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FREE VIBRATION ANALYSIS OF ISOTROPIC AND ORTHOTROPIC CLAMPED PLATES USING ENERGY APPROACH

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Abstract

The free vibration of isotropic, orthotropic clamped rectangular plates using the Rayleigh-Ritz energy approach has been studied. The solution of the eigenvalue problem is obtained by assuming a deflected shape in the form of series functions that satisfy the edge boundary conditions of the plate. Simplified analytical expressions for frequencies are obtained that describe the vibrational characteristics of the plate. To demonstrate the accuracy of the present approach the same plate is discritisated and analyzed using the finite element method and the corresponding frequencies are obtained. The frequencies obtained with energy approach agree closely with those of the conventional finite element method.

Keywords: *Free vibration, Isotropic, Orthotropic, Plate, Natural frequency, fundamental frequency, F.E.M, Clamped.*

Résumé

Les vibrations libres des plaques rectangulaires, orthotropes et isotropes ont été étudiées en utilisant la méthode énergétique de Rayleigh Ritz. La solution du problème aux valeurs propres est obtenue en proposant une déformée de fonction forme de séries qui vérifie les conditions aux limites de la plaque. Ensuite une expression analytique simplifiée pour le calcul de la fréquence fondamentale a été proposée. Pour démontrer la précision de la présente approche, la même plaque a été discrétisée et analysée par la méthode des éléments finis. Les fréquences par l'approche énergétique sont comparées avec celles obtenues par la méthode numérique des éléments finis et les résultats montrent une bonne concordance entre les deux méthodes

Mots clés: *Vibration libre, Isotrope, orthotrope, plaque, fréquence naturelle, fréquence fondamentale, Méthode des éléments finis, Encastrée.*

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ملخص

تمت دراسة الاهتزازات الحرة لصفيحة مستطيلة مثبتة في أربعة جوانب موحدة الخواص وأخرى متعامدة الخواص باستعمال طريقة ريلي ريتز الطاقوية. تم الحصول على حلول مسألة القيم الذاتية بافتراض شكل مشوه للصفيحة باستعمال متسلسلة توافقية تحقق الشروط الحدية للصفيحة. وتم اقتراح أيضا صيغة مبسطة للعبارة التحليلية لحساب التوتر الأساسي. للبرهنة على دقة الطريقة الحالية، نفس الصفيحة تم تكتيمها وتحليلها باستعمال طريقة العناصر المنتهية. مقارنة التوترات المحصل عليها بالطريقة الطاقوية مع التوترات المحصل عليها بطريقة العناصر المنتهية تظهر توافق مقبول بين الطريقتين.

الكلمات المفتاحية: *الاهتزازات الحرة، موحدة الخواص، متعامدة الخواص، صفيحة، تواتر طبيعي، تواتر أساسي، طريقة العناصر المنتهية، مثبت.*

The use of orthotropic material has increased during the last three decades, particularly in the aerospace and civil engineering. Examples are corrugated plates and stiffened plate structures. The need of using composite materials for plate structures intensified the search of finding simplified solutions that predict accurately the frequencies of free vibration orthotropic plates. The finite element method was applied to the free vibration of plates by assuming a displacement field model that satisfies the convergence criteria. This method leads to the calculation of the eigenvalues in a numerical sense and therefore it allows the computations after discretizing the plate domain into finite elements. The results of applying this technique can only be obtained after assuming a large number of elements. For design purposes it is usually preferred to have an idea on the period of the plate structure prior to the estimation of the maximum quantities such as the stresses that can be developed when the plate is subjected to extreme loadings that arise from blast loads. In this sense, analytical expressions of the frequencies present an important parameter rather than the numerical values obtained from the FEM.

The study of the free vibration of orthotropic plates is not a new subject. There is a number of solutions on free vibration of rectangular plates in the natural frequencies with a wide range of support conditions. The most widely known are those of Warburton [1] and Leissa [2, 3]. The work of Warbuton [1] has been extended by Hermon [4] to analyze the free vibration of rectangular orthotropic plates having either clamped or simply supported edges using the Rayleigh method. G. Aksu and R. Ali [5] applied the finite difference method to examine the free vibration characteristics of rectangular stiffened plates having a single stiffener. For conventional homogeneous, orthotropic, plate continuum, a (3-D) exact solution was developed by Srinivas and Rao [6] for bending and vibration analysis with simply-supported conditions. For clamped plates on four edges many approximate solutions have been reported. For example, for the case of all edges clamped, Sakata and Hosokawa [7] proposed a double series solution for free and forced vibration analysis. Gorman [8] analyzed the free vibration frequencies and mod shapes of clamped rectangular orthotropic plates Using the superposition method while Guttérrez *et al.* [9] studied the problem of fundamental frequency of transverse vibration of clamped rectangular orthotropic plate with a free edge hole. Numerical techniques such as the FEM have been widely applied to plate problems, Ahmadian and Sherafati Zangeneh [10] used the concept of super elements for vibration analysis of orthotropic rectangular plates. Lee [11] developed a four-node plate element by using the assumed natural strains on the basis of Reissner-Mindlin to investigate the vibrational characteristics of plates,. The finite difference method is another numerical technique that has been used for free vibration of plates. This technique was used by Karim *et al.* [12] to solve differential equation of motion of free vibration of composite plates with different boundary conditions.

The purpose of the present paper is to develop analytical expressions that predict the fundamental frequency and therefore the fundamental period of thin plate structures with clamped edges. The frequencies are obtained using several terms in the assumed shape function by applying the Rayleigh–Ritz method. Simple analytical expressions for the fundamental natural frequencies are presented for isotropic and orthotropic plates.

1- ANALYTICAL FORMULATION

The Rayleigh Ritz method is an extension of the Rayleigh method which not only provides a means of determining a more accurate value of the fundamental frequency, but it also gives approximation to the higher frequencies and mode shapes. It is always known that fundamental frequencies obtained using the Rayleigh Ritz method are always higher than the exact values, since the plate's mode shape is postulated by a finite number of terms in the shape function which inherently increases the rigidity of the plate. The accuracy of the Rayleigh Ritz method therefore depends on the selection of compatible shape functions.

If the general expressions for deflected shape (Table 1) had been used throughout the above formulation, frequencies for higher modes would have been obtained. These higher mode frequencies can be obtained by taking various integer combinations of *m* and *n*.

Consider a rectangular orthotropic plate of length *a*, width *b* and thickness *h*. For the free vibration case, the frequency equation may be derived using the Rayleigh-Ritz approach. Where the maximum strain energy of bending of orthotropic plate is

$$U_{\max}=\frac{1}{2}\int_{-b}^{+b}\int_{-a}^{+a}\left[D_x\left(\frac{\partial^2 w}{\partial x^2}\right)^2+D_y\left(\frac{\partial^2 w}{\partial y^2}\right)^2+\right.$$
$$\left.+2D_1\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2}+4D_{xy}\left(\frac{\partial^2 w}{\partial y\partial x}\right)^2\right]dxdy \qquad (1)$$

and for isotropic plate is

$$U_{\max}=\frac{D}{2}\int_{-b}^{+b}\int_{-a}^{+a}\left\{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2-\right.$$
$$\left.-2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial y\partial x}\right)^2\right]\right\}dxdy \qquad (2)$$

where the integration is performed over the plate domain.

D_x and D_y are the flexural rigidities, D_{xy} is the torsional rigidity and D_1 is the reduced Poisson's ratio and they are given by :

$$D_x=\frac{E_x h^3}{12(1-\nu_x\nu_y)} \qquad (3)$$

$$D_y=\frac{E_y h^3}{12(1-\nu_x\nu_y)} \qquad (4)$$

$$D_{xy}=\frac{G_{12}h^3}{12} \qquad (5)$$

$$D_1=\nu_y D_x=\nu_x D_y \qquad (6)$$

E_x and E_y are the Young's moduli in the *x* and y directions, ν_x and ν_y are the corresponding Poisson ratios, and G_{12} is the shear modulus.

Table1: Assumed deflected functions.

$w(x,y)$	Ref
$w(x,y)=\left(x^2-\frac{a^2}{4}\right)\left(y^2-\frac{b^2}{4}\right)\sum_{m=1.2...}^{\infty}\sum_{n=1.2...}^{\infty}A_{mn}x^m y^n$	[13]
$w(x,y)=\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)\sum_{m=1.2...}^{\infty}\sum_{n=1.2...}^{\infty}A_{mn}xy\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$	Author

The maximum kinetic energy of the plate freely vibrating with amplitude w and radian frequency ω is given by

$$T_{\max} = \frac{\omega^2}{2} \int_{-b}^{b} \int_{-a}^{a} \gamma(x, y)\, w^2\,(x, y)\, dxdy \quad (7)$$

where $\gamma(x, y)$ is the mass density of the plate material per unit area.

In Eqs. 1 and 2 the function $w(x, y)$ is the assumed deformed shape and ω is the natural frequency of the plate. The total energy of the vibrating plate is formulated as

$$\Pi p = U_{\max} - T_{\max} \quad (8)$$

$$\Pi p_{\max} = \frac{1}{2} \int_{-b}^{+b} \int_{-a}^{+a} \left[D_x \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + D_y \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + 2D_1 \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} + 4\, D_{xy} \left(\frac{\partial^2 w}{\partial y \partial x} \right)^2 - \frac{\omega^2}{2} \gamma(x, y)\, w^2(x, y) \right] dxdy \quad (9)$$

Mathematically the shape function describing the modes of vibration is assumed in the form of a harmonic series as

$$w(x, y) = \sum_{m = 0.1.2...}^{M} \sum_{n = 0.1.2...}^{N} A_{mn} X_m(x) Y_n(y) \quad (10)$$

where $X_n(x)$ and $Y_n(y)$ are appropriate shape functions along x and y axes that must satisfy the geometrical corresponding boundary conditions of the edges normal to the x and y axes respectively. A_{mn} are the unknown numerical coefficients of the functions.

The assumed displacement functions defining the deflection of the plate are given in Table 1 in the form of series functions. These functions satisfy the boundary conditions as follows

1.1- Boundary conditions

Clamped edges:

$$w = 0\,, \quad \frac{\partial w}{\partial x} = 0 \text{ at } x = 0,\ x = a \text{ for } 0 \le y \le b \quad (11)$$

$$w = 0\,, \quad \frac{\partial w}{\partial y} = 0 \text{ at } y = 0,\ y = b \text{ for } 0 \le x \le a \quad (12)$$

Thus the problem can essentially be solved by substituting expression (4) into Eq.(3) and solving the resulting linear equation system for the unknown coefficients A_{mn}; once the latter have been calculated, the approximate response of the plate can be calculated explicitly trough Eq.(4).

1.2- Rayleigh's quotient

By making the ratio of the maximum strain energy to the maximum kinetic energy the Rayleigh's quotient is defined as

$$\lambda_R = \omega^2 = \frac{U_{\max}}{T_{\max}} = \frac{U_{\max}}{\frac{\omega^2}{2} \int_{-b}^{+b} \int_{-a}^{+a} \gamma(x, y)\, w^2(x, y)\, dxdy} \quad (13)$$

where $U_{\max}$ and $T_{\max}$ are given by Eqs.2 and 7 respectively.

Based on the principle of potential energy, and applying the Rayleigh-Ritz method, Eq. 13 is minimized with respect to each unknown coefficient A_{mn} to give a series of homogenous simultaneous equations leads to the necessary conditions

$$\frac{\partial \lambda_R}{\partial A_{mn}} = \frac{\partial(\omega^2)}{\partial A_{mn}} = 0\,, \quad m = 1, 2, ... n = 1, 2, ... \quad (14)$$

Substituting the ratio given by Eq. 13 into Eq. 14 leads to

$$\frac{\partial \lambda_R}{\partial A_{mn}} = \frac{T_{\max} \dfrac{\partial U_{\max}}{\partial A_{mn}} - U_{\max} \dfrac{\partial T_{\max}}{\partial A_{mn}}}{T_{\max}^2} = 0 \quad (15)$$

Substituting the assumed function of Eq. 4 into Eq. 7 yields the eingenvalue equation.

2- FINITE ELEMENT FORMULATION

The governing equation of motion for free vibration can be derived from Hamilton principle, which is a generalization of the principle of virtual displacement in the dynamics of deformable bodies. The equilibrium differential equations of motion can be obtained using Hamilton's principle expressed as

$$\delta \int_{t_1}^{t_2} L dt = \delta \int_{t_1}^{t_2} (T - \Pi p)\, dt = 0 \quad (16)$$

where δ is variational operator, L is the Lagrangian function of the plate, t_1 and t_2 are the arbitrary time limits, T is the kinetic energy, Πp is the potential energy. The Lagrangian equations become

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \{\dot{w}_i\}_e} \right) - \frac{\partial L}{\partial \{ w_i \}_e} = \{0\}, \quad i = 1, 2, ... \quad (17a)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \{\dot{w}_i\}_e} \right) - \frac{\partial T}{\partial \{ w_i \}_e} + \frac{\partial \Pi_P}{\partial \{ w_i \}_e} = \{0\}, \quad i = 1, 2, ... \quad (17b)$$

where $\{ w_i \}$ and $\{ \dot{w}_i \}$ are the displacement and the velocity vectors of the generalized coordinates.

For free vibration analysis, the differential equations for orthotropic plate can be written as

$$[M]\{\ddot{W}\} + [K]\{W\} = \{0\} \quad (18)$$

where $[K]$ and $[M]$ are the global stiffness matrix and the global mass matrix respectively. $\{W\}$ is the global displacement vector.

Assuming that the displacements vary with time in a sinusoidal manner with the natural frequency of ω, the eigenvalues can be obtained from the following equations

$$\left[[K] - \omega^2 [M] \right] \{ W \} = \{0\} \quad (19)$$

Table 2: Values of frequencies ω_i and non dimensional frequency coefficient Ω_i for rectangular isotropic clamped plate using the Rayleigh –Ritz method in eq (1) and the FEM.

	Author		F.E.M		[13]	
Mode (m,n)	Frequency ω_i	Ω_i	Frequency ω_i	Ω_i	Frequency ω_i	Ω_i
(1,1)	ω_1=22596.50	36.98	ω_1=21684.9	36.65	ω_1=21706.94	36.69
(1,2)	ω_2=46094.11	75.43	ω_2=44017.9	74.40	ω_2=44276.21	74.84
(1,3)	ω_3=83127.30	136.04	ω_3=440179.9	74.40	ω_3=76943.46	134.01
(1,4)	ω_4=131772.71	227.63	ω_4=63331.9	107.50	ω_4=137874.49	233.03

Table 3: Values of frequencies ω_i and non dimensional frequency coefficient Ω_i for rectangular isotropic clamped plate using the Rayleigh –Ritz method in Eq(2) and the FEM.

	Author		F.E.M		[13]	
Mode (m,n)	ω_i	Ω_i	ω_i	Ω_i	ω_i	Ω_i
(1,1)	ω_1=22442.35	36.72	ω_1=21684.9	36.65	ω_1=21996.04	35.99
(1,2)	ω_2=45595.11	74.62	ω_2=44017.9	74.40	ω_2=44940.03	73.50
(1,3)	ω_3=81860.32	133.97	ω_3=440179.9	74.40	ω_3=80933.10	132.45
(1,4)	ω_4=139088.57	215.66	ω_4=63331.9	107.50	ω_4=141499.11	231.58

Table 4: Values of frequencies ω_i and non dimensional frequency coefficient Ω_i for orthotropic clamped plate.

	Author		F.E.M		[13]	
Mode (m,n)	ω_i	Ω_i	ω_i	Ω_i	ω_i	Ω_i
(1,1)	ω_1=14829.45	24.27	ω_1=15188	25.67	ω_1=15016.68	24.57
(1,2)	ω_2=19153.67	31.34	ω_2=19912	33.65	ω_2=18990.22	31.08
(1,3)	ω_3=27464.52	44.94	ω_3=28648	48.42	ω_3=25688.76	42.04
(1,4)	ω_4=39861.88	65.23	ω_4=39654	67.02	ω_4=36654.65	59.99

For isotropic plate, the material parameters are

$$\nu_1 = \nu_2 = \nu,$$

$$E_x = E_y = E$$

and $G = \dfrac{E}{2(1+\nu)}$

and therefore $D_x = D_y = D\left(1-\nu^2\right)$,

where $D = \dfrac{Eh^3}{12\left(1-\nu^2\right)}$ is the plate flexural rigidity .

The fundamental natural frequency is presented in term of non dimensional parameter Ω_i defined as

$$\Omega_i = \sqrt{\frac{\rho h}{D}}\omega_i a^2 \tag{20}$$

Using a first approximation given from Ref.[13] as shown in table 1 as

$$w(x,y) = A_{00}\left(x^2 - \frac{1}{4}a^2\right)\left(y^2 - \frac{1}{4}b^2\right) \tag{21}$$

The fundamental natural frequency is given by

$$\omega_1 = \frac{8.48}{\sqrt{\gamma}\, a^2 b^2}\sqrt{D\left(7a^4 + 7b^4 + 4a^2b^2\right)} \tag{22}$$

Using a first approximation of the proposed shape function shown in table 1 as

$$w(x,y) = A_{11}\left(1-\frac{1}{4}x^2\right)\left(1-\frac{1}{4}y^2\right)xy\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right) \tag{23}$$

The fundamental natural frequency is given by

$$\omega_1 = \frac{22.08}{\sqrt{\gamma}\, a^2 b^2}\sqrt{D\left[\,1.09(a^2+b^2) + 0.677a^2b^2\,\right]} \tag{24}$$

3- SIMPLE ANALYTICAL EXPRESSIONS FOR THE FUNDAMENTAL FREQUENCIES

A harmonic series function satisfying the clamped edge boundary conditions of the plate in the form given by table 1 is substituted in the analytical formulation. From the resulting linear system of equations one obtains a determinant whose lowest root constitutes the fundamental frequency. The frequencies values resulting from analytical expressions for isotropic and orthotropic rectangular plates are compared with those of the FEM within table 2, 3 and 4. The formulation has been programmed using Maple6 package to compute the fundamental frequencies of rectangular isotropic and orthotropic plates of thin geometry with clamped edges conditions.

For the FEM the modelling used corresponds to 64 quadrilateral elements in the full plate. For comparison purposes, results have been generated using the SAP90 and the SUPERSAP commercial finite element package for the case of isotropic and orthotropic plates respectively.

3.1- Isotropic rectangular plate

Consider a rectangular isotropic plate of length a, width b and thickness h as shown in figures 1a and 1b. For free vibration the frequency equation may be derived using the Rayleigh-Ritz approach.

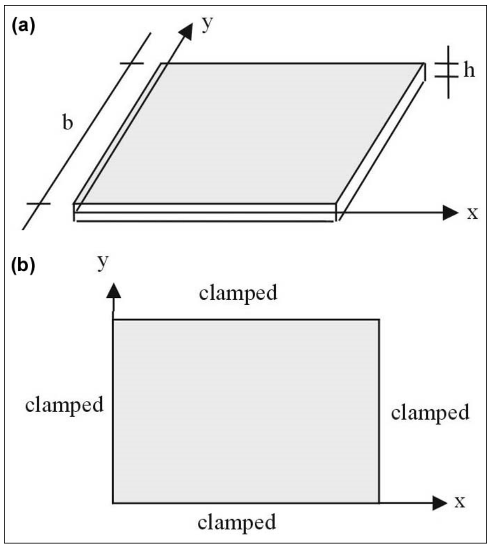

Figure1: **(a)** Rectangular plate, **(b)** clamped plate.

3.2- Orthotropic rectangular plate

Using a first approximation given from Ref.[13] as shown in table 1 as

$$w(x,y)=A_{00}\left(x^2-\frac{1}{4}a^2\right)\left(y^2-\frac{1}{4}b^2\right) \tag{25}$$

The fundamental natural frequency is given by:

$$\omega_1=\frac{8.48}{\sqrt{\gamma\, a^2b^2}}\sqrt{\left(7D_xb^4+4.D_1a^2b^2+8D_{xy}a^2b^2+7D_ya^4\right)} \tag{26}$$

Using a first approximation of the proposed shape function shown in table 1 as

$$w(x,y)=A_{00}\left(1-\frac{1}{4}x^2\right)\left(1-\frac{1}{4}y^2\right)xy\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right) \tag{27}$$

The fundamental natural frequency is given by:

$$\omega_1=\frac{22.08}{\sqrt{\gamma a^2b^2}}\sqrt{\left[1.09(D_xb^4+D_ya^4)+0.67a^2b^2(2D_{xy}+D_1)\right]} \tag{28}$$

4- NUMERICAL RESULTS

The obtained results by applying the present formulation combined respectively with the proposed displacement function and that of Ref.[13] are very close resulting from the FEM The present results compare very closely with the FEM in Table 2.

4.1- Isotropic Plate

For this example, a rectangular plate panel of length a=0.25m and width b=0.25m and a thickness of 0.005m is considered. Young's modulus E=525000 N/mm^2, Poisson's ratio ν=0.25 and density ρ=800 kg/m^3. In generating the finite element results a mesh of 8x8 quadrilateral elements is used.

4.2- Orthotropic plate

For this problem, the plate dimensions are the same as the isotropic plate. The results of applying the present formulation with the proposed displacement function and that of Ref. (13) and the FEM are presented in Table 2. The present formulation results are very close with the FEM results. The fundamental natural frequency is presented in term of non dimensional parameter Ω_i defined as

$$\Omega_i=\sqrt{\frac{\rho h}{D_x}}\omega_i a^2 \tag{29}$$

5- DISCUSSION

The free vibration of orthotropic plates, based on the linear, 2-D elasticity theory have been investigated. The simple Rayleigh –Ritz method was applied to orthotropic clamped plates to derive the eigenvalue equation and determine the natural frequencies. As it is well known, the Raleigh-Ritz method can provide accurate solutions. However, its efficiency depends greatly on the choice of shape functions. Although the result from the finite element analysis gave the best correlation with the predicted ones, however they require much longer computing times. In this respect approximate analytical tools such as the present method and Rayleigh quotients are better. The comparative results indicate that one term approximation is not sufficient to predict the first lowest frequencies of the plate. For the modes percentage error is quite satisfactory ie for CCCC orthotropic plate ω_{11} (2.3%), ω_{12} (3.8 %), ω_{13} (4.13 %) and ω_{14} (0.52 %). For CCCC isotropic plate ω_{11} (3.37%) , ω_{12} (3.46%). But if higher modes are required percentage error is increasing considerably. Practically, only the lowest modes are considered in the dynamic analysis. The frequencies obtained could be improved by increasing the number of terms of admissible functions in the computation and hence solution of any accuracy can be obtained in theory. However, a practical limit to the number of terms used always exists because of the limited speed, the capacity and the numerical accuracy of computers. In the technical literature there is no exact solution for the clamped orthotropic plate. However, the above study, show that a good shape functions are suitable for engineering analysis to determine the first lowest frequencies in the free vibration of orthotropic plate.

CONCLUSION

The free vibration of isotropic, orthotropic clamped rectangular plates using the Rayleigh-Ritz energy approach has been studied. Using mechanical characteristics of the plate, the fundamental frequency has been determined by a proposed simple analytical formula. For isotropic and orthotropic plates, the values of natural frequency obtained by Rayleigh-Ritz method agree well with those given by the finite element method in case of low frequencies only. However, for high frequencies, the percentage error increases with increasing the frequency mode. Since only the lowest modes are important, so it 's sufficient to design our structure.

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