NUMERICAL INVESTIGATION FOR NATURAL CONVECTION BOUNDARY LAYER FLOW OF A NANOFLUID ALONG AN INCLINED PLATE EMBEDDED IN A DARCY POROUS MEDIUM

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Résumé

Dans ce présent travail, l'analyse d'un écoulement en convection naturelle de type couche limite est présentée pour une plaque inclinée semi infinie immergée dans un milieu poreux saturé d'un nanofluide, sous des conditions aux limites classique.

Le modèle utilisé pour le nanofluide comporte l'effet de diffusion brownienne et la thermophorèse, tandis que le modèle Darcy est utilisé pour décrire le comportement du milieu poreux.

La formulation du problème est obtenue par des transformations de similarité appropriées. Les équations de similarité sont résolues numériquement en utilisant la méthode des différences finies via bvp4c.

Une étude paramétrique des paramètres physiques est menée pour afficher leur influence sur les différents profils vitesse, température et fraction volumique des nanoparticules. Les autres quantités d'intérêt sont calculées.

Mots clés *: convection naturelle, milieu poreux, nanofluide, mouvement Brownien, thermophorèse*

Abstract

In this paper, we examine the steady natural convection boundary layer flow of an incompressible viscous nanofluid along an inclined plate at an angle α in a porous medium.

The model used for the nanofluid includes the effects of Brownian motion and thermophoresis, while the Darcy model is used for describe the porous medium. The resulting similarity equations are solved numerically using finite difference method via bvp4c routine. Many results are obtained and representative set is displayed graphically to illustrate the influence of the various parameters on different profiles.

The conclusion is drawn that the flow field, temperature and nanoparticle volume fraction shapes are significantly influenced by nanofluid Lewis number, Brownian motion parameter and thermophoresis parameter.

Keywords: *natural convection, porous medium, nanofluid, Brownian motion, thermophoresis*

ملخص

في هذه المقالة، الانتقال الحراري عن طريق الحمل الطبيعي في الأوساط المسامية المشبعة بمائع قد درست بواسطة المعالجة الرقمية. الشكل الهندسي لهذه الدراسة هي عبارة عن لوحة مائلة مغموسة في وسط مسامي مشبع بسائل نانوني.

نموذج درسي قد استعمل لنمذجت الوسط المسامي. فما استعملت تأثير الحركة البرونية والترموفرز لنمذجت المائع النانوني. الإشكالية المطروحة تمت صياغتها عن طريق التحويلات المتشابهة المناسبة.

هذه الاخيرة حلت بواسطة الطرق الرقمية، بمساعدة طريقة BVP ، النتائج المحصل عليها قادتنا إلى الأخذ بعين الاعتبار التأثير الحركة البرونية والترموفرز على التدفق الحملي لاسيما في سرعة التدفق، توزيع درجة الحرارة و نسبة الحجم للجسيمات النانونية.

الكلمات المفتاحية: *حمل طبيعي، وسط مسامي، سائل نانوني، حركة برونية، ترموفرز*

Study of natural convective heat in porous media has been the interest of several researches owing to its wide applicability in engineering and geophysical problems such as in oil recovery technology, in the use of fibrous materials for thermal insulations, in the design of aquifers as an energy storage system, and also in resin transfer molding process, in which fibre-reinforced polymeric parts are produced in final shape. Excellent reviews of the natural convection flows in porous media have been presented by many authors. [1-4].

Nanofluid refer to a liquid containing a dispersion of nanoparticles. The nanoparticles are different from conventional particles in that they keep suspended in the base fluid without sedimentation [5]. A nanofluid is a new class of heat transfer fluids that contain a base fluid and nanoparticles. The use of additives is a technique applied to enhance the heat transfer performance of base fluids. The thermal conductivity of the ordinary heat transfer fluids is not adequate to meet today's cooling rate requirements. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base liquids [6]. Beside, nanofluids find numerous applications in various fields of science and engineering as convective heat transfer fluids, ferromagnetic fluids, superwetting fluids and detergents, biomedical fluids, polymer nanocomposites, gain media in random lasers, and as building blocks for electronic and optoelectronic devices [7]. The investigations of boundary layer flow, heat and mass transfer over a flat plate embedded in porous media containing nanofluids are important due to its applications in industries and many manufacturing processes, in this field of study, Nield and Kuznetsov have presented similarity solutions for the Cheng–Minkowycz problem for the double-diffusive natural convective boundary layer flow in a porous medium, they have used the model for the nanofluid incorporates the effects of Brownian motion and thermophoresis [8], the same authors studied thermal instability in a porous medium layer saturated by a nanofluid, they found that for a typical nanofluid (for which the Lewis number is large) the primary contribution of the nanoparticles is via a buoyancy effect coupled with the conservation of nanoparticles, with the contribution of nanoparticles to the thermal energy equation being a second-order effect [9]. Natural convective flow of a nanofluid over a vertical plate with a constant surface heat flux is investigated by Khan and Aziz, they used the transport model which includes the effect of Brownian motion and thermophoresis, they have concluded that velocity, temperature and nanoparticle volume fraction profiles in the respective boundary layers depend on five dimensionless parameters [10]. Mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids is investigated by Syakila and Pop using different types of nanoparticles as Cu (cuprom), Al_2O_3 (aluminium) and TiO_2 (titanium) [11].

The principal aim of the present paper is to study the combined effect of Brownian motion and thermophoresis of nanofluid on steady free convection heat over inclined flat plate embedded in a Darcy porous medium Based on the literature survey only the papers by Nield and Kuznetsov [12]. The resulting similarity solutions of the governing equations are obtained. Many results are obtained and representative set is displayed graphically to illustrate the influence of the various dimensionless parameters. It is in the main objective to determine the influence of the simultaneous effects on heat and mass transfer from the plate to the porous medium.

1. PROBLEME STATEMODELE MATHEMATIQUE

We consider the Darcy natural convection of two dimensional flow, heat transfer of an incompressible viscous nanofluid past an inclined plate embedded in Darcy porous medium as illustrated in figure 1. The x-axis is taken and measured along the plate and y-axis is normal to it. The temperature T and the nanoparticle fraction ϕ at the plate surface takes T_w and ϕ_w, respectively. The ambient values, attained as y tends to infinity, of T, ϕ are denoted by T_∞ and ϕ_∞, respectively. The oberbeck–boussinesq approximation for the nanofluid [13] is employed. homogeneity and local thermal equilibrium in the porous medium is assumed. Under the above of these assumptions, the boundary layer equations governing the flow, temperature and concentration field can be written in dimensional form as

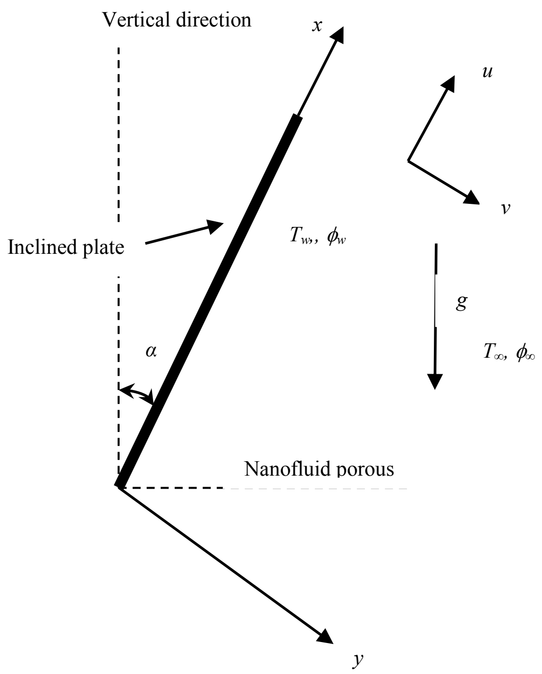

Figure 1 : Physical model and coordinate system.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial p}{\partial x}=-\frac{\mu}{K}u+\begin{bmatrix}(1-\phi_\infty)\rho_{f\infty}g\beta(T-T_\infty)\\-(\rho_p-\rho_{f\infty})g(\phi-\phi_\infty)\end{bmatrix}\cos(\alpha) \tag{2}$$

$$\frac{\partial p}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \tau\left[D_B \frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{4}$$

$$\frac{1}{\varepsilon}\left(u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y}\right) = D_B\frac{\partial^2 \phi}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} \tag{5}$$

Here p is the pressure, T is the temperature and ϕ nanoparticle volume fraction. K is the permeability constant of the porous medium, ρ_f, μ and β_T are the density, viscosity, and volumetric thermal expansion coefficient of the fluid, while ρ_P is the density of the particles and α_m represent thermal diffusivity of the porous medium. The gravitational acceleration is denoted by g. The coefficients that appear in Eqs. (4) and (5) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T. τ is a parameter defined as $\tau = \varepsilon(\rho c)_p/(\rho c)_f$. The flow is assumed to be slow so that an advective term and a Forchheimer quadratic drag term do not appear in the momentum equation.

The corresponding wall and for stream boundary conditions are defined as follow

$$\text{At} \quad y = 0 : v = 0, T = T_w, \phi = \phi_w \tag{6}$$

$$\text{As} \quad y \to \infty : u = 0, T \to T_\infty, \phi \to \phi_\infty \tag{7}$$

We now introduce the local Rayleigh number Ra_x defined by

$$Ra_x = \frac{(1-\phi_\infty)Kg\beta_T(T_w - T_\infty)x}{\alpha_m \nu} \tag{8}$$

And the similarity variable

$$\eta = \frac{y}{x}Ra_x^{1/2} \tag{9}$$

We also introduce the dimensionless variables f, θ and h defined by

$$f = \frac{\psi}{\alpha_m Ra_x^{1/2}};\ \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)};\ h = \frac{(\phi - \phi_\infty)}{(\phi_w - \phi_\infty)} \tag{10}$$

Where ψ is the classical stream function, and from the definition of the stream function, the velocity components become

$$u = \frac{\alpha_m}{x}Ra_x f';\ v = -\frac{\alpha_m}{2x}Ra_x^{1/2}(f - \eta f')$$

Equations “1, 2, 3, 4 and 5” with the above appropriate transformations and algebraic combining for “Equations 1 and 2” and “Equation 3” can be further reduced to a set of ordinary differential equations for which numerical solutions are more easily determined

$$f'' = (\theta' - Nrf')\cos(\alpha) \tag{11}$$

$$\theta'' + Nbh'\theta' + Nt\theta'^2 + \frac{1}{2}f\theta' = 0 \tag{12}$$

$$h'' + \frac{Nt}{Nb}\theta'' + \frac{1}{2}Le_p fh' = 0 \tag{13}$$

The boundary conditions become

$$f(0) = 0 \quad \theta(0) = h(0) = 1 \quad f'(\infty) = \theta(\infty) = h(\infty) \tag{14}$$

Where the various parameters are defined by

$$Nr = \frac{(\rho_p - \rho_{f\infty})(\phi_w - \phi_\infty)}{(1-\phi_\infty)\rho_{f\infty}\beta_T(T_w - T_\infty)} \tag{15}$$

$$Nb = \frac{\varepsilon(\rho c)_p D_B(\phi_w - \phi_\infty)}{(\rho c)_f \alpha_m} \tag{16}$$

$$Nt = \frac{\varepsilon(\rho c)_p D_T(T_w - T_\infty)}{(\rho c)_f \alpha_m T_\infty} \tag{17}$$

$$Le_p = \frac{\alpha_m}{\varepsilon D_B} \tag{18}$$

Here Nr, Nb, Nt, denote the nanofluid buoyancy ratio, the Brownian motion parameter, the thermophoresis parameter, respectively, while Le_p is a nanofluid Lewis number.

Quantities of practical interest in thermal engineering design applications are the local Nusselt number Nu_x and the local Sherwood number Sh_x, which take the form

$$Nu_x = \frac{q_w}{T_w - T_\infty}\frac{x}{k},\quad Sh_x = \frac{q'_w}{\phi_w - \phi_\infty}\frac{x}{D_m} \tag{11}$$

Here, q_w and q'_w are the heat flux and mass flux at the surface (plate), respectively. Whereas k is the effective thermal conductivity and Dm is molecular solutal diffusivity of the fluid. Using (10) we obtain dimensionless versions of these key design quantities:

$$Nu_x Ra_x^{-1/2} = -\theta'(0), \quad Sh_x Ra_x^{-1/2} = -h'(0)$$

2. PROBLEME STATEMODELE MATHEMATIQUE

The set of the coupled ordinary differential "Equations 11 – 13" is highly nonlinear and cannot be solved analytically. Together with the boundary conditions "Equation 14", they form a two point boundary value problem (BVP) which can be solved using the routine bvp4c of the symbolic computer algebra software MATLAB, this routine is based on the finite differences method that implements the 3-stage Lobatto collocation formula and the collocation polynomial provides a continuous solution that is fourth-order accurate uniformly in the interval of integration. Mesh selection and error control are based on the residual of the continuous solution. The collocation technique uses a mesh of points to divide the interval of integration into subintervals. The flow region is controlled by thermophysical parameters, namely Nr, Nb, Nt and Le_p. Numerical computations are carried out for different values of the parameters shown in all figures. Preliminary calculations are conducted to check the numerical results. It is interesting to show the influence of all the control parameters on the velocity, temperature and nanoparticale volume fraction profiles respectively.

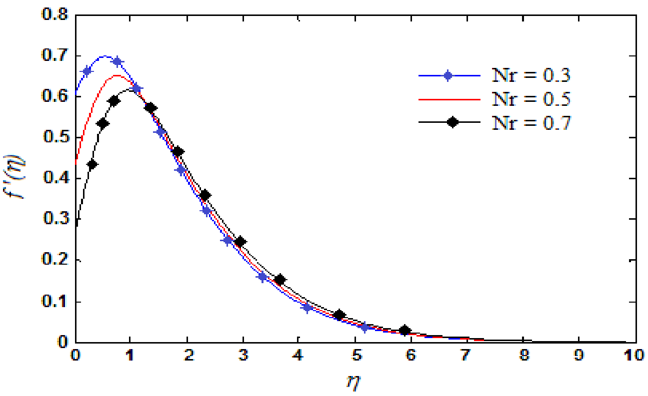

Figure 2 : Variation of velocity profiles with similarity variable η, (Nb =1.0, Nt =1.0, Le_p =10.0, α =30°).

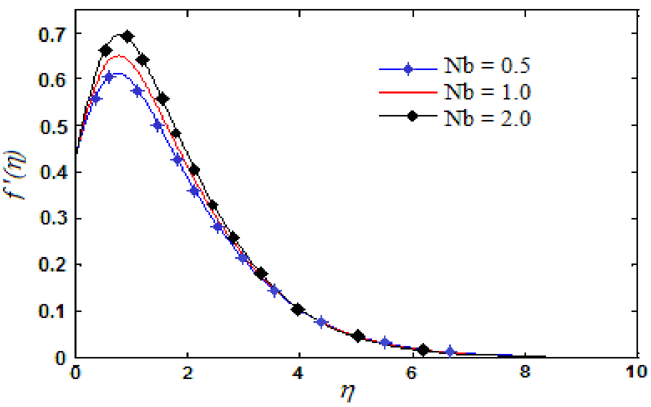

Figure 3 : Variation of velocity profiles with similarity variable η, (Nr =0.5, Nt =1.0, Le_p =10.0, α =30°).

The figures 2, 3 represent the behaviours of the dimensionless velocity along the inclined plate with similarity variable η for different values of buoyancy ratio parameter Nr, Brownian motion parameter Nb, respectively. From figure 2 that the component of the velocity along of the plate increases initially, reaches a maximum and then decreases asymptotically to zero at the edge of the hydrodynamic boundary layer. This pattern is essentially the same as in the natural convective boundary layer of a regular fluid. Kuznetsov and Nield [14] noted the same pattern for the velocity profiles. The local dimensionless velocity is seen to decrease with an increase in the buoyancy-ratio parameter Nr. Results in the Brownian motion of nano-particale there is a sensible effect from parameter Nb to the velocity increases near the surface, as seen in figure 3.

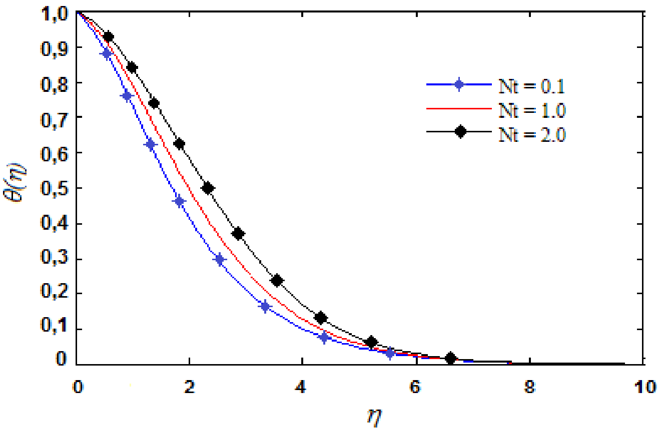

Figure 4 : Variation of temperature profiles with similarity variable η, (Nr = 0.5, Nb =1.0, Le_p =10.0, α=30°).

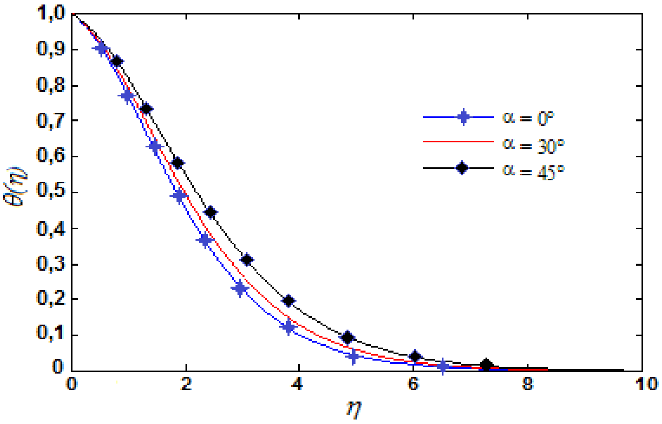

Figure 5 : Variation of temperature profiles with similarity variable η, (Nr = 0.5, Nb =1.0, Nt = 1.0, Lep =10.0).

Figure 4 show the effect of thermophoresis parameter *Nt* on the some profiles. Here, again, it is seen that the thermophoretic of particles effect obviously affects the temperature distributions. However, the thermal boundary layer thickness increases, when the of thermophoresis parameter increases. Then the effects of thermophoresis parameter play an importance role on the surface heat transfer. The greatest difference in temperature profiles occurs some distance from the wall. In Figure 5, the influence of the angle of inclination from the vertical, α, ranging from 0° to 45°, on the temperature $\theta(\eta)$ profile are displayed. Similar to the effects of the thermophoresis parameter *Nt*, it is observed that increase in the inclination angle (α) increases the fluid temperature. This is due to the reduction in the thermal buoyancy effect $\left[(1-\phi_\infty)\rho_{f\infty}g\beta(T-T_\infty)-(\rho_p-\rho_{f\infty})g(\phi-\phi_\infty)\right]\cos(\alpha)$ caused by increases in $\boldsymbol{\alpha}$.

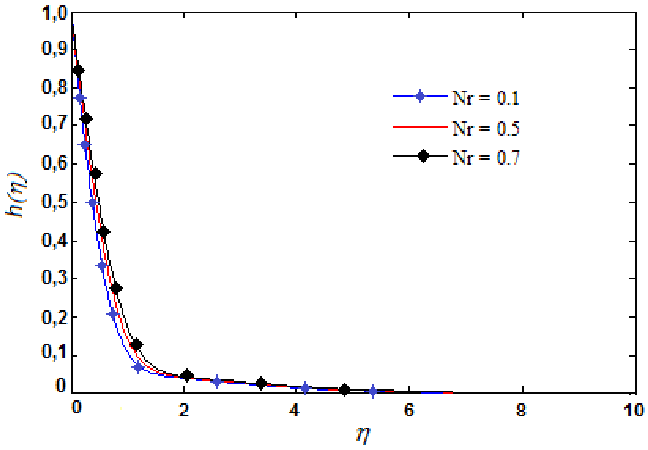

Figure 6 : Variation of volume fraction profiles with similarity variable η, (Nb =1.0, Nt =1.0, Le_p =10.0, α = 30°).

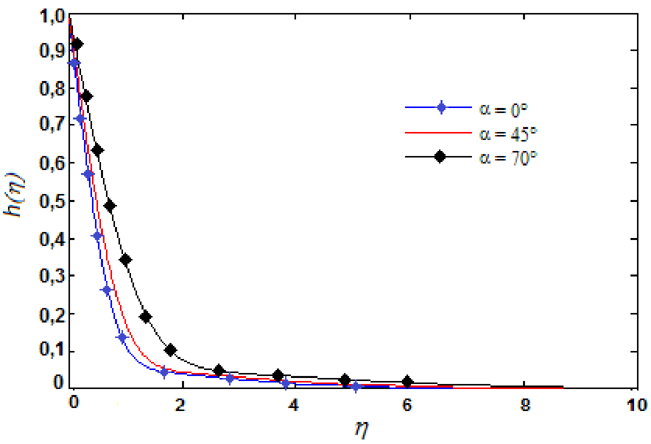

Figure 7 : Variation of volume fraction profiles with similarity variable η, (Nr = 0.5, Nb =1.0, Nt =1.0, Le_p =10.0).

Figures 6 and 7 illustrate the effect of the buoyancy ratio parameter Nr and the angle of inclination parameter α on the Nanoparticle volume fraction distributions through the boundary layer regime, With an increase in the angle of inclination parameter from 0° to 70° and the buoyancy ratio from 0.1 to 0.7, leads to increasing of the volume fraction of the nanoparticle in the fluid, and the boundary layer becomes larger of the nanoparticle.

The variations of the dimensionless heat transfer rates $Nu_x/Ra^1{}_x{}^{/2}$ and mass transfer rates $Sh_x/Ra^1{}_x{}^{/2}$ with the thermophoretic Nt parameter and the Brownian motion parameter Nb are shown in Table I. The table indicates the effects of the Brownian motion parameter Nb on the dimensionless heat transfer rates for Le_p = 10 (the thermal diffusivity is 10 times the mass diffusivity) and for Le_p = 20 (the thermal diffusivity is 20 times the mass diffusivity).

Table 1 : Values of $Nu_x/Ra^1{}_x{}^{/2}$ and $Sh_x/Ra^1{}_x{}^{/2}$ for selected values of Le_p, Nt and Nb.

		$Nu_x/Ra^1{}_x{}^{/2}$			$Shx_x/Ra^1{}_x{}^{/2}$		
Le_p	Nt	Nb=0.5	Nb=1.0	Nb=2.0	Nb=0.5	Nb=1.0	Nb=2.0
10	0.1	0.2373	0.1580	0.0667	1.2726	1.3008	1.3294
	0.5	0.2021	0.1348	0.0568	1.2933	1.3338	1.3562
	1.0	0.1679	0.1121	0.0472	1.3409	1.3771	1.3862
20	0.1	0.2379	0.1541	0.0623	1.8479	1.8738	1.9005
	0.5	0.2023	0.1311	0.0530	1.8867	1.9150	1.9304
	1.0	0.1676	0.1087	0.0438	1.9480	1.9641	1.9622

It is clear that the dimensionless heat transfer rates decrease with increasing thermophoresis parameter Nt, and also decrease with increasing Brownian motion parameter Nb. However, the dimensionless mass transfer rates increase with an increase in both the thermophoresis parameter Nt and the Brownian motion parameter Nb. Also, an increase in dimensional mass transfer rates accompany an increase in Lewis number.

CONCLUSION

In this paper, we have studied the natural convection boundary layer flow, heat and mass transfer in a porous medium saturated by a nanofluid past a semi infinite inclined plate, via a model in which Brownian motion and thermophoresis are accounted for.

We have used the Darcy model for the momentum equation and we have assumed the simplest possible boundary conditions. the differential partial equations are transformed in to the ordinary differential equations using the local similarity solution which depends on four dimensionless parameters, namely, a nanofluid Lewis number Le_p, a buoyancy ratio parameter Nr, a Brownian motion parameter Nb, a thermophoresis parameter Nt. One can conclude with the following note.

The buoyancy ratio Nr reduces the velocity and expands the volume fraction in nanofluid . Concerning, the thermophoresis parameter Nt, we note that the augmentation of this parameter enhances the temperature fields and thicken the thermal boundary layer. More than that, increasing the Brownian motion number Nb, thermophoresis number Nt, and the angle of inclination α reduces the local heat transfer rate (local Nusselt number) and improves the dimensionless mass transfer rates (local Sherwood number).

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