

A NEW NUMERICAL APPROACH FOR MODELLING OF SILICON PIEZORESISTIVE SENSORS

Reçu le 04/12/2005 – Accepté le 23/11/2007

Résumé

Dans cet article, nous présentons une réponse statique d'une membrane fine au silicium de forme carré ou rectangulaire, soumise à une pression constante et uniforme, dans ce cas de faibles perturbations. Nous utilisons la méthode de Galerkin avec des fonctions de base trigonométrique pour obtenir une solution $w(x,y)$ précise et simple, en tout point de la membrane.

Cette dernière est utilisée comme corps d'épreuve sur laquelle quatre jauge piézorésistives sont implantées et connectées en pont de Wheatstone. Nous avons déterminé les caractéristiques du capteur tel que le facteur de jauge (K), la sensibilité (S_p) et la tension de sortie (ΔV). Les résultats obtenus montrent que la variation de la résistance en fonction de la pression est linéaire sur une gamme de pressions de [0 à 10] bar.

Mots clés : Déformation, méthode de Galerkin, membrane, jauge piézorésistive, capteur de pression, sensibilité.

Abstract

In this paper, we present a static response of a square or rectangular silicon membrane under uniform and constant pressure, in the case of weak perturbations. We employ Galerkin method with trigonometrical basis functions to provide an accuracy and simple solution $w(x,y)$ for the silicon membrane.

We then investigated a piezoresistive pressure sensor that have a silicon diaphragm with rectangular or square shape and employ piezoresistive gages. These gages are linked together in the form of Wheatstone bridge. The gages factor (K), sensitivity (S_p) and output voltage (ΔV) is analysed. The results obtained show that the change of resistance due to pressure was linear over a pressure range of [0 to 10bar].

Keywords : Deformation, Galerkin method, membrane, piezoresistive gages, pressure sensor, sensitivity.

F. KERROUR

F. HOBAR

Laboratoire Microsystèmes et Instrumentations - Faculté des Sciences de l'Ingénieur - Université Mentouri Constantine - Algérie

ملخص

في هذا البحث (المقال)، نقدم الاستجابة الثابتة لغشاء دقيق من سيليسيوم مربع أو مستطيل الشكل، أخضع لضغط ثابت و منتظم، في إطار الاضطرابات الضعيفة.

نستعمل طريقة Galerkin مع دوال أساسية مثلثية قصد الحصول على استجابة بسيطة و دقيقة $w(x, y)$ لغشاء السيليسيوم.

قمنا بعدها بتحصص لاقط ذو مقاومة متغيرة مع الضغط و متكون من غشاء دقيق من السيليسيوم مربع أو مستطيل الشكل، و هذا باستعمال 4 مسابر، ركبت على شكل جسر Wheatston معامل المسobar (K)، حساسيته (S_p) و توتر المخرج (ΔV) فحصوا و حلوا.

النتائج المتحصل عليها تظهر بأن التغير في المقاومة نتيجة الضغط المطبق هو تغير خطى داخل المجال [0 ، 10] بار.

الكلمات المفتاحية: تشوه، طريقة Galerkin، غشاء، مسobar ذو مقاومة متغيرة مع الضغط، لاقط الضغط، حساسية

Several fields require the application of pressure sensors with increasingly high performances. Silicon has good properties like material for piezoresistive pressure sensors, which are improved by micromechanical technology allowing manufacture of mechanical structure three-dimensional miniaturized. A piezoresistive pressure sensor is composed of a micro machined silicon membrane on which four piezoresistive gauges are implanted (or diffused) by a planar process technology [1]. The variations of treatment on the substrate cause fluctuations of the profile of doping and geometry gages bridge. The principle of the piezoresistive sensor is based on the variations of the resistance of the gages when they are subjected to a pressure, in general 60% of the membrane deformation is found on the gages if they are well positioned [2]. This change in they stresses, produces a sensor output voltage proportional to applied pressure. The study of the sensors response leads to determine the silicon membrane deflection $w(x, y)$ as a function of the applied pressure P . Thus in the first time we will study the membrane mechanical behaviour, by determining the relation between the deflection $w(x, y)$ and pressure applied to a square or rectangular membrane having a weak deflection with clamped boundaries. And the influence of the membrane dimension ratio r is analysed. In the second time the different mechanical stress at any point and the center of the membrane deflection w_{00} are also determined by using elasticity theory [3]. Then we locate the gages positions and evaluate their performances. Finally we deduce the sensor output voltage $\Delta V = f(P)$ and pressure sensitivity S_p .

1. MODELLING OF MEMBRANE

A rectangular or square Silicon membrane is described in figure 1. It is a micro structure directed according to the crystallographic plan (110) deposited on a substrate of orientation (100) [4, 5], whose dimensions are (figure 2) :

Length: $2a$ according to the x axis ;
 Width: $2b$ according to the oy axis ;
 height: h according to the oz axis, with $h \ll 2a$ and $h \ll 2b$ (assumptions of thin plate).

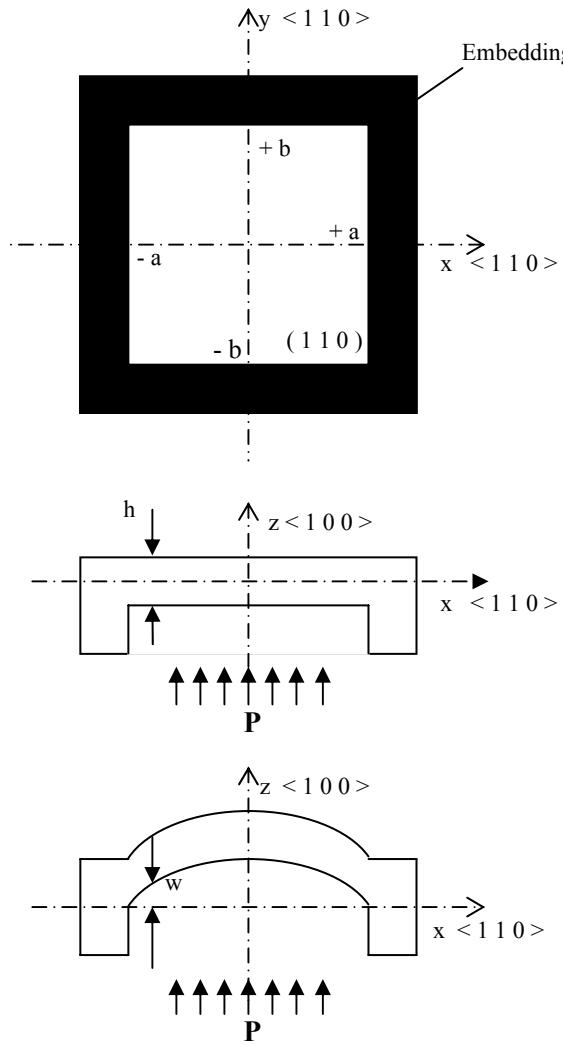


Figure 1 : Schematic top- view of the diaphragm.

We define r ($r = \frac{b}{a}$) as the ratio of membrane dimensions. According to the theory of the thin elastic and anisotropic silicon membrane [3-6], the membrane behaviour is described by the partial differential equation :

$$\frac{\partial^4 w(x, y)}{\partial x^4} + 2\alpha_{si} \frac{\partial^4 w(x, y)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x, y)}{\partial y^4} = \frac{P}{D} \quad (1)$$

Where $w(x, y)$ is the membrane deflection in the case of the weak disturbances, $w \ll h$ and P is a uniform and static pressure. With α_{si} is the anisotropy coefficient and D is the membrane stiffness, they are defined by [7]:

$$\alpha_{si} = \nu + \frac{2G}{E}(1-\nu^2) \quad (2)$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (3)$$

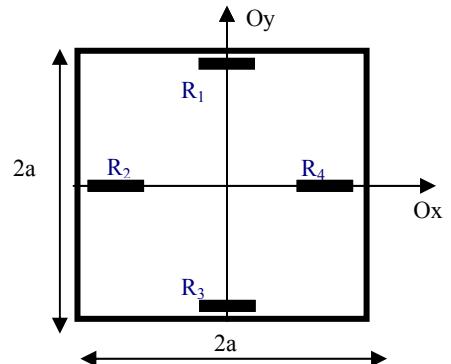


Figure 2a : The gages location on the square membrane

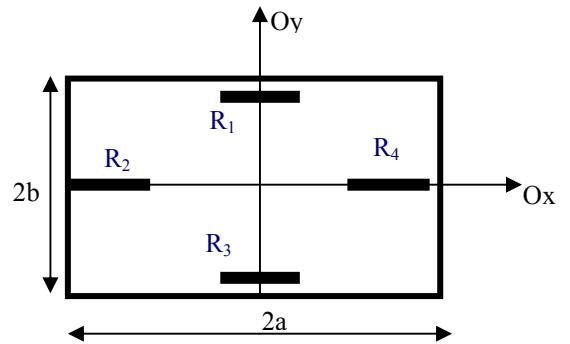


Figure 2b : The gages location on the rectangular membrane

R_1, R_3 are the longitudinal gages
 R_2, R_4 are the perpendicular gages

Figure 2 : Schematic top-view of the diaphragm with piezoresistive gages.

Where ν is the Poisson's ratio, E the Young modulus and G the Coulomb (shear) modulus, P is applied pressure. The boundary conditions imposed by the embedding of the membrane on its edges [3] are:

$$w(x = \pm a, \forall y) = 0 \quad (4a)$$

$$w(\forall x, y = \pm b) = 0 \quad (4b)$$

$$\frac{\partial w}{\partial x}(x = \pm a, \forall y) = 0 \quad (4c)$$

$$\frac{\partial w}{\partial y}(\forall x, y = \pm b) = 0 \quad (4d)$$

This is a fourth equation for $w(x, y)$, the exact solution of this type of problem does not exist.

However several approaches were proposed, in particular the Ritz method and the Galerkin method [8-14].

We use the membrane deflection expression $W(x, y)$ obtained by F.Kerrou and F. Hobar [13, 14] given by:

$$w(x, y) = \frac{w_{00}}{4} \sum_{i=0}^n \sum_{j=0}^n k_{ij} \cos^2\left(\frac{(2i+1)\pi x}{2a}\right) \cos^2\left(\frac{(2j+1)\pi y}{2b}\right) \quad (5)$$

From this expression we can deduce the deflection membrane $w(x, y)$ at any point.

And we define the maximum deflection in the center of the membrane at $x=0, y=0$ w_{00} as [6]:

$$w_{00} = k \frac{Pa^2 b^2}{D} \quad (6)$$

With $k = 0.0224$ and the coefficients k_{ij} are summarized in table1.

Table 1 : Value of the standardized coefficients k_{ij} for $n=3$ and $r=1$

	k	k_{00}	$k_{02}=k_{20}$	k_{22}	$k_{24}=k_{42}$	$k_{40}=k_{04}$	k_{44}
Our model	0.0224	1	0.0284	0.0123	0.0030	0.0038	0.0016

We obtain w_{00} about $4.7529 \mu\text{m}$ which is 2% higher than polynomial results [13-14]. We also found the membrane deflection for different value of the ratio ($r=a/b$).

2. RESULTS AND DISCUSSION

2.1 Membrane modelling

From the expression (5) we plot the normalized deflection $\left[\frac{w(x, y)}{w_{00}}\right]$, $\left[\frac{w(x)}{w_{00}}\right]$ and $\left[\frac{w(y)}{w_{00}}\right]$ curves in figure 3.

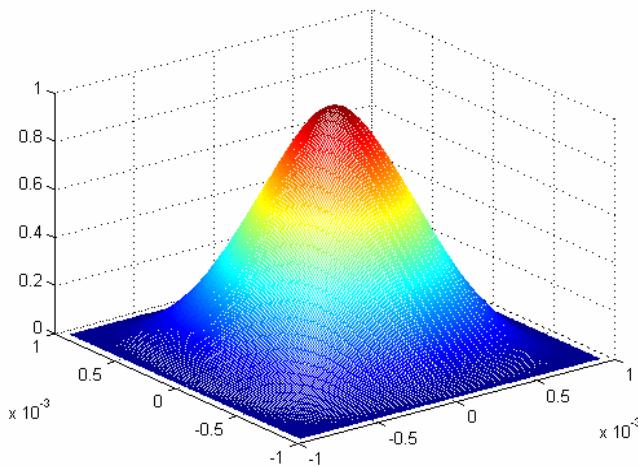


Figure 3a : Normalized Deflection $\left[\frac{w(x, y)}{w_{00}}\right]$

The figure 4 shows the variation of the membrane deflection versus the dimension ratio r , and we can note that it's proportional to the value of the ratio r .

Thus the rectangular membrane is more robust than the square one.

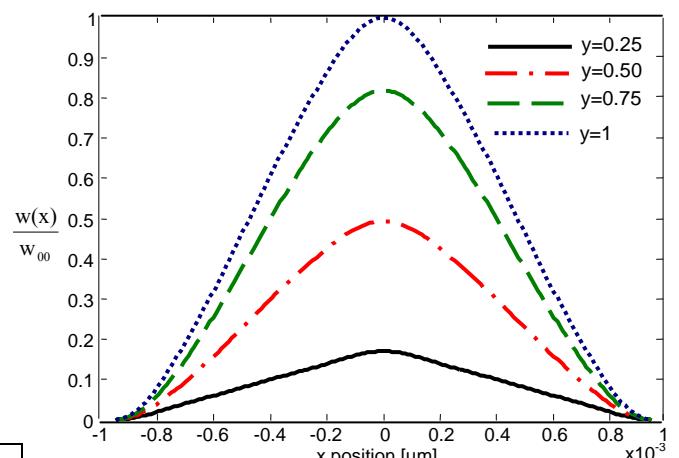


Figure 3b : Normalized deflection $\left[\frac{w(x)}{w_{00}}\right]$ vs x position for different value of y

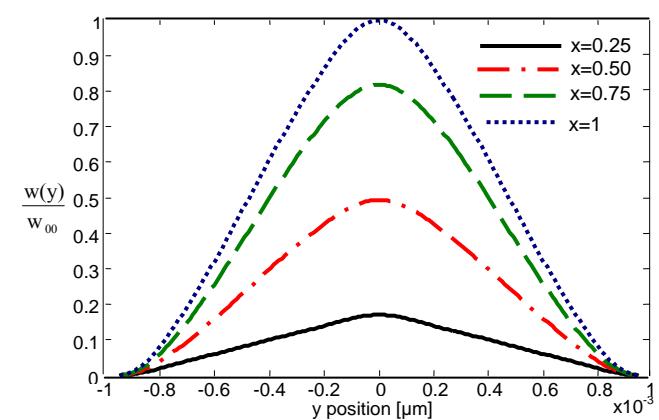


Figure 3c : Normalized deflection $\left[\frac{w(y)}{w_{00}}\right]$ vs x position for different value of x

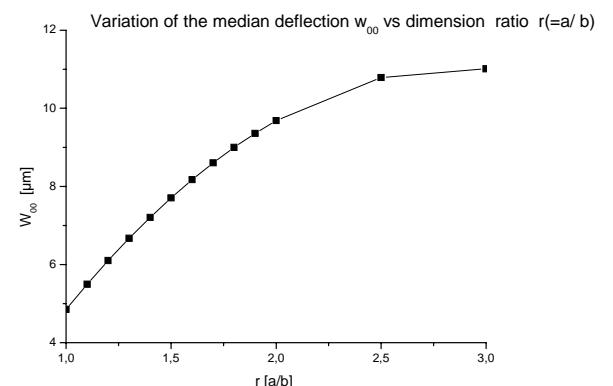


Figure 4 : Variation of the median deflection vs dimension ratio r $[a/b]$

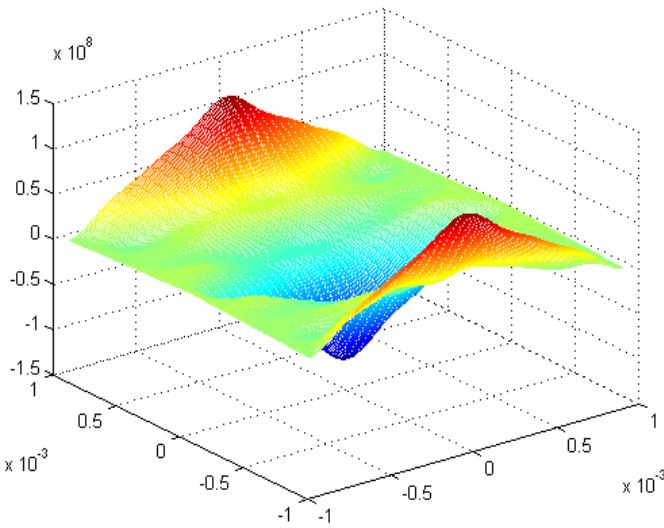


Figure 5 a : Trigonometrical Model : Normal stress $\sigma_{xx}(x, y)$

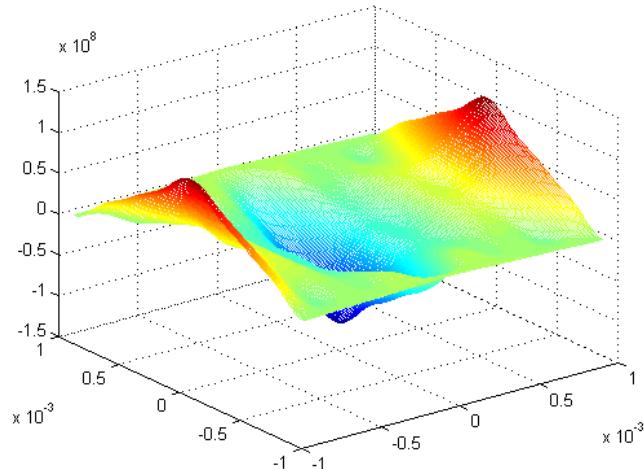


Figure 5 b : Trigonometrical Model : Normal stress $\sigma_{yy}(x, y)$

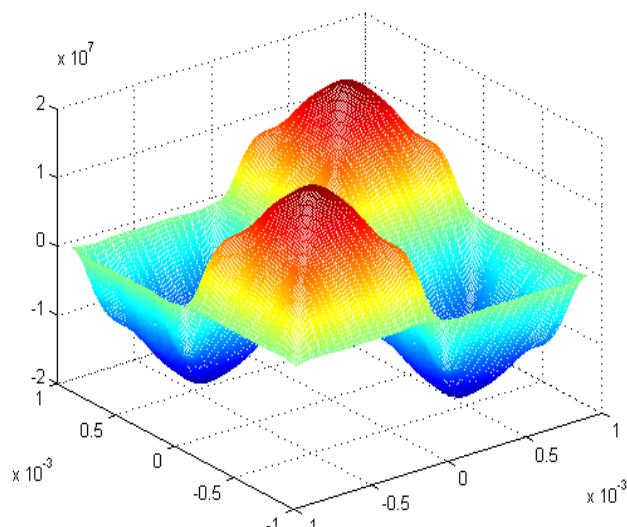


Figure 5 c : Trigonometrical Model : Shear stress $\sigma_{xy}(x, y)$

Substituting expression (5) of the membrane deflection $w(x, y)$ in the equations defining the constraints according to the deformation [9, 10], we obtain the different expressions of the components of the mechanical stress σ_{xx} ; σ_{yy} ; σ_{xy} in the upper of the membrane is given by equations:

$$\sigma_{xx} = -\frac{Ez}{1-\nu^2} \left[\frac{\partial^2 w(x, y)}{\partial x^2} + \nu \frac{\partial^2 w(x, y)}{\partial y^2} \right] \quad (7)$$

$$\sigma_{yy} = -\frac{Ez}{1-\nu^2} \left[\nu \frac{\partial^2 w(x, y)}{\partial x^2} + \frac{\partial^2 w(x, y)}{\partial y^2} \right] \quad (8)$$

$$\sigma_{xy} = -2Gz \frac{\partial^2 w(x, y)}{\partial x \partial y} \quad (9)$$

By carry out all calculations and for $z=-h/2$ we have:

$$\sigma_l = \sigma_{xx} = +\frac{1}{32} \times \frac{Eh}{(1-\nu^2)} \times \frac{Pka^2b^2}{D} \times \mu_{ij1}(x, y) \quad (10)$$

$$\sigma_t = \sigma_{yy} = +\frac{1}{32} \times \frac{Eh}{(1-\nu^2)} \times \frac{Pka^2b^2}{D} \times \mu_{ij2}(x, y) \quad (11)$$

$$\sigma_{xy} = \frac{1}{16} \times \frac{Gh}{(1-\nu^2)} \times \frac{Pka^2b^2}{D} \times \mu_{ij}(x, y) \quad (12)$$

with:

$$\mu_{ij1} = \sum_0^n \sum_0^n k_{ij} P_i^2 (1 + \cos Q_j y) (\cos P_i x) + \nu \sum_0^n \sum_0^n k_{ij} Q_i^2 (1 + \cos P_i x) (\cos Q_j y) \quad (13)$$

$$\mu_{ij2} = \nu \sum_0^n \sum_0^n k_{ij} P_i^2 (1 + \cos Q_j y) (\cos P_i x) + \sum_0^n \sum_0^n k_{ij} Q_i^2 (1 + \cos P_i x) (\cos Q_j y) \quad (14)$$

$$\mu_{ij} = \sum_0^n \sum_0^n k_{ij} P_i Q_j (\sin P_i x) (\sin Q_j y) \quad (15)$$

From equations (6 ÷ 8) we plot the curves constraints $\sigma_{xx}(x, y)$; $\sigma_{yy}(x, y)$; $\sigma_{xy}(x, y)$; $\sigma_{xx}(x)$ and $\sigma_{yy}(y)$ for $n=16$, which are illustrated in figures 5 and 6. The obtained results are comparable in the form but more accurate to those found in literature [9] which validates our calculations.

We note that the mechanical stresses are maximal in middle edges membrane.

The mechanical stress expressions at these positions are

1- For : $x = 0$; $y = \pm b$

$$\sigma_l = \sigma_{xx}(x = 0, y = \pm b) = \nu \frac{1}{16} \times \frac{Eh}{(1-\nu^2)} \times \frac{Pka^2b^2}{D} \times \sum_0^n \sum_0^n k_{ij} Q_j^2 \quad (16)$$

$$\sigma_l = \sigma_{xx}(x = 0, y = \pm b) = \nu * 0.4646 * P \left(\frac{a}{h} \right)^2 = 0.0298 * P * \left(\frac{a}{h} \right)^2 \quad (16-1)$$

$$\sigma_t = \sigma_{yy}(x = 0, y = \pm b) = \frac{1}{16} \times \frac{Eh}{(1-\nu^2)} \times \frac{Pka^2b^2}{D} \times \sum_0^n \sum_0^n k_{ij} Q_j^2 \quad (17)$$

$$\sigma_t = \sigma_{yy}(x = 0, y = \pm b) = 0.4646 * P * \left(\frac{a}{h} \right)^2 \quad (17-1)$$

$$\sigma_{xy}(x = 0, y = \pm b) = 0 \quad (18)$$

2- For : $x = \pm a ; y = 0$

$$\sigma_{xx}(x = \pm a, y = 0) = \frac{1}{16} \times \frac{Eh}{(1-\nu^2)} \times \frac{Pka^2b^2}{D} \times \sum_0^n \sum_0^n k_{ij} P_i^2 \quad (19)$$

$$\sigma_{xx}(x = 0, y = \pm b) = 0.4646 * P * \left(\frac{b}{h}\right)^2 \quad (19-1)$$

$$\sigma_{yy}(x = \pm a, y = 0) = \nu \frac{1}{16} \times \frac{Eh}{(1-\nu^2)} \times \frac{Pka^2b^2}{D} \times \sum_0^n \sum_0^n k_{ij} P_j^2 \quad (20)$$

$$\sigma_{yy}(x = \pm a, y = 0) = \nu * 0.4646 * P * \left(\frac{b}{h}\right)^2 = 0.0298 * P * \left(\frac{b}{h}\right)^2 \quad (20-1)$$

$$\sigma_{xy}(x = \pm a, y = 0) = 0 \quad (21)$$

Where P_i and Q_j :

$$P_i = \left(\frac{(2i+1)\pi}{a} \right) \quad (22)$$

$$Q_j = \left(\frac{(2j+1)\pi}{b} \right) \quad (23)$$

These mechanical stress expressions have a high amplitude (0.4646 ; 0.0298) if they are compared to A.Boukaabache [15] (0.316 ; 0.0202) and C. Plantier [16] (0.3654 ; 0.0664)

2.1 Gages Localization and gages Sensitivity

From the mechanical stress distributions along x and y respectively, we can note that its maximum are in the mediums of the membrane edges, as shown in figures 5-6, and due to the symmetry they change sign while passing from the edge in the membrane center along y axis. For this reason, two of the four piezoresistive gages 1, 3 are placed on the line of centers with ($x = 0 ; y = +/-b$) and two other gauges 2, 4 are placed in the medium of the edges such as their centers are at the points ($x = -a, y = 0$) and ($x = a, y = 0$). In order to get high sensitivity, we choose the conventional orientations of gages two are placed parallel and the two others perpendicular to the membrane edges, and their length should be as small as possible [17-19].

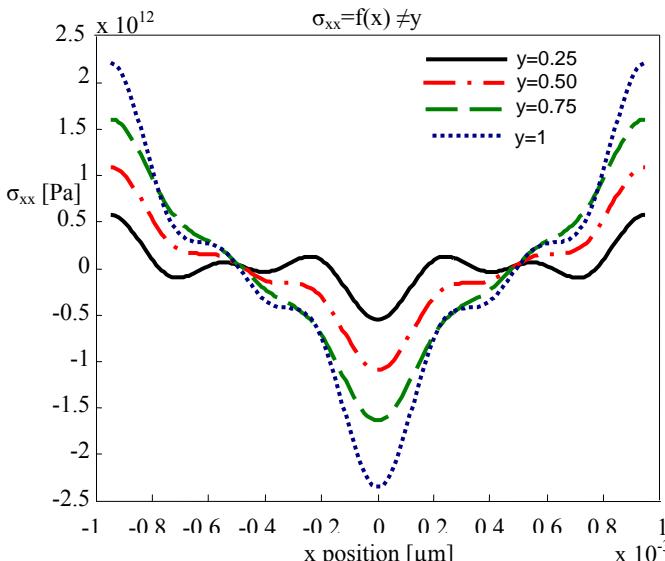


Figure 6a : Longitudinal stress $\sigma_{xx}(x) = f(x)$ for different value of y

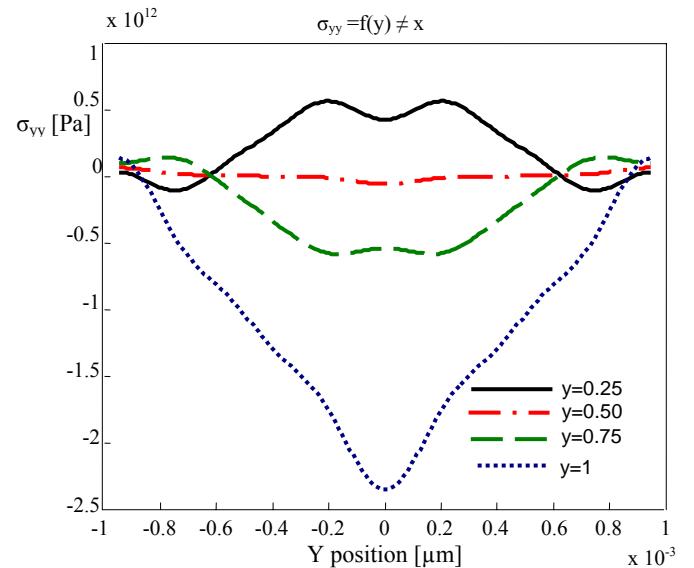


Figure 6 b : Perpendicular stress $\sigma_{yy}(y) = f(y)$ for different value of x

The basic features of the layout of pressure sensor are shown in figure 2, and the simplest sensor circuit can be built using piezoresistors is the Wheatstone bridge.

The figure 7 presents the stress distribution along the resistor for two value of ($n=3$; $n=16$), and it shows the accuracy of the trigonometrical model [13-14]. The change of piezoresistance is given by:

$$\Delta R/R = \pi_l \sigma_l + \pi_t \sigma_t + \pi_c \sigma_c \quad (24)$$

With π_l ; π_t ; π_c are longitudinal, transverse and shearing piezoresistives coefficients respectively and σ_l ; σ_t ; σ_c are longitudinal, transverse and shearing constraints respectively.

Work of Y.Kanda [20-21] showed that these coefficients (π_l , π_t , π_c) depend on the temperature and doping, and can be expressed according to the fundamental piezoresistives coefficients (π_{11} , π_{12} , π_{44}) establish by Smith [22] at 300°K (table 2).

Table 2 : Value of the fundamentals piezoresistive coefficients of silicon

	$\pi_{11}(E-11 [\text{Pa}])$	$\pi_{12}(E-11 [\text{Pa}])$	$\pi_{44}(E-11 [\text{Pa}])$
Sc-P	6.6	-1.1	138.1
Sc-N	-102.6	53.4	-13.6

The values of the piezoresistives coefficients of (π_l , π_t , π_c) are gathered in the table 3, what enables us to write for a Sc-type P (and assumed that π_{44} is the dominant coefficient) :

$$\pi_l = (\pi_{11} + \pi_{12} + \pi_{44})/2 = \pi_{44}/2 \quad (25)$$

$$\pi_t = (\pi_{11} + \pi_{12} - \pi_{44})/2 = -\pi_{44}/2 \quad (26)$$

$$\pi_c = 0 \quad (27)$$

By substituting the equation (25-27) in equation (24), in the case where the mechanical stress is constant over resistor, we can deduce the expressions simplified of the longitudinal ($\Delta R/R$)_{//} and perpendicular ($\Delta R/R$)_⊥ piezoresistors [19]:

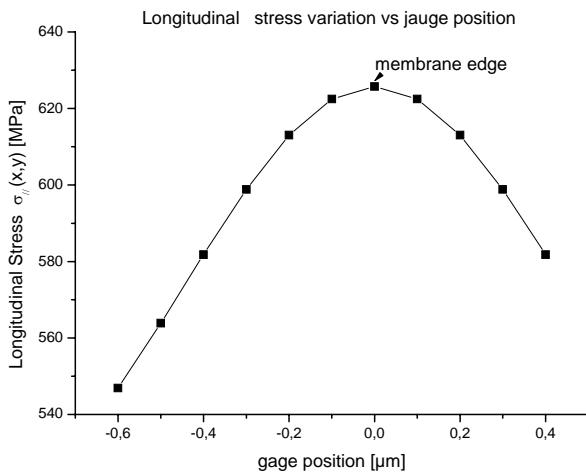


Figure 7a : Longitudinal stress $\sigma_{||}(x, y)$ vs gage position along resistor R₁

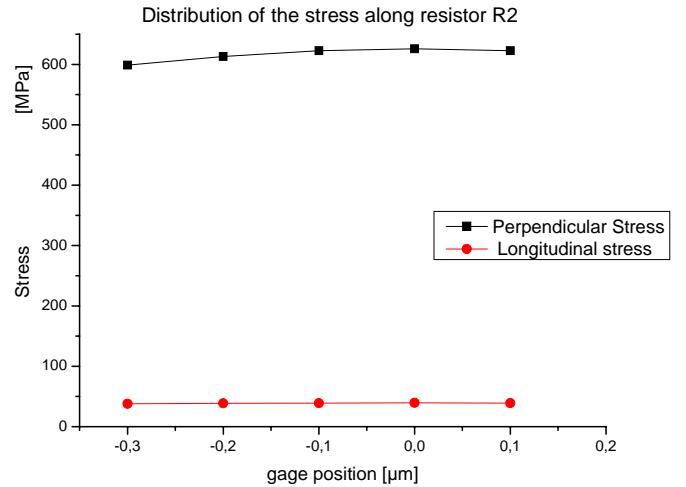


Figure 7d : Distribution of the stress along resistor R₂ ($=R_T$)

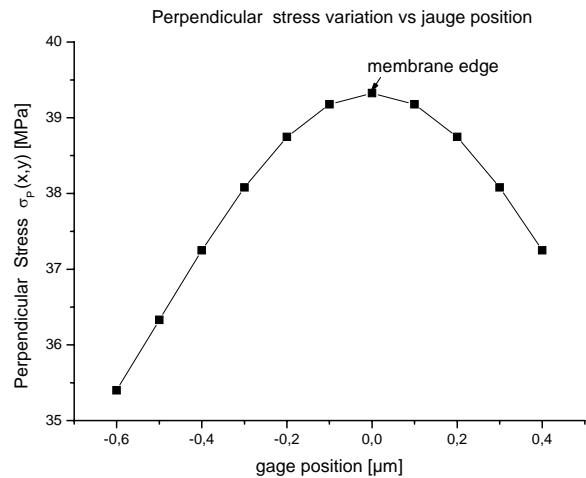


Figure 7b : Perpendicular stress $\sigma_p(x, y)$ vs gage position R₁

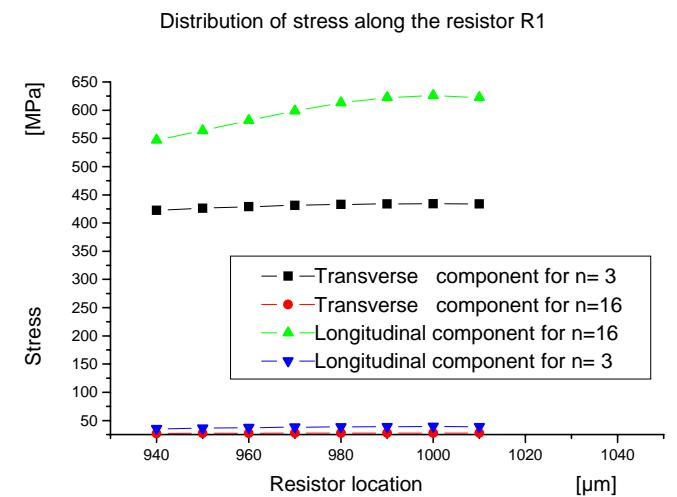


Figure 7e : Distribution of the stress along resistor R₁(R_{||})

$$(\Delta R/R)_{||} = \pi_l \sigma_l + \pi_t \sigma_t = \pi_{44} (\sigma_l - \sigma_t) / 2 \quad (28)$$

$$(\Delta R/R)_{\perp} = \pi_l \sigma_t + \pi_t \sigma_l = -\pi_{44} (\sigma_l - \sigma_t) / 2 \quad (29)$$

The gages sensitivity to the applied pressure is expressed by [19]:

$$S_{||} = (\Delta R/R)_{||} \times \frac{1}{P} = \frac{1}{2P} \times \Pi_{44} (\sigma_l - \sigma_t) \quad (30)$$

$$S_{\perp} = (\Delta R/R)_{\perp} \times \frac{1}{P} = -\frac{1}{2P} \times \Pi_{44} (\sigma_l - \sigma_t) \quad (31)$$

Due to the piezoresistive effect the pressure P applied to a sensor changes its output voltage ΔV according to [19]:

$$\Delta V = V_a * (\Delta R/R) \quad (32)$$

Where V_a is the bridge supply tension.

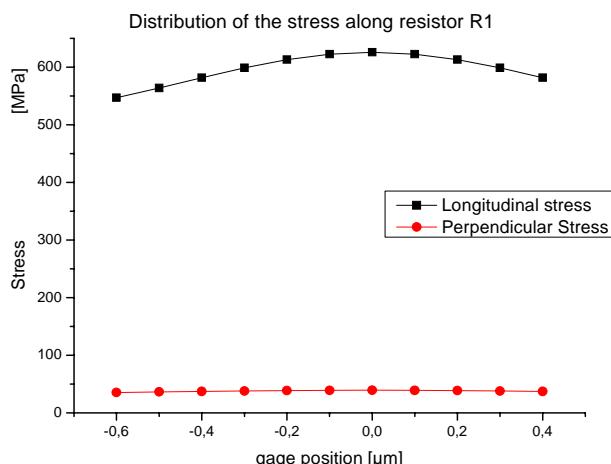


Figure 7c : Distribution of the stress along resistor R₁ ($=R_{||}$)

We can thus define the sensor sensitivity to the applied supply V_a noted S_p [19]:

$$S_p = \frac{\Delta V}{P} \times \frac{1}{V_a} = \frac{\Delta R}{P} \times \frac{1}{R} = \frac{1}{2P} \times \Pi_{44}(\sigma_l - \sigma_t) \quad (33)$$

This coefficient of proportionality or sensitivity S_p relies on mechanical, electrical and geometrical parameters. Obviously, it is desirable to select a diaphragm configuration that will enhance the difference $(\sigma_l - \sigma_t)$. The figure 8 presents the variation $\frac{\Delta V}{V_a}$ according to the applied pressure P.

It shows that the sensor response is linear and procures sensitivity S_p about 280 [mV/V/bar] and a gage factor K ($=\Delta R/R$) about 105.

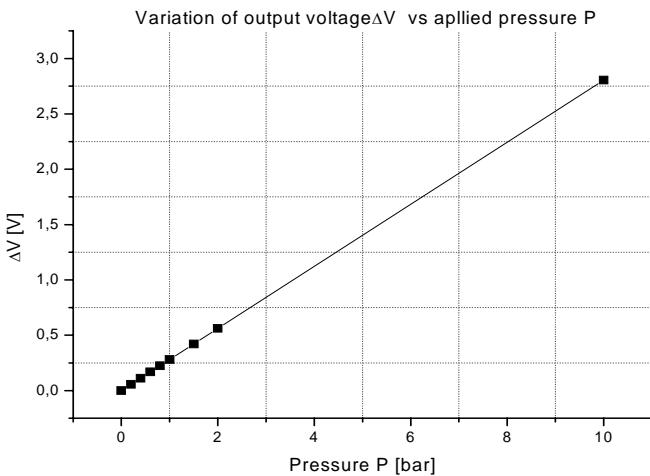


Figure 8 : Output voltage ΔV according to the applied pressure P

Table 3 : Expression of the piezoresistive coefficients π_l , π_t , π_c versus P type silicon in function of crystallographic orientation.

plan	Orientations	π_l	π_t	π_c
(100)	[100]	π_{11}	π_{12}	0
	[110]	$(\pi_{11} + \pi_{12} + \pi_{44})/2$	$(\pi_{11} + \pi_{12} - \pi_{44})/2$	0
(011)	[011]	$(\pi_{11} + \pi_{12} + \pi_{44})/2$	π_{12}	0
	[100]	π_{11}	π_{12}	0
	[111]	$(\pi_{11} + 2\pi_{12} + \pi_{44})/3$	$(2\pi_{11} + \pi_{12} - \pi_{44})/3$	$0.16(\pi_{11} - \pi_{12} + \pi_{44})$
(111)	All directions	$(\pi_{11} + \pi_{12} + \pi_{44})/2$	$(5\pi_{11} + \pi_{12} - \pi_{44})/6$	0

This presents a moderate discrepancy compared with experimental data [15-16], due to the process induced variation in membrane thickness, size, resistor size and its alignment. The obtained results show that the change of resistance due to pressure was linear over a pressure range of [0 to 10 bar].

CONCLUSION

In this work we present a method for determination of silicon pressure membrane deflection $W(x, y)$ using the Galerkin method with trigonometrical basis function. The maximum membrane deformation is about $4.7529\mu\text{m}$ and it is slightly higher than to say an inaccuracy about 2% tolerated by the polynomial model. The influence of the membrane shape has been analyzed.

This trigonometrical model allows to obtain more accurate results and with a reduced computing time. Using elastic theory we carry out the expression of mechanical stress as a function of applied pressure. The determination of the optimum position of piezoresistance allows us to have a good sensitivity. The obtained results show that the change of resistance due to pressure was linear over a pressure range of [0 to 10 bar]. The gage factor K is about 105 and the sensor sensitivity is about 280 [mV/V/bar].

REFERENCES

- [1]- S. Mir : "Conception des microsystèmes sur Silicium", 2002 Hermès Sciences.
- [2]- L. Cao, T. Song Kim, S.C. Mantell, D.L. Polla: "Simulation and fabrication of piezoresistive membrane type MEMS strain sensors" Sensors And Actuators 80(2000) pp 273-279.
- [3]- S.P. Timchenko, S. Woinoiwsky-Krieger. "Theory of plates and shells". Mc Graw-hill; 1982.
- [4]- Y.L. Inden, L. Tenerz, J. Tiren and B. Hok. "Fabrication of three dimensional Silicon structure by means of doping selective etching". Sensors And Actuators 1989; (16): p 67.
- [5]- X.Y. Ye, J.H. Zhang, Z.Y. Zhou and Y. Yang "Measurement of Young's Modulus and Residual Stress of Micromembranes" Seven international Symposium on Micro Machine and Human Science" (1996) IEEE, pp 125-P.R.
- [6]- C. S. Sander, J.W. Knutti and J.D. Meindel. A monolithic capacitive pressure sensor with pulse -period output. IEEE transaction on electron devices 1980; ED 27 (5): 927.
- [7]- J.J. Wortmans, R.A. Evans. "Young's modulus, Shear modulus and Poisson's ratio in Silicon and Germanium". Journal of Applied Physics. 1965; 36 (1):153.
- [8]- D. Maier-Schneider, J. Maibach, and E. Obermeier. "A new Analytical solution for the load-deflection of square Membranes". Journal of Micro electromechanical systems 1995; (4): 238-241.
- [9]- G. Blasquez, Y. Naciri, P. Blondel, N. Benmoussa and P. Pons, "Static response of miniature pressure sensors with square or rectangular diaphragm", Revue Phys appl. 22, (1987), pp 505-510
- [10]- R. Steinmann, H. Friemann, C. Presher and R. Schellin. "Mechanical behaviour of micro machined sensor membrane under uniform external pressure affected by in plane stresses using a Ritz method and Hermite polynomials". Sensors And Actuators 1995; A48: 37-46.
- [11]- H.E. Elgamel. "Closed form expressions for the relationships between stress, diaphragm deflection and resistance change with pressure in silicon piezoresistive pressure sensors". Sensors and Actuators 1995; A50: 17-22.
- [12]- D. Young, "Vibration of rectangular plates by the Ritz method". Journal of Applied Mechanical: 1950; (17): 448.
- [13]- F. Kerrou, F. Hobar : Nouvelle approche pour la résolution de l'équation différentielle de Lagrange régissant la déformation d'une membrane fine au silicium" ISESC'05 june 19-21, 2005, Jijel University, Algeria, IEEE.
- [14]- F. Kerrou F. Hobar: "A novel numerical approach for modelling of the square shaped silicon membrane" Semiconductor Physics, Quantum Electronics & Optoelectronics, 2006.V9, N4.pp 52-57.

- [15]- A. Boukaabache ‘Conception, modélisation et réalisation d’un capteur de pression piezoresistif à faible dérive thermique’ thèse de doctorat d’état de l’université de Constantine, 1993.
- [16]- C. Plantier “Etude de faisabilité de capteurs de pression piezoresistifs à jauge en silicium poly cristallin” thèse de doctorat de l’université Paul Sabatier de Toulouse 1992.
- [17]- S. Mir “silicium Dispositif et physique des MEMS au Silicium”, pp 19-27, 2002. Hermes Sciences.
- [18]- H. Johari “Development of MEMS sensors for measurements of pressure, relative humidity and temperature” thesis of degree of master sciences in mechanical engineering, Worcester Polytechnic institute. April 2003.
- [19]- A.V. Gridchin, V.A. Gridchin,” The four-terminal piezotransducer: theory and comparison with Piezoresistif Bridge” Sensors And Actuators A58 (1997) pp 219-223.
- [20]- Y. Kanda “A Graphical Representation of the Piezoresistance Coefficient in Silicon” IEEE transaction on electron devices1980; ED-29, N°.1 1982 pp 64-74.
- [21]- Y. Kanda “Piezoresistance Effect of Silicon” Sensors and Actuators Vol A28 p83 (1991)
- [22]- C.S. Smith “Piezoresistivity in Silicon and Germanium” Phys. Rev. 94, pp 42-49, (1954).