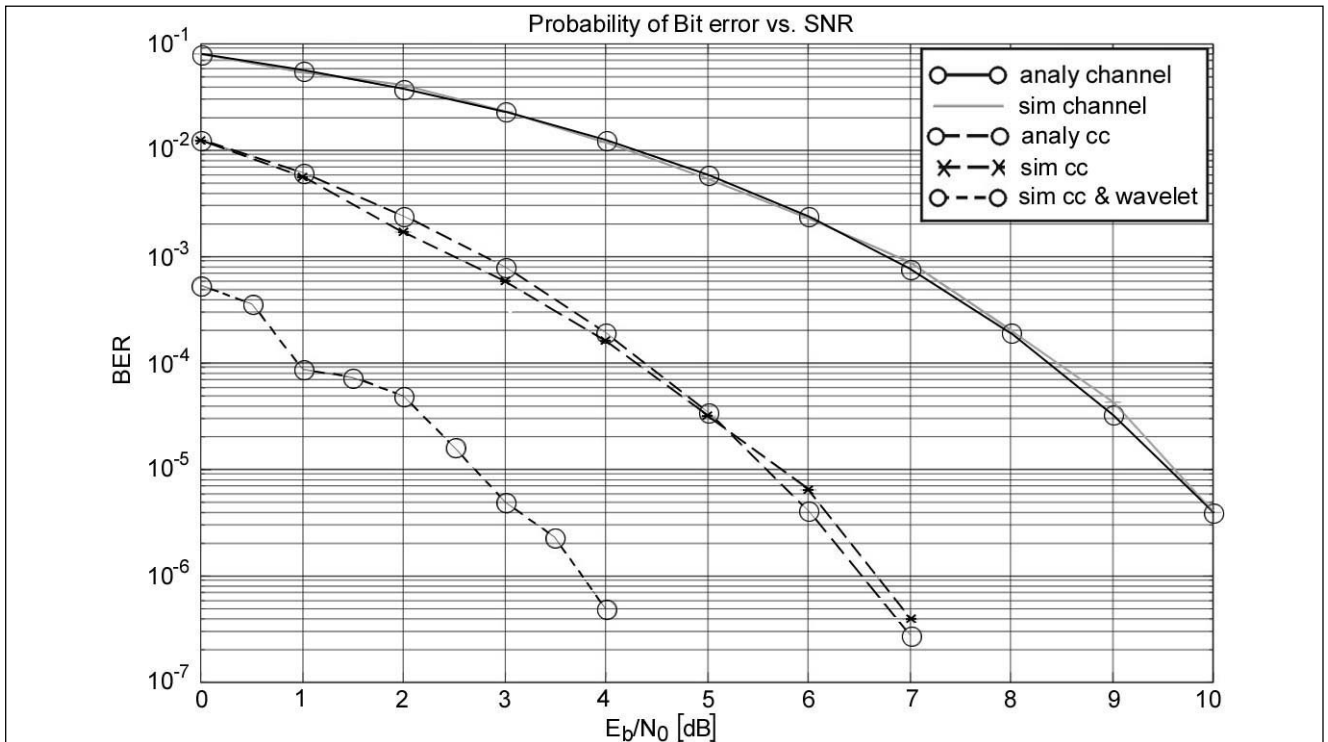
$$\Psi(x) = \begin{cases} 1 & 0 \leq x \leq \frac{1}{2} \\ -1 & \frac{1}{2} \leq x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

The level of decomposing with discrete wavelet is 2. A Soft thresholding is used.

The simulation results are shown from figure 3.

For valid simulation we made a comparison between a channel uncoded simulates and analytical, thus with for a channel codes [23], according to this figure one obtains the same performance, we can see that using convolutional code with discrete wavelet give better performance than convolutional code signal. The result is given in this table:

<b>BER</b>	$10^5$	$10^6$
<b>Es/No</b>	2.7	3.7



**Figure 3:** Performance of the proposed method.

We can obtain the better performance when we take the length of constraint 7 or 8.

### CONCLUSION

In this paper, we have developed a new approach to solve the problem of spectral occupation, this approach is association of discrete wavelet with convolutional code, the wavelet is used to denoising the receiver signal also, with the place of the second coding who decrease the redundancies, the performance of this approach is applied in AWGN channel.

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