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ELASTO-PLASTIC ANALYSIS OF CONFINED COLUMNS USING FINITE ELEMENTS

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Abstract

Finite element program using solid cube elements for concrete and bar elements for steel reinforcement was developed in order to analyze the behavior of confined reinforced concrete columns with circular cross section under increasing compression load. The concrete was assumed as elastic- plastic material and follow Von Mises failure criterion with associated flow rule. The steel reinforcement was considered as linear strain hardening material. Experimental results were used to verify the numerical results.

***Keywords**: Finite elements, confinement, reinforced concrete column, transverse reinforcement.*

Résumé

Un programme en éléments finis a été élaboré pour analyser l'effet de confinement, à l'aide des armatures transversales, sur le comportement des colonnes en béton armé soumises à un chargement de compression monotone. Les éléments à trois dimensions et les éléments à une dimension ont été utilisés pour la discrétisation du béton et des armatures, successivement. Le comportement du béton est considéré comme étant élasto - plastique et celui des armatures comme étant élasto - plastique avec écrouissage. Le critère de rupture de Von Mises pour un écoulement associé a été adopté Les résultats numériques obtenus ont été comparés avec d'autres résultats publiés dans la littérature.

***Mots clés**: Eléments finis, confinement, colonnes en béton armé, armatures transversales.*

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ملخص

يقوم برنامج الحساب العددي على طريقة العناصر المنتهية وقد تم إعداده من أجل دراسة سلوك الأعمدة المكونة من الخرسانة مع وجود القضبان الحديدية الطولية والعرضية، ذات المقطع الدائري والمعرضة لقوة انظغاط طولية.

لقد استعملت العناصر المكعبة الصلبة، ثلاثية الأبعاد، لتمثيل الخرسانة، والعناصر الطولية، ذات البعد الواحد، لتمثيل القضبان الحديدية الطولية والعرضية. كما اعتبرت الخرسانة كمادة مرنة - لدنة وتتبع قانون Von Mises with associated flow rule .

لقد تم مقارنة النتائج المتحصل عليها من خلال هذا البرنامج العددي مع بعض ما تم نشره عن دراسات سابقة.

***الكلمات المفتاحية** : العناصر المنتهية، الإنضغاط، أعمدة الخرسانة المسلحة، القضبان الحديدية العرضية.*

Ductility of reinforced concrete members allows a structure to undergo large plastic deformations with little decrease in strength. It is an important factor related to the safety of structure. Much effort has done into evaluating the effect of lateral reinforcement steel for improving the strength and ductility of reinforced concrete columns under different loads. It has been shown that high ductility can be achieved in reinforced concrete members if the volume of stirrups and the reinforcement configurations are provided sufficiently [1-5]. It is commonly agreed that confinement enhances both strength and ductility in several respects [6, 7]. Much effort has done into evaluating the effect of lateral reinforcement steel for improving the strength and ductility of reinforced concrete columns under different loads. In recent years, the effect of steel reinforcement on strength and deformation capacity of concrete has been extensively studied from both experimental and theoritical point of view Mander *et al.* [1] Irawan and Maekawa [6].

Design codes (ACI 318 [8], CEB-FIP [9]) incorporates minimum requirements of the transverse steel reinforcement hoop spacing, in order to improve ductility and strength of the concrete members and avoid a brittle failure.

In this study, a finite element program is developed to evaluate the effect of lateral reinforcement on strength and ductility of reinforced concrete columns having a circular cross section. The concrete is discretised using solid cube and triangular prism - three dimensional elements (8 node and 6 node elements). The steel reinforcement is discretised with bar elements (2 nodes element). The uniaxial compressive stress - strain curve of concrete is represented in figure 1a and Von Mises failure criterion with associated flow rule has been taken into consideration. The steel reinforcement is considered as linear strain hardening material (Fig. 1b).

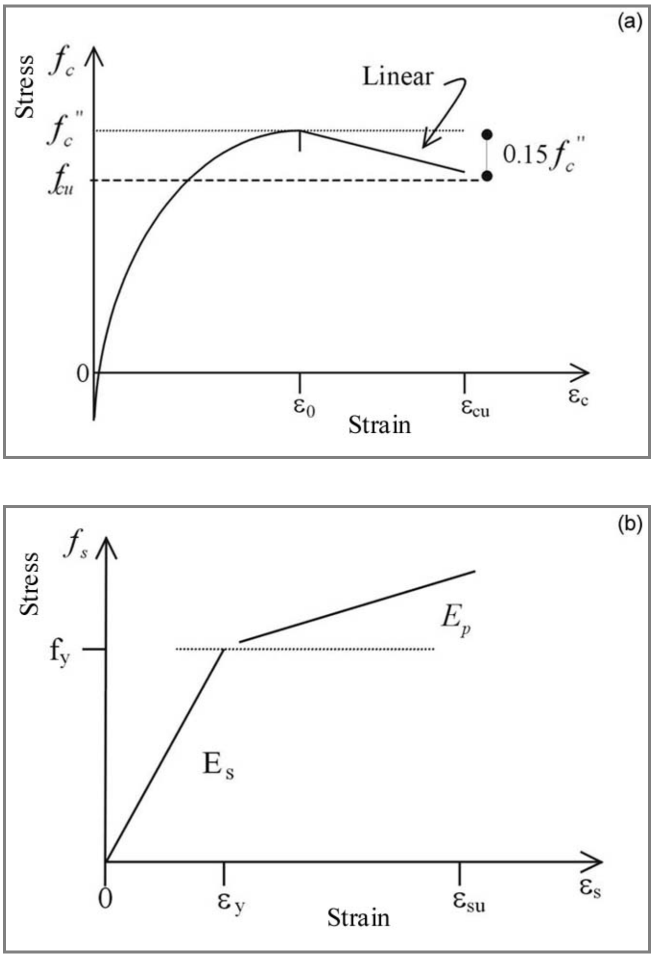

Figure 1: Constitutive relations for materials: **(a)** concrete; **(b)** steel reinforcement.

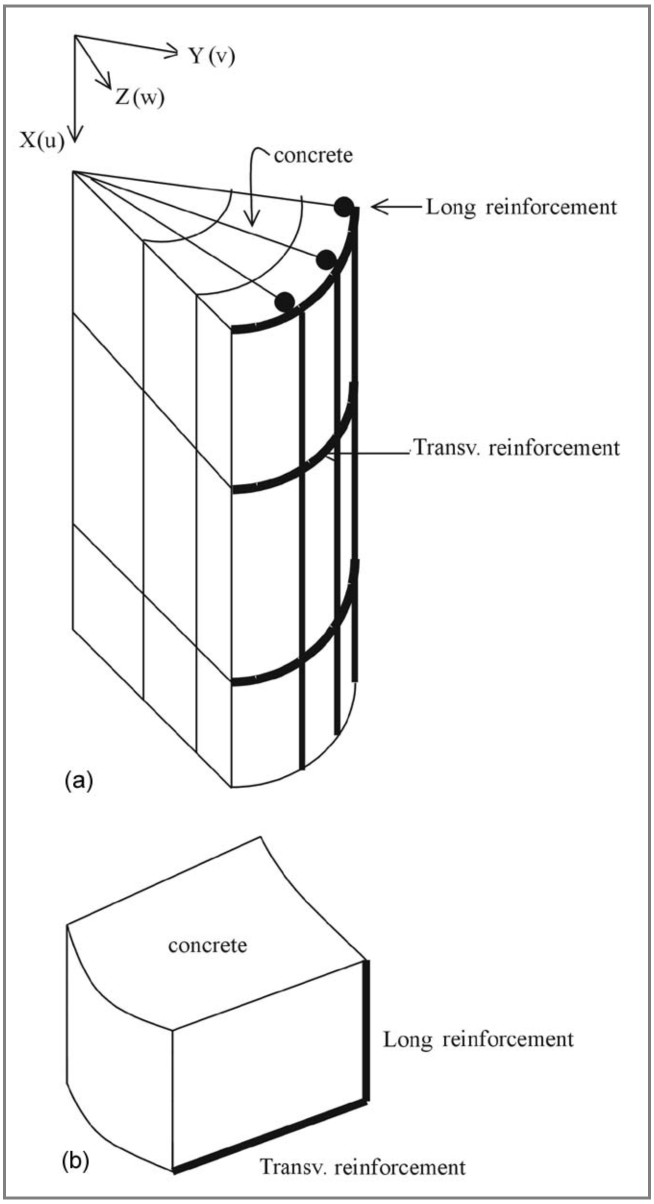

Figure 2: Discretisation of reinforced concrete column.
(a) representation of fourth half column.
(b) brick (hexahedron three dimensionnel element).

FORMULATION

In order to apply the incremental theory of plasticity to concrete, these basic assumptions must be made: The yield surface that defines the beginning of plastic flow, the hardening rule that describes the evolution of the subsequent loading surface and the flow rule that is related to a plastic potential function and gives an incremental plastic stress – strain relation.

The finite element analysis starts with the subdivision of the columns into an assemblage of discrete elements. Isoparametric hexahedron (brick) and triangular prism three dimensional elements with three degrees of freedom per node are used for concrete. Longitudinal and transverse steel reinforcement are assumed as truss members carrying axial forces only, so one dimensional bar element with one degree of freedom per node is used [10] (Fig. 2a). Due to symmetry of the column geometry and reinforcement, only one fourth of the section was taken in discretisation (Fig. 2b). The stiffness matrix of the composite element is obtained by superposition of material stiffness of the individual material components, concrete and reinforcement. Perfect bonding between concrete and reinforcement is assumed.

The behavior of concrete is defined by the following equations [11] (Fig. 1a):

$$f_c = f_c''\left[\frac{2\varepsilon_c}{\varepsilon_0} - \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2\right] \qquad \text{for } 0 \le \varepsilon_c \le \varepsilon_0 \qquad (1)$$

$$f_c = f_c'' + \left(f_c'' - f_{cu}\right)\left(\frac{\varepsilon_c - \varepsilon_0}{\varepsilon_{cu} - \varepsilon_0}\right) \qquad \text{for } \varepsilon_0 \le \varepsilon_c \le \varepsilon_{cu} \qquad (2)$$

with $f_c'' = 0.85 f_c'$

$f_{cu} = 0.85 f_c''$

f_c' : ultimate uniaxiale compressive strength

f_c'' : maximum concrete compressive stress

Transverse and longitudinal steel reinforcements are assumed to be a linear strain hardening material (Fig. 1b). They are defined by the following equations:

$$f_s = E_s \varepsilon_s \qquad \text{for } \varepsilon_s \le \varepsilon_y \qquad (3)$$

$$f_s = \left(E_s - E_p\right)\varepsilon_y + E_p\varepsilon_s \qquad \text{for} \qquad \varepsilon_s > \varepsilon_y \tag{4}$$

E_s and E_p are successively elastic and plastic modulus;

ε_y the strain corresponding to the yield stress.

A material behaves elastically when a yield surface is not exceeded, and equilibrium will be gained by default. Beyond the elastic limit, an iterative procedure is needed until convergence is occurred.

LINEAR ELASTIC REGION

The stress strain increment relation for concrete is given as:

$$\{d\sigma\} = [D]\{d\varepsilon\} \tag{5}$$

with

$$\{d\sigma\}^T = \left\{d\sigma_x,\, d\sigma_y,\, d\sigma_z\,,\, d\tau_{xy},\, d\tau_{yz},\, d\tau_{zx}\right\} \tag{6}$$

$$\{d\varepsilon\}^T = \left\{d\varepsilon_x,\, d\varepsilon_y,\, d\varepsilon_z,\, d\gamma_{xy},\, d\gamma_{yz},\, d\gamma_{zx}\right\} \tag{7}$$

where

$\{d\sigma\}$ and $\{d\varepsilon\}$ are the stress and strain increment vectors;

$[D]$: elastic material stiffness matrix [12].

PLASTIC HARDENING REGION

The concrete yielding occurs, according to Von Mises criterion, when the second invariant of deviatoric stress (J_2) reaches a critical value. After yielding, the stress-strain behavior of concrete material can be separated into recoverable elastic component $\left\{d\varepsilon^e\right\}$ and irrecoverable plastic component $\left\{d\varepsilon^p\right\}$ so that:

$$\{d\varepsilon\} = \left\{d\varepsilon^e\right\} + \left\{d\varepsilon^p\right\} \tag{8}$$

and the incremental constitutive relationship will be expressed [13] as:

$$\{d\sigma\} = [D]^{ep}\{d\varepsilon\} \tag{9}$$

where $[D]^{ep}$: elasto-plastic stiffness matrix, for associated plasticity it is given by :

$$[D]^{ep} = [D] - \frac{[D]\{a\}\{a\}^T[D]}{A + \{a\}^T[D]\{a\}} \tag{10}$$

with : $\{a\} = \left\{\dfrac{\partial f(\sigma)}{\partial \sigma}\right\}$

$f(\sigma)$: yield function.

For Von Mises failure criterion :

$$\{a\} = \frac{1}{2\sqrt{J_2}}\left\{\ \sigma_x, \sigma_y, \sigma_z, 2\tau_{xy}, 2\tau_{yz}, 2\tau_{xz}\ \right\} \tag{11}$$

A: Hardening parameter. It is considered to be the slope of the uniaxial stress – plastic strain curve [13].

The composite material stiffness is obtained by superposition of matrices of the individual material components, concrete and reinforcement.

NONLINEAR SOLUTION PROCEDURE

A Newton Raphson non-linear method is adopted in which the stiffness matrices for concrete and reinforcement are updated at the beginning of each iteration. The iterative cycles are repeated until the convergence is reached.

The convergence criterion is given by :

$$\|R\| = \|F_{ext} - F_{\text{int}}\| \le tolerance \tag{12}$$

$\{R\}$: Residual load vector,

$\{F_{ext}\}$: Applied force vector,

$\{F_{\text{int}}\}$: Internal force vector.

The algorithm used in the analysis is as fellows:

1- Initialization and input data;

I = 0 (counter over all load increment)

Iter = 0 (iter = iteration. counter over all iteration process)

$\{F_{ext}\} = \{F_0\}$, $\{F_0\}$: initial external load vector.

2- Increment the applied load.

I = I + 1

$\{F_{ext}\} = \{F_{ext}\} + \{\Delta F\}$

3- Increment iterations.

iter = iter + 1

4- Compute the residual loads.

$\{R\} = \{F_{ext}\} - \{F_{\text{int}}\}$

5- Update and assemble the element stiffness matrices in the global stiffness matrix $[K_T]$.

6- Solve equilibrium equation.

$[K_T]\{\Delta d\} = \{R\}$

7- Add displacement increment $\{\Delta d\}$ to the current displacement $\{d\}$.

8- For all elements and all integration points update the strains and stresses:

$\left\{\sigma^e\right\} = \{\sigma\} + [D]\left\{d\varepsilon^e\right\}$, (exponent 'e' for elastic state).

9- evaluate the yield function f using failure criterion.

if $f < 0$ gauss point is elastic goto step 10.

if $f \ge 0$ gauss point is plastic. Compute the stresses using elasto plastic approach to return to the yield surface [12].

10- Compute $\{F_{\text{int}}\}$.

11- Check convergence.

a/ If convergence occurs print results.

b/ If (I) ≥ maximum load steps goto step 12 else goto step 2.

c/ If convergence does not occur goto step 3.

12- stop.

NUMERICAL RESULTS

Numerical results were compared to the results obtained by Irawan and Maekawa [6] for normal strength concrete. The characteristics of the columns are shown in table 1.

Figure 3 and figure 4 show the same tendency observed between the curves of the present analysis and that obtained

Table 1: Mechanical properties of columns.

ρ_s (%)	E_s (MPa)	E_p (MPa)	f_c'' (MPa)	f_y (MPa)	E_c (MPa)
2.5	2.10^5	4.10^3	25	240	2.10^3

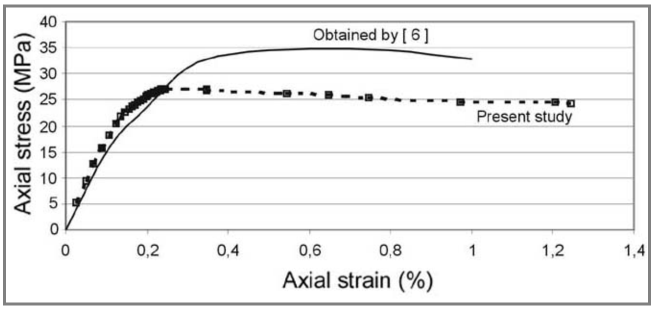

Figure 3: Comparison of present study with data from Irawan and Maekawa [6].

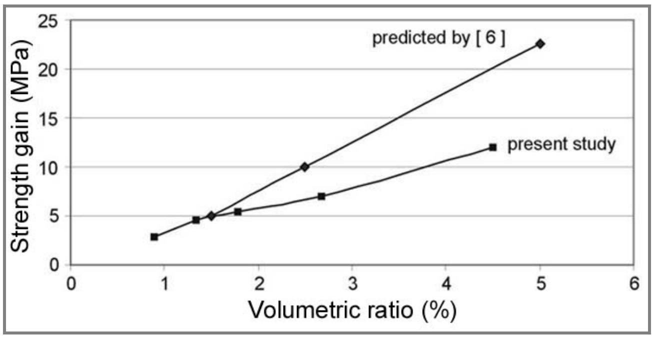

Figure 4: Effect of volumetric ratio of transverse reinforcement to strength gain.

by Irawan and Maekawa [6]. Nevertheless, the present analysis seems to be more conservative.

The results indicate that an increase in volumetric ratio of transverse reinforcement leads to an increase in strength, that is due to the confinement effect. This same behavior has been observed by Irawan and Maekawa [6]. However the difference observed in comparing these results can be attributed to the discretisation methods, models assumed for concrete material and analysis approach used in both studies.

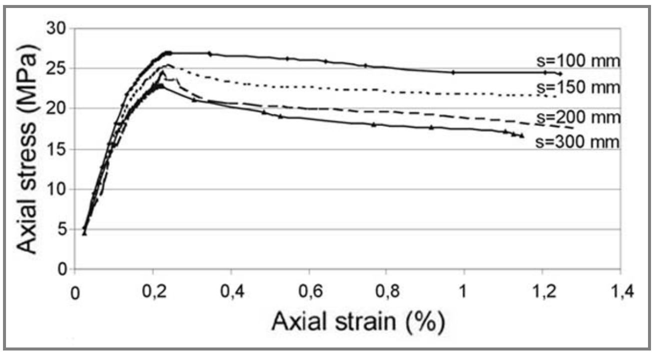

Figure 5: Stress-strain curves for different hoop spacing (s).

Figure 5 shows the evolution of the stress-strain curves for different hoop spacing of transverse reinforcement. It is confirmed that more ductile behavior can be achieved by decreasing the hoop spacing. However, the strength gain is lightly increased by increasing the volume ratio of transverse steel (ρ_s). At the beginning, the curves are identical until the stress reaches about 40% of concrete yield stress, where the effect of lateral steel reinforcement appears. The slope of ascending branch of stress-strain curve becomes a bit greater in the case of reduced hoop spacing.

CONCLUSION

The present work is devoted to the development of a numerical computer code based on incremental plasticity theory, using the finite element method. The proposed algorithm has been developed to compute the response of a reinforced concrete column subjected to the axial compressive load. Numerical results are presented and compared to those of Irawan and Maekawa [6]. The same tendency has been observed. However, some differences are observed, that can be attributed to several possible reasons, as the discretisation methods, models assumed for concrete material and analysis approach used in both studies.

It has been confirmed that ductile behavior of confined columns and gain strength can be achieved by decreasing the hoop spacing. The effect of lateral reinforcement appears at about 40% of concrete yield stress.

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