STRENGTH AND DUCTILITY ENHANCEMENT IN CONFINED REINFORCED CONCRETE COLUMNS
Proposed Stress Strain Model

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Résumé

Le comportement flexionnel des éléments structuraux en béton armé est intimement lié aux lois contraintes déformations des matériaux constitutifs. Le comportement du béton confiné est l'outil essentiel qui devra permettre au concepteur de calculer et d'adopter, par utilisation de l'armature transversale, la ductilité de courbure exigée dans les sections critiques dissipatives d'énergie. Dans ses prescriptions actuelles, le RPA 99 enregistre un déficit réglementaire, notamment l'absence d'un modèle de confinement. L'objectif de cette étude est de développer un modèle analytique prenant en compte le mécanisme de confinement dans les poteaux en béton armé. Le modèle proposé est basé sur des résultats expérimentaux issus de différents travaux réalisés durant les deux dernières décennies. Le modèle tient compte des différents paramètres influents, en particulier, le rapport volumétrique des aciers transversaux, les caractéristiques mécaniques de l'acier et de béton et le coefficient efficace de confinement.

***Mots clés**: Béton confiné, Ductilité, Modèles de confinement, Gain de résistance, Gain de déformation.*

Abstract

In order to estimate the flexural behavior of reinforced concrete members, the stress strain behavior of the constituent materials must be well established. The behavior of confined concrete is important to the designer in order to determine the quantity of the confining steel required in reinforced concrete column sections to achieve the ultimate curvatures required in seismic design for ductility. The confinement steel requirements for confined concrete are subject of deficiency in current seismic code RPA 99. The objective of this study is to develop an analytical model for the confinement mechanism in reinforced rectangular concrete columns. The model is based on some existing experimental evidence and on a number of test results reported in the past 20 years. The main variables examined in the model include the volumetric ratio of lateral reinforcement, the characteristics of steel and concrete and the effectiveness confinement coefficient.

***Keywords**: Confined concrete, Ductility, Confinement models, Gain in stress, Gain in strength.*

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ملخص

يتعلق سلوك الخرسانة المسلحة مباشرة بقانون اجهاد - تشوه للمواد المكونة. يعتبر سلوك الخرسانة المحاطة أداة هامّة حيث تسمح للمّصمم حساب واعتماد ماطلية التّقوس المفروضة فى المقاطع الحرجة الماصة للطاقة وذلك باستعمال التسليح المستعرض الملائم. المتفحص فى القواعد الجزائرية الخاصة بتوصيات الحساب ضد الزلازل (RPA 99) يكتشف غياب النموذج الخاص بالخرسانة المحاطة. الهدف من هذه الدراسة هو تطوير نموذج تحليلي يأخذ بعين الاعتبار ميكانيزم الاحاطة في الاعمدة الخرسانية المسلحة. يرتكز النموذج المقترح على النتائج التجريبية الناجمة عن الأعمال الخاصة بالعشريتين الأخرتين. في هذه الدراسة، أخذت بعين الاعتبار العوامل المؤثرة بصفة مباشرة، منها النسبة الحجمية للتسليح المستعرض، الخصائص الميكانيكية للتسليح والخرسانة وعامل الاحاطة الفعال.

***الكلمات المفتاحية:** الخرسانة المحاطة، الماطلية، نماذج للاحاطة، الربح في المقاومة، الربح في التشوه.*

The most important design consideration for ductility in plastic hinge region of reinforced concrete columns is the provision of transverse reinforcement that confines the core of the compressed concrete [1]. Although the RPA (1999) [2] satisfies in some way the approach that correlate demand with capacity, the examination of its content shows that the aspect of confinement and its positive influence towards the enhancement of the section moment capacity is still not considered in the design of reinforced concrete columns. Though, the prediction of the ultimate behaviour of reinforced concrete columns subjected to large seismic lateral forces relies on the relationships of the constituent materials.

The objective of this is paper is to develop, a confinement stress-strain model of concrete, based on an analysis of existing experimental evidence. Several factors are taken into account, concrete strength, amount and strength of transverse reinforcement, and the distribution of longitudinal bars hence the resulting configuration of transverse reinforcement. The investigation showed that the three parameters that are significant in the development of the stress strain confined concrete relationship are the peak stress, the corresponding strain and the softening rate. Based on the available test data these factors were analyzed using regression analysis and a set of constitutive laws to asses them were provided. Experimental data was compared with both proposed and existing models. The comparison with well established models illustrates the validity of the proposed equations.

Reference	Nb	Section (mm^2)	Test specimen parameters				
			ρ_l %	f_{co} (MPa)	s (mm)	ρ_{sh} %	f_{yh} (MPa)
Sheikh *et al.* [8]	24	305×305	2.2-4.8	26.6-34.8	29-102	0.76-2.4	265-798
Moehle *et al.* [9]	8	305×305	2.44	32.3	38	1.21-2.07	440
Shah *et al.* [10]	17	ϕ 75	0	20.7-65.5	12.7-102	0.28-2.97	414-1434
Mander *et al.* [11]	15	ϕ 450	1.23-3.7	24-32	36-119	0.6-2.5	307-340
	12	150×700	1.1-3.1	28-41	25-72	1.62-7.87	310-360
Beni Assa *et al.* [12]	24	ϕ 145	0	25-85	20.4-75	1.13-4.15	909-1296
Hoshikuma *et al.* [7]	6	ϕ 200	0	18.5	12.5-150	0.39-4.66	235
	10	ϕ 500	1.01	28.8	50-300	0.19-0.58	295
	6	200×200	0	23.2	12.5-150	0.39-4.66	235
	5	500×500	0.95	24.3	40-75	1.73-4.10	295
	1	350×700	0.97	24.3	65	1.72	295
	1	300×900	1.03	24.3	67	1.74	295
	2	250×1000	0.95	24.3	75	1.77-2.45	295

Table1: Summary of some available tests on confined concrete.

1- SUMMARY OF EXISTING MODELS

Several theoretical stress-strain models for the prediction of the confined concrete have been proposed. The following section provides a summary of these analytical models that cover both rectangular and circular cross sections.

1.1- Sheikh and Uzumeri [3]

The proposed model assume that the effectively confined concrete area is less than the core area and is determined by introducing the effectiveness confinement coefficient that takes in account the distribution of longitudinal steel, the tie configuration and the spacing of ties. The complete stress-strain curve was calibrated against their own test results.

1.2- Modified Kent and Park [4]

Park *et al.* modified the original model by making an allowance for the enhancement in the concrete strength and the peak strain due to confinement. The increase in strength and the corresponding strain was assumed to be equal to $\rho_{sh} f_{yh}$. The slope of the descending part of the curve remained the same as in the original model up to a stress of 20 % of the maximum, beyond which a horizontal line represented the curve.

1.3- Shah *et al.* [5]

The analytical model derived by the authors was based on the result of experiments conducted on small cylinders loaded eccentrically. The model is based on two parameters controlling, respectively, the ascending and the descending path of the strain stress curves, which were expressed in terms of the concrete strength and a confinement index.

1.4- Mander *et al.* [6]

A unified stress strain approach applicable to both circular and rectangular transverse reinforcement was developed. A fractional expression to represent both the ascending and falling branches of the stress strain curve was proposed. The influence of various types of confinement is taken into account by defining an effective lateral confining stress, which is dependent on the configuration of the transverse and longitudinal reinforcement. This approach was similar to the one adopted by Sheikh and Uzumeri.

1.5- Hoshikuma *et al.* [7]

The stress strain model was based on the results of a series of compression loading tests of reinforced concrete columns with various shapes and reinforcement arrangements so as to cover practical bridge column sections designed in Japan. The experimental results have shown that the three parameters that define the proposed model - peak stress, peak strain and deteriorating rate - were significant factors for the stress strain curve of confined concrete.

2- EXPERIMENTAL INVESTIGATION

Experimental evidence of a total of 131 compressed reinforced concrete columns confined by either rectangular or circular transverse lateral steel was selected and investigated herein and used to determine a representation of the stress strain relationship of confined concrete. A summary of this previous experimental work, with details of specimens and variables considered is given in Table 1.

3- MODELING OF STRESS STRAIN RELATIONSHIP OF CONFINED CONCRETE

Experimental evidence shows that the stress strain curve of confined concrete is made up of three parts: the ascending branch, falling branch and the sustaining branch. A typical stress strain curve of confined concrete determined by the proposed model is schematically shown in figure 1.

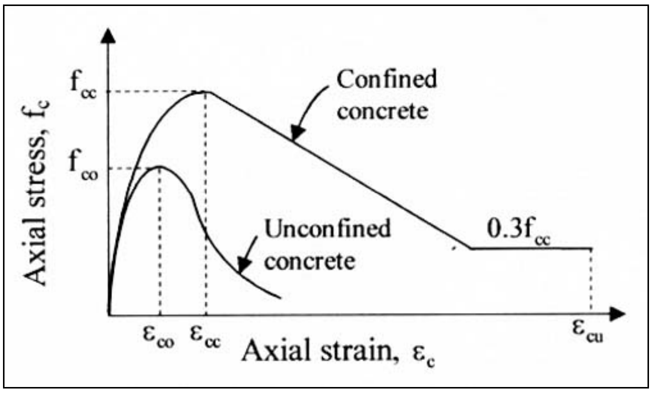

Figure 1: Proposed Stress Strain curve for confined normal strength concrete.

The ascending part of the curve is formulated by adopting Popovics [13] equation, also adopted by Mander *et al.* [6] which is as follows:

$$f_c = \frac{f_{cc} x n}{n - 1 + x^n} \qquad (1)$$

where: $x = \dfrac{\varepsilon_c}{\varepsilon_{cc}}$ and $n = \dfrac{E_c \varepsilon_{cc}}{E_c \varepsilon_{cc} - f_{cc}}$ (2)

with $E_c = 11000\sqrt[3]{f_{co}}$ [14], concrete modulus of elasticity

Examination of the coordinates of points on the line shows that the equation for the falling branch of Figure 1 may be written as:

$$f_c = f_{cc} - E_{soft}\left(\varepsilon_c - \varepsilon_{cc}\right) \geq 0.3 f_{cc} \qquad (3)$$

where E_{soft} is the value that controls the slope of the descending branch.

It is clear that the factors controlling the stress strain relationship of confined concrete materialized by equations 1, 2 and 3, are: the maximum compressive stress f_{cc}, the corresponding strain ε_{cc} and the softening rate E_{soft}.

Tests have shown that the residual stress following the falling branch is 20 to 30 % the peak stress [3, 4, 15]. In this study the sustained stress is assumed to be 30 % the peak stress.

The stress strain curve is terminated at the ultimate compressive strain ε_{cu}. Scott *et al.* [16] observed that it is reasonably conservative to define the limit of useful concrete compressive strain ε_{cu}, where the first hoop fractures occurs due to serious buckling of longitudinal bars. Recently, Bing *et al.* [17] proposed values of the maximum concrete strain according to the nature of concrete strength and the shape of the cross section. Due to the shortage of data concerning the experimental values of the compressive strain corresponding to hoop fractures, values proposed by Bing *et al.* [17] have been adopted in the proposed model and are as follows:

* For circular confinement:

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + \left(1.145 - 1.48 f_{co}\right)\sqrt{\frac{f_l'}{f_{co}}} \qquad (5)$$

with: $f_l' = 0.5 k_e \rho_{sh} f_{yh}$.

* For rectangular confinement:

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + \left(122.5 - 0.92 f_{co}\right)\sqrt{\frac{f_l'}{f_{co}}} \qquad (6)$$

with: $f_l' = 0.5 k_e \left(\rho_{shx} + \rho_{shy}\right) f_{yh}$

3.1- Evaluation of confinement parameters

The proposed model was developed on the basis of the observations derived from several experimental studies conducted in past years. A summary of the testing parameters for different data sets is given in Table 1. Tests have shown that the confinement of concrete by transverse reinforcement results in a significant increase in both the strength and the ductility of the compressed concrete.

The mathematical formulation of the stress strain relationships proposed herein involves particularly the

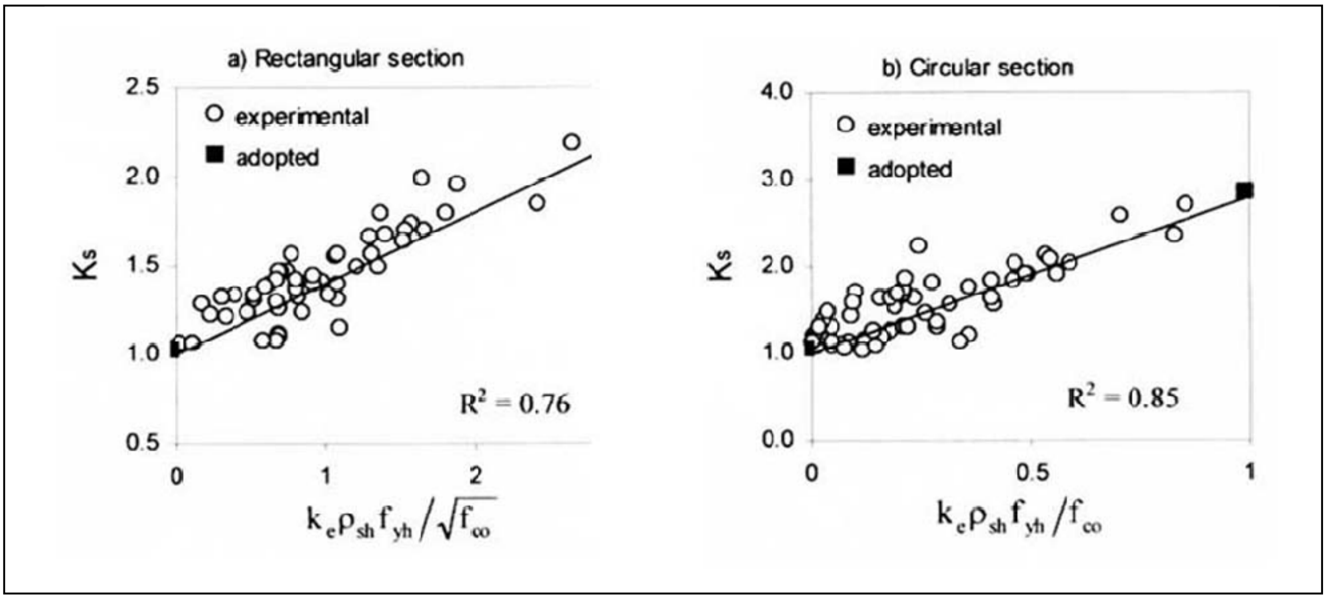

Figure 2: Strength enhacement factors and K_s relationships.

following features:

1. The gain in strength defined by the parameter K_s where $K_s = f_{cc}/f_{co}$ is the ratio of peak stress to the strength of unconfined concrete.

2. The gain in strain defined by the parameter K_d where $K_d = \varepsilon_{cc}/\varepsilon_{co}$ is the ratio of strain of confined concrete to the strain of unconfined concrete both corresponding to the peak stresses.

3. The slope of the descending branch defined by the value E_{soft}.

The three influencing parameters were first analyzed statistically using test data and the governing equations were developed.

3.2- Gain in strength

Figure 2 shows the relation between the strength enhancement factors $k_e \rho_{sh} f_{yh} / \sqrt{f_{co}}$ and $k_e \rho_{sh} f_{yh} / f_{co}$ and the parameter K_s for the rectangular and circular cross sections respectively.

It can be seen that the parameter K_s is directly proportional to the volumetric ratio ρ_{sh}, the yield strength f_{yh} of hoop reinforcement and k_e the effectiveness confinement coefficient and inversely proportional to f_{co}. The coefficient k_e introduced reflects the efficiency of reinforcement arrangement and approaches unity when the confinement pressure is near uniform as in the case of closely spaced circular spirals. This approach was first proposed by Sheikh *et al.* [3] and later adopted by Murat *et al.* [15] and Mander *et al.* [6]. In evaluating the parameter K_s, the effectiveness confinement coefficients adopted by Mander *et al.* [6] were used in this study and given as follows:

Section	Confinement type	
	Circles	spires
circular	$k_e = \dfrac{\left(1-0.5 s'/d_s\right)^2}{1-\rho_{cc}}$ (7)	$k_e = \dfrac{\left(1-0.5 s'/d_s\right)}{1-\rho_{cc}}$ (8)
rectangular	$k_e = \dfrac{\left[1-\sum_{i=1}^{m} \dfrac{C_i^2}{b_c h_c}\right] \times \left[1-0.5\dfrac{s'}{bc}\right] \times \left[1-0.5\dfrac{s'}{h_c}\right]}{1-\rho_{cc}}$ (9)	

Figure 2 illustrates that the function relating strength enhancement factor with the parameter K_s for rectangular sections can be approximated as linear. The correlation factor obtained is relatively less than unity, this is mainly attributed to the greater number of parameters that affect the behavior of rectangular section confined by different type of transverse reinforcement and may have a variable aspect ratio.

3.3- Gain in strain

The between the strain enhancement factor $k_e \rho_{sh} f_{yh} / \sqrt{f_{co}}$ and the parameter K_d is indicated in Figure 3.

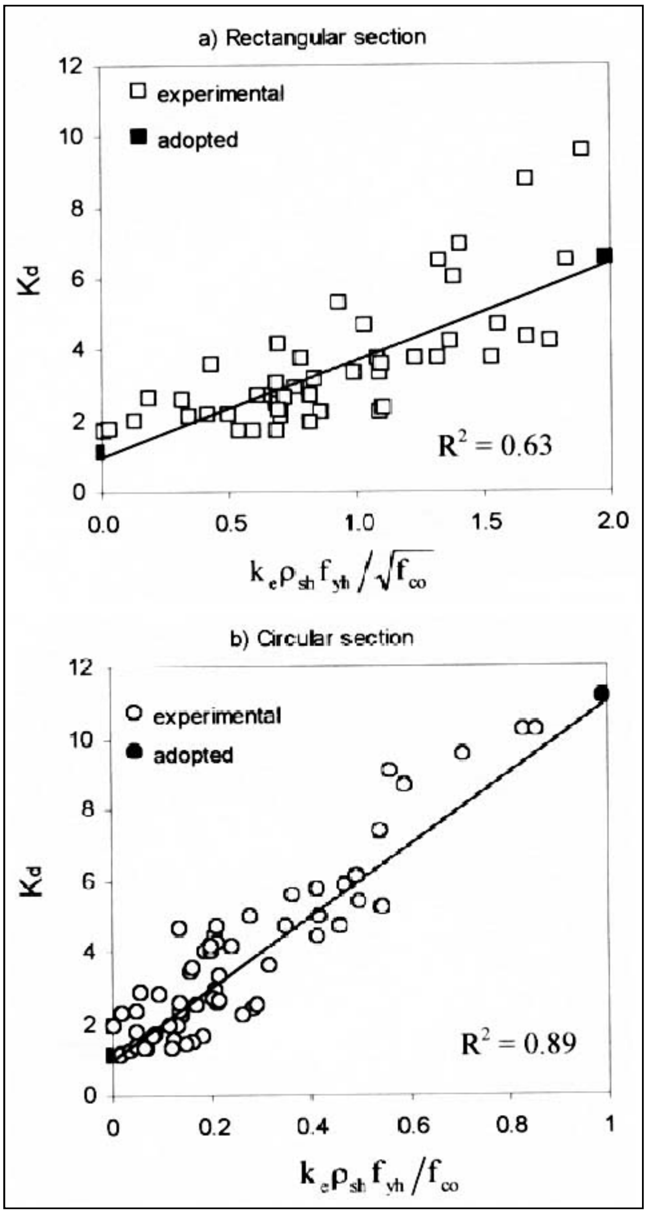

Figure 3: Strain enhancement factors and K_d relationships.

It is clear that the strain enhacement factor for the two shapes can be approximated by a linear function and it becomes more evident in the case of circular sections where the correlation factor attains a value of 0.89. The correlation factor for rectangular sections is lower and the reasons evocated above remain valid.

The derived equations based on regression analysis are given in Table 2 where it can be seen that the influence of confinement on the stress and strain enhancement is more pronounced in circular sections than in rectangular ones. The relative gain in strain / stress enhancement is about 7 times in the case of rectangular sections and about 6 times in the case of circular sections. This observation show that confinement may cause an increase in strength, but not as significant as the increase in the ductility.

Cross section	K_s	K_d
Rectangular	$1+0.4\frac{k_e \rho_{sh} f_{yh}}{\sqrt{f_{co}}}$	$1+2.7\frac{k_e \rho_{sh} f_{yh}}{\sqrt{f_{co}}}$
Circular	$1+1.8\frac{k_e \rho_{sh} f_{yh}}{f_{co}}$	$1+10.5\frac{k_e \rho_{sh} f_{yh}}{f_{co}}$

Table 2: Stress – strain enhancement factors Relationships.

3.4- Softening rate

The approach adopted for determining the factor that controls the slope of the descending branch is similar as that proposed by Hoshikuma *et al*. In this study the effect of confinement types has been considered by introducing the confinement effectiveness coefficient k_e in the calculation of the softening rate. It can be noticed from Figure 4. that the softening rate increases with the decrease of the factor $k_e \rho_{sh} f_{yh} / f_{co}^2$.

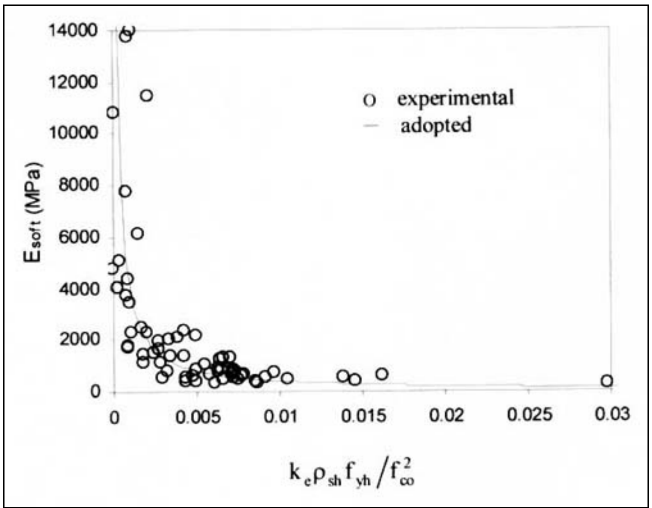

Figure 4 : Variation of softening rate with the factor $k_e \rho_{sh} f_{yh} / f_{co}^2$.

Test results indicate that this rate seems to be not affected considerably by the cross section shape. The best equation approximating the test data is given by the following relation:

$$E_{soft} = 4k_e f_{co}^2 / \rho_{sh} f_{yh} \quad (10)$$

4- COMPARISON WITH THE EXPERIMENTAL RESULTS

The five various confinement models discussed above along with the proposed model were used to predict the experimental tests results reported by different researchers (Table 1).

4.1- Peak stress and Peak strain

The experimental and analytical strength and strain values represented respectively by K_s and K_d are compared in Figure 5 considering models for rectangular and circular transverse reinforcement. A brief statistical analysis was conducted; the results, which include the mean difference (MD) between the predictions by various models and the experimental data and the associated correlation factor (R^2), are summarized in Table 3. The value of MD illustrates the tendency of a model to overestimate or underestimate the experimental data set.

The results indicated in Table 3 reveal that all models give better predictions for the peak stress than for the peak strain. Referring to the information considered in Table 1 and in Figure 5; the results obtained can be divided into three groups depending on the accuracy of the predictions.

In the case of rectangular section, the first group including Sheikh, Mander and the proposed models give a good prediction for the peak stress respectively (-1.31, 3.30 % and 5.27 % MD); (0.66, 0.47 and 0.60 R^2) but a moderate predictions for peak strain (16.52, 17.13 % and 12.66 % MD); (0.48, 0.62 and 0.56 R^2). Hoshikuma model representing the second group, give a moderate prediction for both peak stress (12.89 % MD and 0.59 R^2) and for the peak strain (-8.02 % MD and 0.47 R^2). The third group is the Modified Kent and Park model which gives a moderate prediction for the stress (10.37 % MD and 0.59 R^2) and the worst prediction for the peak strain (-56.04 % MD and 0.52 R^2).

In circular sections case, the first group includes the proposed and Shah models. The proposed model gives a very good prediction for the peak stress (5.25% MD and 0.81 R^2) and a moderate prediction for the peak strain (22.38 % MD and 0.82 R^2) where Shah model gives a poor prediction for the peak stress (36.77 % MD and 0.82 R^2) and the best prediction for the peak strain (0.4 % MD and 0.78 R^2). In the second group represented by Mander model, the prediction is good for the peak stress (10.90 % MD and 0.76 R^2) and poor prediction for the peak strain

Analytical Models	Rectangular section				Circular section			
	K_s		K_d		K_s		K_d	
	MD %	R^2	MD %	R^2	MD %	R^2	MD %	R^2
Sheikh *et al.* [9]	3.30	0.47	17.13	0.62	-	-	-	-
Mander *et al.* [13]	5.27	0.60	12.66	0.56	10.90	0.76	39.53	0.73
Hoshikuma *et al.* [8]	12.89	0.59	-8.02	0.47	42.60	0.82	78.95	0.78
Kent & Park [11]	10.37	0.59	56.04	0.52	-	-	-	-
Shah *et al.* [12]	-	-	-	-	63.77	0.82	78.95	0.78
Proposed	-1.31	0.66	16.52	0.48	5.25	0.81	22.38	0.82

Table3: Statistical results of predicted experimental and analytical values for peak stress and peak strain.

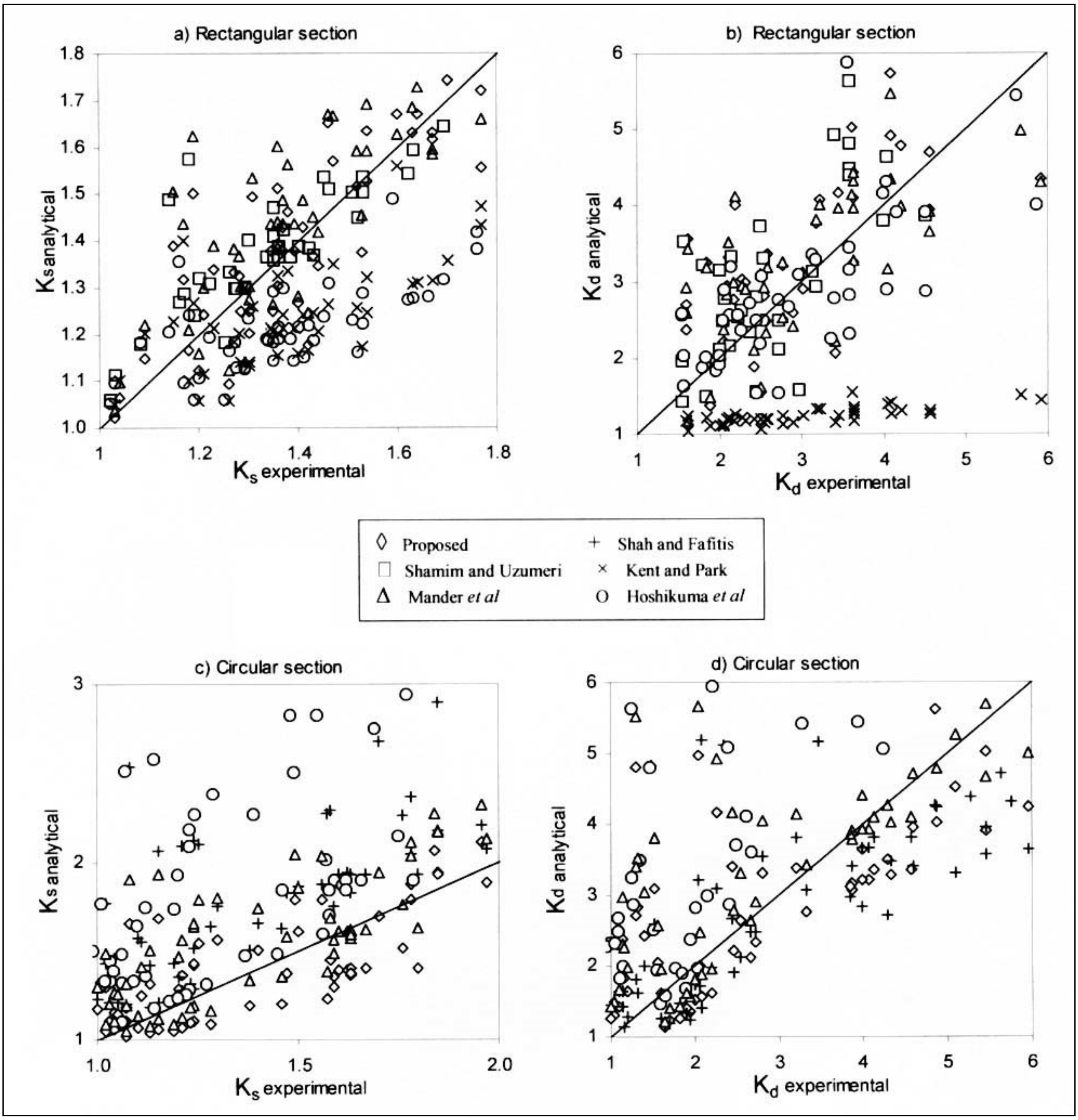

Figure 5: Comparison of experimental and analytical values of peak stresses and strains respectively for rectangular and circular sections.

(39.53 % MD and 0.73 R^2). Hoshikuma model gives the worst predictions for both peak stress (42.5 % MD and 0.82 R^2) and peak strain (78.95 % MD and 0.78 R^2).

4.2- Stress strain curves

The analytical curves derived from the considered models representing the three groups are compared with some of the experimental curves [9, 12] as illustrated in Figure 6 considering both rectangular and circular cross sections respectively for four representative columns, with different amount of lateral steel, the purpose is to see how well each model predicted the pre-peak and post-peak behavior of the experimental stress strain curves.

The analytical initial slopes of the ascending branch predicted by all models agree well with the experimental stress strain curves except for the specimen 30M50 [12] data set where the model's initial slopes were slightly stiffer than the experimental curves.

Regarding the case of rectangular sections, in almost all cases the predicted post peak portion of the curves was below the experimental curves except for the Proposed and Mander models where the analytical curves were slightly under predicted for the specimen A2 [9] and were in close agreement with experimental curve of specimen D2 [9].

In the case of circular sections, the post peak behavior

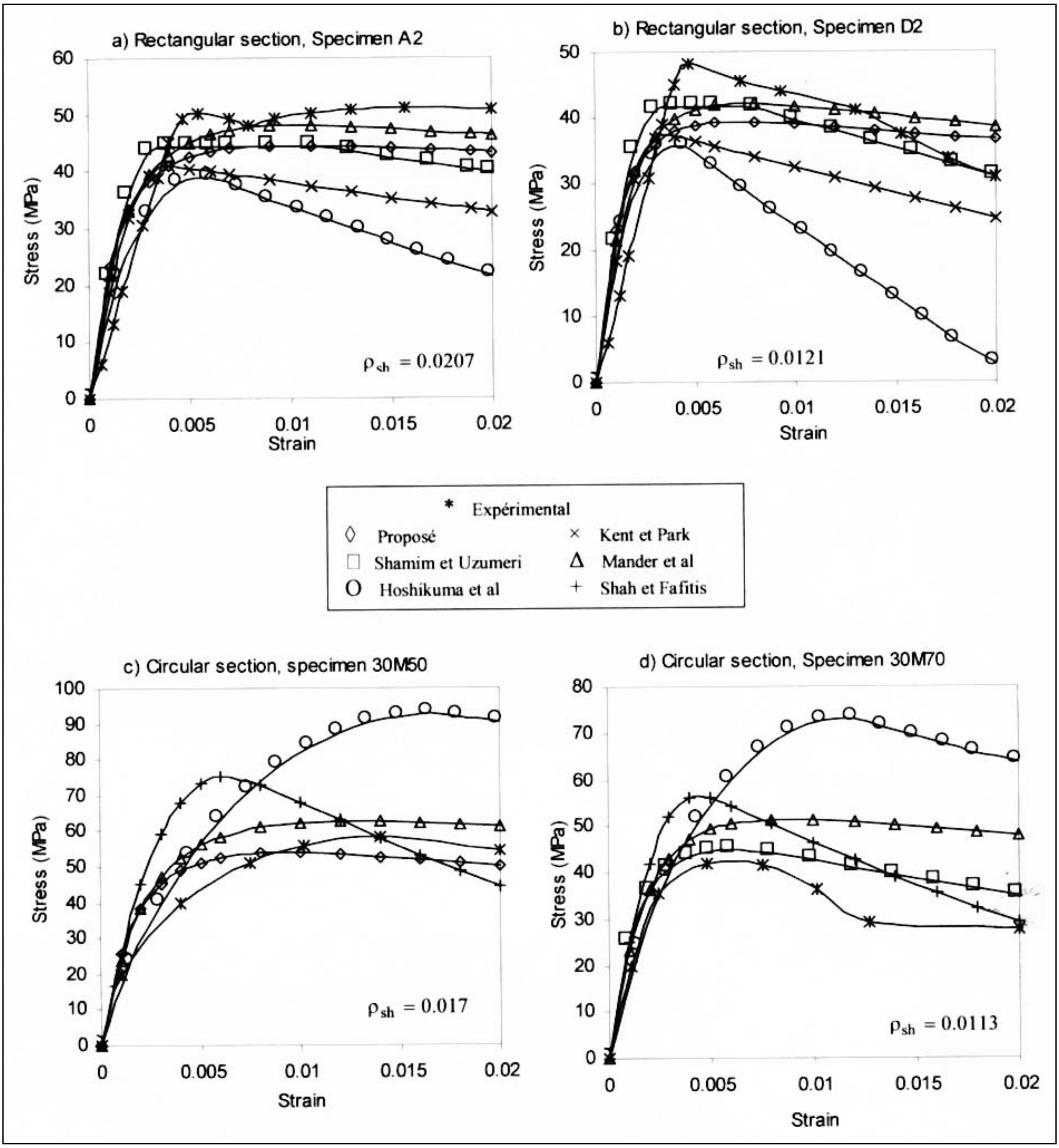

Figure 6: Comparison of experimental and predicted stress-strain curves for specimen A2 – D2 [9] et 3050 – 30M70 [12].

of the two experimental curves with different amount of lateral steel is over predicted by all the models considered except for the proposed model as clearly illustrated in Figure 6. The over prediction, is more unconservative in the case of moderate volumetric ratio.

CONCLUSIONS

A stress strain model for confined concrete was developed for both rectangular and circular transverse reinforcement on the basis of the observations issued from several experimental studies conducted in past years. The following conclusions can be drawn:

* Accurate prediction of the co-ordinates of the peak point is important, since it has a major influence on the proximity of the prediction of stress strain curve to the experimental results.

* The proposed model was established by introducing the confinement parameters namely

- the gain in strength defined by the factor K_s ;
- the gain in strain defined by the factor K_d ;

- The slope of the descending branch defined by the value E_{soft}.

* The most significant parameters affecting the form of the stress strain curve of confined concrete is the volumetric ratio of transverse reinforcement, the yield strength of confinement reinforcement, the concrete strength and the confinement effectiveness factor.

* None of the available analytical models could accurately predict the stress strain curves for the full range of the experimental data considered. However the analytical stress strain model proposed in this study was generally in closest agreement with the experimental results.

* The present RPA 99 code expression for the quantity of transverse steel is based on the of the value of the shear force and then checked by using the geometrical slenderness ratio. A detailed study using the confinement model proposed herein could result in a better assessment of the confining steel required in reinforced concrete column sections to achieve the ultimate curvatures required in seismic design for ductility.

NOTATIONS

b_c, h_c	:	concrete core dimensions measured to the center line of the perimeter tie
C	:	distance between the laterally supported longitudinal bars
d_s	:	diameter of spiral
E_c , E_{soft}	:	respectively concrete modulus of elasticity and the softening rate in the descending branch
f_{co} , f_{cc}	:	respectively compressive strength of unconfined and confined concrete
$f_l^{'}$, f_{yh}	:	respectively lateral confining stress and yield strength of transverse reinforcing steel
k_e	:	confinement effectiveness coefficient
K_s, K_d	:	confinement parameters expressing respectively the gain in stress and strain
m	:	number of laterally supported longitudinal bars
$s^{'}$	:	spacing between transverse reinforcement
ε_{co}, ε_{cc}	:	respectively the strain at peak stress of unconfined and confined concrete
ε_{cu}	:	ultimate confined concrete strain corresponding to first hoop fracture
ρ_{cc}	:	ratio of area of longitudinal steel to area of core of section
ρ_{sh}	:	ratio of volume of transverse confining steel to volume of confined concrete core
ρ_{shx} ,ρ_{shy}	:	ratio of volume of transverse confining steel to volume of confined concrete core respectively in x and y directions

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