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# LOCAL DETERMINATION OF VELOCITY AND DISPERSIVITY IN GROUNDWATER FLOW

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**Abstract**

Velocities and dispersivities are both pre-requisite in view to simulate tracer or contaminant spreading in the field as a dispersion phenomenon. Moreover they should be determined under field conditions. According to these premisses, we conducted two experiments to perform their measurements. The first one consisted of injecting water in the flow field from a well and then observing evolution towards the new steady state. The hydraulic diffusivity was evaluated by fitting the experimental heads $h(r,t)$ to the computed ones. Once the injection cutoff we supposed all water discharging in the aquifer originates from the well in view to deduce the hydraulic conductivity and the the specific yield . Then the velocity was computed directly on use of Darcy's equation. The second experiment is a single well injection test with two observation wells. It was monitored by measuring the electrical resistivity of the salt tracer in the piezometers. Horizontal dispersivity is determined by adjusting experimental and numerical data.The value thus obtained is close to the one estimated with the analytical models. Transverse dispersivity is computed with the semi-analytical formulae.

***Keywords**: Groundwater, dispersion, pollution, tracer injection.*

**Résumé**

Aussi bien le champ des vitesses que la dispersivité sont nécessaires pour simuler la dispersion d'une substance dissoute dans l'eau d'une nappe aquifère. On se propose de les collecter à partir de deux expériences préalables conduites avec un dispositif expérimental constitué d'un puits d'injection et de deux piézomètres. Lors de la première expérience, on assimile l'injection brutale d'eau claire dans le puits à une condition initiale en échelon dont la réponse indicielle est mesurée à l'endroit des piézomètres. La diffusivité hydraulique est déterminée en ajustant la charge hydraulique fournie par le modèle d'écoulement à celle mesurée aux points d'observation. La perméabilité et la porosité efficace s'en déduisent en supposant qu'après arrêt de l'injection, toute l'eau débitée dans la nappe provient de la vidange du puits d'injection. La vitesse d'écoulement est alors donnée par application de la loi de Darcy. La seconde expérience est réalisée en couplant injection de saumure et injection d'eau claire et en mesurant la résistivité de l'eau aux points d'observation. La dispersivité longitudinale est alors estimée en identifiant la réponse indicielle à la courbe fournie par le modèle. La valeur obtenue est de l'ordre de grandeur de celle donnée par les modèles analytiques. La dispersivité transversale est, elle, calculée à l'aide de formules semi-analytiques.

***Mots clés**: Hydrogéologie, dispersivité, pollution, traçage.*

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**ملخص**

إن السرعة والتشتت شرطان أساسيان في المعاينة لتقليد مرسام أو انتشار ملوث في حقل. وتبعا لهذه المقدمات قمنا بتجربتين لإنجاز القياسات لهما. لقد تم في التجربة الأولى حقن ماء في الحقل الجاري من بئر ثم ملاحظة التطور بعد ذلك في اتجاه حالة الاستقرار الجديدة. و تم حساب تقييم السنة K/ϕ (التوصيل الهيدروليكي K إلى المردودية النوعية ϕ) بواسطة تناسب المقدمة التجريبية h (r,t) إلى تلك المحسوبة. عندما يتوقف الحقن، نفترض أن كل الماء المفرغ في الحقل الجاري نشأ من بئر المعاينة لتمييز K من $\phi_c$ ثم نحسب السرعة مباشرة باستخدام معادلة دارسي. أما في التجربة الثانية، فقد تم اختبار بئر منفرد باختبار نبضي مع بئرين للملاحظة. وتم التحكم فيه بقياس التوصيل الكهربائي للآثار الملحية في بئر الملاحظة. وعين التشتت الأفقي بواسطة ضبط المعطيات أو المعلومات التجريبية والعددية. إن القيمة المحصل عليها تماثل تلك المقدرة بتحاليل النماذج حسب التشتت العابر بواسطة تحليل فريد.

***الكلمات المفتاحية**: علم النابيع، إنتشار، تلوث.*

A large body of experimental studies showed that (1) velocities based on Darcy's law have large inherent uncertainties associated with both gradient determination and uncertainty in hydraulic conductivity [1]; (2) dispersivity is elusive and its estimates even in field conditions is time-dependant [2,3]. These non-constant values of dispersivity means a scale – dependance which may be due to incomplete spatial averaging or to the size of the sample volume as discussed by [4, 5]. Recent works [6,7] invoked and validated the notion of macro dispersivity [8] as a useful conceptual model to account for the influence of the distance data. They underline the limitations in extending classical theory of hydrodynamic dispersion missing spreading induced by heterogeneity at the upper scale [9,10].

Herein, we will describe a two-steps experiment to (1) estimate velocity by use of Darcy's equation and (2) evaluate horizontal dispersivity by interpreting the single-well injection test with two piezometers. Accordingly, dispersivity thus computed could be compared with its assymptotic value evaluated from the geostatistical model of dispersion described in [7] to validate the homogeneous nature of the aquifer [11] .

## PRESENTATION OF THE SITE

The test site lies in the Campus of University of Louvain–la-Neuve in Belgium. It is located in the aquifer of the "Plateau de Lauzelle" in the south region of Wavre. Groundwater is contained in the unconsolidated sand deposits of the Bruxellian strata and in the Landenian. The thickness of the aquifer is increasing towards the North. In the neighborhood of the site, the aquifer is unconfined. Elsewhere, a silty sand layer may form an impervious cover confining the main aquifer.

## AQUIFER RESPONSE TO A STEP INJECTION

Analysis of the unsteasteady state expressing the aquifer responses following a step injection is the core of the first test. The measurements are monitored at the well and at two observation points located five meters and fiveteen meters away. In table 1 is reported the evolution of the head $h(r,t)$ measured at a radial distance $r$ from the center of the well for different times $t$. The water is injected at a rate of five cubic meters per hour since a starting time $t$ = 0.

| Time mn | h(5,t) m | Time mn | h(10,t) m |
|---|---|---|---|
| 0 | 26.10 | 0 | 25.84 |
| 29 | 25.98 | 32 | 25.83 |
| 152 | 25.89 | 154 | 25.80 |
| 544 | 25.84 | 545 | 25.74 |
| 1954 | 25.67 | 1955 | 25.65 |
| 2819 | 25.64 | 2820 | 25.62 |
| 5684 | 25.59 | 5685 | 25.58 |
| 7079 | 25.56 | 7080 | 25.55 |

**Table 1a:** Water levels measurements at $r$ = 5 m and $r$ = 15 m.

| Time mn | h(r$_w$,t) m | Time mn | h(r$_w$,t) m |
|---|---|---|---|
| 0 | 27.81 | 6.67 | 26.39 |
| 0.25 | 27.51 | 7.17 | 26.36 |
| 0.58 | 27.31 | 8.25 | 26.31 |
| 1.08 | 26.98 | 9.50 | 26.26 |
| 1.25 | 26.91 | 11.17 | 26.21 |
| 1.67 | 26.81 | 13.75 | 26.16 |
| 2.25 | 26.71 | 17.81 | 26.11 |
| 2.75 | 26.66 | 27.00 | 26.06 |
| 3.34 | 26.61 | 37.25 | 26.02 |
| 4.08 | 26.56 | 148.17 | 25.87 |
| 4.92 | 26.51 | 544.41 | 25.75 |
| 5.67 | 24.46 | 1954.41 | 25.63 |

**Table 1b:** Water levels measurements at the well.

### Analysis of test data

The injection test is governed by the polar coordinate form of the flow equation [12] :

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rh\frac{\partial h}{\partial r}\right)=\frac{\phi_e}{K}\frac{\partial h}{\partial t} \tag{1}$$

where $\phi_e$ refers to a storage parameter termed specific yield and defined as the volume of water an unconfined aquifer releases from storage per unit surface area of aquifer per unit decline in the water table. $K$ is known as the hydraulic conductivity; it expresses the ease with which a fluid is movinng through a porous medium.

Equation (1) is more tractable under the form :

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\ \frac{\partial(h^2/2)}{\partial r}\right]=\ \frac{\phi_e}{K}\frac{\partial h}{\partial t} \tag{2}$$

subject to the following initial conditions :

$$h(r_w,\ 0)=h_w$$

$$h(r_1,\ 0)=h_1$$

$$h(r_2,\ 0)=h_2$$

where $r_w$ is the radius of the well, $r_1$ and $r_2$ the radial distances of the piezometers $P_1$ and $P_2$ measured from the center of the well.

Into solving equation (2), we may determine only the ratio $\phi_e/K$ . In view to segregate $\phi_e$ and $K$, we have to provide an additional equation. This may be derived if we suppose that at the well :

* the inflow is given by Darcy's law:

$$q=-\ 2\pi r h_w K\left(\frac{\partial h}{\partial r}\right)_w$$

* after the injection stopped this inflow is due to the dewatering of the well i.e :

$$q=-\ \frac{d}{dt}\ (\pi r_w^2 h_w)=-\pi r_w^2\frac{dh_w}{dt}$$

The seeking equation is then obtained by equating the lattice two expressions of $q$ :

$$\frac{2K}{r_w}h_w\left(\frac{\partial h}{\partial r}\right)_w\ =\ \frac{dh_w}{dt}$$

Thus we establish finally the equations to be solved numerically :

$$\frac{dh_i}{dt}\ =\ \frac{K}{2r_i\phi_e}\left[\frac{\partial}{\partial r}\left(r\frac{\partial h^2}{\partial r}\right)\right]_i \tag{3}$$

$$\frac{dh_w}{dt}=\frac{2K}{r_w}h_w\left(\frac{\partial h}{\partial r}\right)_w \tag{4}$$

The starting point of the calculations is to get an initial guesses of $R$ the radius of influence of the well, the hydraulic conductivity and the specific yield.

Let :

- $R$ = 60 m
- $K = 2.75\times10^{-6}$ m/s
- $\phi_e$ = 5%

According to the initial conditions :

- $h_w = 25.41$ m
- $h_1 = 25.61$ m
- $h_2 = 25.44$ m

and a space increment $\Delta r$ equal to 2.5 m, we re-estimated the ratio $\phi_e/K$ to adjust the experimental curves to the model responses $h(r,t)$.

Determination of the fitting values is realized via a digital model [4] which allows to compute the fitting ratio i.e. $\phi_e/K = 2.10^3$ , then the value of the hydraulic conductivity $K = 2.10^{-5}$ m/s and the specific yield $\phi_e$ = 4% are evaluated by use of the additional equation.

This value of $K$ agrees with the one obtained from the pumping tests wheras those of $\phi_e$ is slightly different. This difference is imputed 1) to the difference between the governed equations, 2) to the linearization technique.

Moreover, measurements performed in the well were very noisy. One way to remediate this drawback would be to consider $h(r_w,t)$ as a missing initial condition and to use invariant imbedding approach to evaluate it. That is a subject of a future paper.

## SINGLE WELL INJECTION

A salt solution is injected in the flow from previous well. The migration of the tracer is due almost to the inflow gradient which is greater than the natural groundwater gradient.

The theoretical analysis of the levels concentration data monitored at the observation wells are based on the physics of flow of tracer around a well. Under the specific conditions of the experiment, the dispersion equation [9, 13] is a suitable model to compute a concentration response $C(r,t)$ in view to determine the local longitudinal dispersivity $\alpha_r$. The following equations are requisite :

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{\rho D_r r \frac{\partial}{\partial r}\left(C/\rho\right)\right\} - \frac{1}{r}\frac{\partial}{\partial r}\left(rV_r C\right) = \frac{\partial C}{\partial t} \qquad (4a)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rh\frac{\partial h}{\partial r}\right) = \frac{\phi_e}{K}\frac{\partial h}{\partial t} \qquad (4b)$$

$$V_r = -\frac{k\rho g}{\phi_e \mu}\frac{\partial h}{\partial r} \qquad (4c)$$

where $C(r,t)$ is the concentration of the tracer spreading at a radial groundwater pore velocity $V_r$ ; $\rho$ , $g$ and $\mu$ are respectively its mass density, its viscosity and the acceleration of gravity ; $k$ is the permeability of the aquifer i.e. a geometrical characteristic of the aquifer. $D_r$ is the hydrodynamic coefficient of dispersion which lumps in a single term transport by diffusion and transport by mechanical dispersion.Their respective contributions are depicted by the Peclet number i.e. a ratio expressing advective to diffusive transport [7]. In a radial flow field, it takes the form : $D_r = \alpha_r V_r + D^*$. $\alpha_r$ stands for the dispersivity i.e. a length characteristic of the medium [13].

According to the actual single - well injection test, mechanical dispersion dominates the mixing process. The prescripted conditions are :

$$C(r_w,t) = C_0 \quad , \qquad 0 \le t \le T$$
$$C(r_w,t) = 0 \quad , \qquad t \ge T$$
$$C(r_w,\infty) = 0$$
$$C(r_w,0) = 0$$
$$h(\infty,t) = h_0$$
$$h(r,0) = h_0$$

where $C_0$ is the initial concentration of the tracer imposed during the laps of time $T$, and $h_0$ the initial steady-state head previous the test.

### Steady state conditions

The salt solution is injected simultaneously with clear water at a respective rates $Q_1$ = 0.96 m$^3$/h and $Q_2$ = 8.38 m$^3$/h during ninety minutes. Then clear water is injected solely at a rate $Q = Q_1 + Q_2 = 9.31$m$^3$/h.

The data level concentrations monitored at $P_1$ and $P_2$ are reported in table 2. There the heads at steady state are respectively $h_1 = 28.28$ m and $h_2 = 27.93$ m h$_2$ = 27.93 m whereas at the well $h_w = 31.7$ m.

| Time mn | Resistivity Ω/cm | Time mn | Resistivity Ω/cm |
|---|---|---|---|
| 0 | 1393 | 14430 | 1399 |
| 12660 | 1436 | 14565 | 1424 |
| 12885 | 1439 | 14850 | 1509 |
| 13065 | 1391 | 15480 | 1489 |
| 14070 | 1481 | 16740 | 1526 |
| 14250 | 1397 | 19230 | 1701 |

**Table 2a:** Resistivity values ρ($r$,$t$) at $r$ = 15 m.

| Time mn | Resistivity Ω/cm | Time mn | Resistivity Ω/cm |
|---|---|---|---|
| 0 | 2150 | 2640 | 757 |
| 1170 | 1991 | 2760 | 884 |
| 1245 | 1963 | 2880 | 955 |
| 1410 | 1640 | 3000 | 1010 |
| 1485 | 1622 | 3180 | 1003 |
| 1530 | 1593 | 3360 | 974 |
| 1695 | 1524 | 3930 | 1085 |
| 1905 | 1490 | 4440 | 1199 |
| 1980 | 1332 | 4710 | 1205 |
| 2040 | 997 | 5550 | 1275 |
| 2160 | 632 | 6900 | 1325 |
| 2220 | 529 | 8220 | 1332 |
| 2340 | 527 | 8850 | 1341 |
| 2460 | 576 | 9720 | 1366 |
| 2520 | 628 | 11220 | 1388 |

**Table 2b:** Resistivity values ρ($r$,$t$) at $r$ = 5 m.

We may notice in table 2a, that the resistivity monitored at $P_2$ is quasi-time-insensitive in the laps of time devoted to the measurements. The variability observed is accounted to noise monitoring. On the contrary, at point $P_1$, the values of the resistivity sharply decline since time $t$ = 2040 mn due to the arrival of the cloud of tracer. This is in agreement with the estimation of the transit time given in [14] which is roughly proportionnal to $r^2$ for the specific conditions of the actual site. The transit times thus estimated are respectively:

- $t_1 = 25\ h$ = 1500 mn for $P_1$,
- $t_2 = 225\ h$ = 13500 mn for $P_2$.

It express the mean-time [15] to a particle tracing to travel between the well and observation points $P_i$.

### Determination of the longitudinal dispersivity

The successive steps towards the determination of the longitudinal dispersivity are :

- (1) to discretize equation (4),
- (2) to solve the discrete analogs for a given value of $\alpha_r$,
- (3) to adjust the numerical and the experimental curves $C(r,t)$,
- (4) repeat steps 2 and 3 till adequation is obtained between the two curves.

The application of this chart to the actual data gives a value of the longitudinal dispersivity equal to 69 cm. This value is close to the one obtained from the semi-analytical formula of Fried [9] in the case of a groundwater aquifer of thickness $b$ :

$$\alpha_r = \frac{r\frac{\partial C}{\partial t} + A\frac{\partial C}{\partial r}}{A\frac{\partial^2 C}{\partial r^2}}$$

where the quantity $A = 2\pi Q b \phi_e$ is evaluated from the data of the test whereas the derivatives $\partial C/\partial t$, $\partial C/\partial r$ and $\partial^2 C/\partial r^2$ are approximated from the data of table 2. Indeed such approximation means that for a small increment $\Delta t$, the tracer moves with the velocity stated by Darcy's law and corrected for flow through the pores. The dispersivity $\alpha_r$ thus estimated is equal to 65 cm.

### Determination of the transversal dispersivity

Despite its importance to evaluate transverse macrodispersivity from the relations derived by Gehlar and Axness [8,11], the local transverse dispersivity is just estimated with the current formulae [9] according to the characteristic length $d_{10}$ of the porous medium: $\alpha_T = \frac{0.5 d_{10}}{15}$. For $d_{10} = 0.3$ cm , $\alpha_T = 0.01$ cm.

## CONCLUSIONS

Field observations of dispersion under the controlled conditions reported above allowed an evaluation of the longitudinal dispersivity in agreement with the range encountered in the field [2]. The dispersivity ratio $\alpha_L/\alpha_T$ is very high. It is not consistent with the statement reported in the litterature [16,17] which indicates that for large range of velocities this ratio is close to 20. We suspect this is due to the semi analytical formula used to estimate the transverse dispersivity. This value must be removed once renewal estimations of longitudinal dispersivity were performed with the dispersivities equations provided in [1, 2, 18]. This is crucial since [8,11,19] state that longitudinal macrodispersivity is often convectively controlled whereas transverse macrodispersivety is determined by the local dispersion. Comparing the values obtained in the tracer test with those estimated with the stochastic model of dispersion may serve in the future to validate the hypothesis of aquifer-homogeneity inferred by the grain size distributions.

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