AN ATTITUDE CONTROL LAW A SPACE CRAFT BASED ON SWITCHING FUNCTION SELECTION

Meriem BENENIA, Malek BENSLAMA

Electromagnetic and Telecommunication laboratory,
University of « Frères Mentouri Constantine », Algeria

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Résumé

Cet article est consacré au problème de contrôle d'attitude non linéaire pour un satellite rigide basé sur une commande en mode glissant. La fonction de sortie qui doit être suivie représente les paramètres du quaternion d'attitude. Un dispositif de commande en mode glissant a été conçu pour forcer les variables d'état du système en boucle fermée à converger vers les valeurs de consigne avec la discussion de deux cas.

L'approche proposée est définie de manière à ce que les paramètres de la commande sélectionnés puissent conduire l'état du système à atteindre la surface de glissement et le maintenir dans cette région. La commande est également simplifiée avec des performances de suivi stable. La structure de contrôle de base est présentée. Les résultats de simulation montrent que la commande précise d'attitude est effectuée en dépit de l'incertitude dans le système.

Mots clés *: Contrôle d'attitude d'un engin spatial, paramètres de quaternion, commande par mode glissant, surface de glissement*

Abstract

This paper addressed the problem of nonlinear attitude control for a rigid satellite using a sliding mode control, in which the output function to be tracked is the quaternion attitude parameter. A sliding mode controller has been designed to force the state variables of the closed loop system to converge to the desired values with discussing the two cases.

The proposed approach is defined in such a way that the selected controller parameters can drive the state to hit the sliding surface and keep it in this state. The controller is also simplified with stable tracking performances. The basic design of the controller is presented in this paper, and simulation results show that precise attitude control is accomplished in spite of the uncertainty in the system.

Key words*: Spacecraft attitude control, quaternion parameter, output tracking control, sliding mode control, sliding surface.*

ملخص

وتناولت هذه الورقة مشكلة التحكم غير الخطي على توجيه القمر الصناعي باستخدام التحكم في وضع الانزلاق، دالة المخرج الواجب اتباعها هي معاملات كواتيرنيون التوجيه. وقد تم تصميم وحدة تحكم وضع انزلاق لإجبار متغيرات حالة النظام في حالة حلقة مغلقة لتؤول جميعها الى نقطة القيم المستهدفة مع مناقشة حالتين.

يتم تعريف المنهجية المقترحة بحيث معاملات قانون التحكم المحدد يؤدي النظام للوصول إلى سطح انزلاق والاحتفاظ بها في هذه المنطقة. يتم تبسيط قانون التحكم أيضا مع أداء تتبع مستقر. وقد قدمت بنية قانون التحكم الأساسي في هذا البحث. وتبين نتائج المحاكاة انه تم تنفيذ دقيق لقانون التحكم في التوجيه على الرغم من عدم اليقين في النظام

الكلمات المفتاحية *: التحكم في التوجيه لمركبة فضائية، معاملات كواتيرنيون ، المراقبة بوضع الانزلاق، سطح الانزلاق*

Many studies have been written on control engineering, describing new techniques for controlling systems, or new and better ways of mathematically formulating existing methods to solve the ever increasing complex problems faced by practicing engineers [1].

However, control problem of a spacecraft is an important topic in automatic control engineering. A body orbiting the Earth has instabilities in attitude dynamics and disturbances caused by the Earth, the Moon, the Sun and other bodies in space. These effects force the body to lose initial orbit and attitude. Here the control system takes important part of spacecraft missions where it keeps the body in designed orbit and desired attitude.

The sliding mode theory has an attention in the aerospace field. The technique permits the use of a lower order system model for generating control commands. On the other hand, the system is robust to the external disturbances and includes unmodelled dynamics [2], as well. In general, the spacecraft motion is governed by the so-called kinematic and dynamic equations [3].

Actually, mathematical descriptions are highly nonlinear and thus, the conventional linear control techniques are not suitable for the controller design, especially when large-angle spacecraft maneuvers are required. Variable structure systems with nonlinear control techniques and dead-band on switching function for sliding mode controllers are introduced [4].

A variable structure control design for rigid body spacecraft attitude dynamics with quaternion representation for optimal sliding mode control which consists of three parts [5]: equivalent control, sliding variable, and relay control where simulation results illustrate that the motion along the sliding mode is insensitive to parameter variations and unmodeled effects is given [6].

A smooth sliding mode control which requires well-estimated initial condition for quaternion based spacecraft attitude tracking maneuver is studied [7] where the chattering is eliminated by replacing saturation instead of signum function. A class of uncertain nonlinear systems decoupled by state variable feedback with sliding mode approach for attitude control of an orbiting spacecraft is considered [8].

The paper is organized as follows. Section 1 gives an introduction to sliding mode control of a satellite. Section 2 gives the system description of satellite's dynamics. Section 3 evaluates the control design with designing two cases of sliding surface. Section 4 includes simulation results. Section 5 concludes the paper.

1. SYSTEM DESCIPTION

The equations of motion of a spacecraft attitude dynamics can be divided into two sets [5], the kinematic equations of motion and dynamic equation of motion [7].

1.1. Dynamic equations of motion

The attitude dynamic equations of a rigid spacecraft are given by:

$$I\dot{\omega} + \omega \times I\omega = u + d \tag{1}$$

Where:

$\omega = [\omega_1 \quad \omega_2 \quad \omega_3] \in R^{3x1}$, is the angular rate of the spacecraft;

$\mathbf{u} = [u_1 \quad u_2 \quad u_3] \in R^{3x1}$, represents the control vector;

$\mathbf{d} = [d_1 \quad d_2 \quad d_3] \in R^{3x1}$, are bounded disturbances acting on the spacecraft body;

and, I: is the inertia matrix.

For simplicity, let : $f = -\mathbf{I}^{-1}[\omega \times \mathbf{I}\omega]$, $\mathbf{b} = \mathbf{I}^{-1}$ therefore,

$$\dot{\omega} = \mathbf{f} + \mathbf{b}\square + \mathbf{d} \tag{2}$$

1.2. Kinematic equations of motion

The attitude kinematics part of a spacecraft can be represented by using various attitude parameters, but representation through quaternion parameter has the property of non-singularity and it is free from the trigonometric component. Therefore, this representation is widely used to study the attitude behavior of spacecraft. The kinematics of the satellite model is the part which expresses the relation between the attitude and angular velocities of the body. The kinematic equation through unit quaternion representation is given as [9]:

$$\dot{q} = \frac{1}{2}\Omega(\omega)q = \frac{1}{2}\Xi(q)\omega \tag{3}$$

For which

$$\Omega(\omega) = \begin{bmatrix} -[\tilde{\omega}] & \omega \\ -\omega^T & 0 \end{bmatrix} , \quad \Xi(q) = \begin{bmatrix} q_4 I_{3\times3} + [\tilde{q}_{13}] \\ -q_{13} \end{bmatrix}$$

The notations $[\tilde{\omega}], [\tilde{q}]$ represents the matrixes:

$$[\tilde{q}_{13}] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} , \quad [\tilde{\omega}] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

ω is the body coordinate angular velocity vector represented in body frame with respect to the orbit frame.

We note that another element of the attitude quaternion ''q_4'' is automatically determined from equation:

$$q^T q = q_{13}^T q_{13} + q_4^2 = 1 \tag{4}$$

The temporal derivative of the attitude quaternion is:

$$\dot{q}_{13} = -\frac{1}{2}\omega \times q_{13} + \frac{1}{2}q_4\omega \tag{5}$$

In a step further and deriving the output one more time we obtain the following:

$$\ddot{q} = \frac{1}{4}\omega \times (\omega \times q_{13}) - \frac{1}{4}\omega^T q_{13}\omega + \frac{1}{2}([\tilde{q}_{13}] + q_4 I_{3\times 3})$$
$$(J^{-1}\omega) \times (J\ \ \omega) + \frac{1}{2}([\tilde{q}_{13}] + q_4 I_{3\times 3})J^{-1}u$$
$$\equiv \alpha(\omega, q) + \beta(\omega, q)u \qquad (6)$$

1.3. System errors

Let define the signal of the error by:

$$e(t)=q_{13}(t)-r(t) \qquad (7)$$

Where r: is the reference trajectory for the corresponding output control. In general, the follow up of the asymptotic behavior of the control law is more practical than the exact one using the initial error between the reference and the actual system.

1.4. Reference determination

The reference trajectory for the altitude quaternion should be determined. To do so we assume that the initial state quaternion (q^0_{13}) and final state quaternion (q^f_{13}) are given by:

$q_{13}(t_0) = q_{13}^0$ at $t = 0$, and $q_{13}(t_f) = q_{13}^f$ at $t \to \infty$

$$r(t) = q^0_{13} + (q^f_{13} - q^0_{13})(1 - e^{-t/\tau}) \qquad (8)$$

Where, $r = [r_1, r_2, r_3]^T \in R^3$ and $(\tau > 0)$ is the time constant that we need to select. We notice that the higher order derivatives of this reference trajectory do exist.

The temporal derivatives of the signal are:

$$\dot{r}(t) = (q^f_{13} - q^0_{13})(1/\tau)e^{-t/\tau} \qquad (9)$$

$$\ddot{r}(t) = (q^f_{13} - q^0_{13})(1/\tau^2)e^{-t/\tau} \qquad (10)$$

Controller objective is to drive the attitude states (q, ω), from the initial states (q(0),$\omega(0)$) to the desired states (q^f, ω^f), while the constraint $|\omega| \le \omega_{max}$ are met during the attitude maneuver.

Assumption 1: In spacecraft model equations (2) and (3), the unit-quaternion q and the body angular velocity ω are available in the feedback control design.

Assumption 2: The external disturbance d is assumed to be bounded and to satisfy the following condition: $\|d(t)\| < d_{max}$

Assumption 3: initial angular velocity of the satellite body is zero.

2. SLIDING MODE DESIGN

The design of a sliding mode controller (SMC) involves designing of a sliding surface that represents the desired stable dynamics and a control law that makes the designed sliding surface attractive [4,10]. The phase trajectory of a SMC can be investigated in two parts representing two modes of the system. The trajectories starting from a given initial condition of the sliding surface tend towards the sliding surface. This is known as reaching or hitting phase and the system is sensitive to disturbances in this part of the phase trajectory. When the hitting to the sliding surface occurs, the sliding phase starts **[5, 9, 10]**. In this phase the trajectories are insensitive to parameter's variations and disturbances.

2.1. Sliding Surface Design

The sliding surface is defined by:

$$s(t) = \dot{e} + c_1 e - c_0 \int e dt = 0 \qquad (11)$$

Where: c_1, c_0 are constants of a stable closed loop system.

We assume that the sliding condition determined as follows is satisfied:

$$\frac{1}{2}\frac{d^2 s}{dt^2} \le -\eta\ |s|, \ \eta > 0 \qquad (12)$$

The sliding mode control can be constructed from the vector equation of the sliding surface.

Note that $e = [e_1, e_2, e_3]^T$ represents the error vector, and $s = [s_1, s_2, s_3]^T$ the sliding surface. The sliding mode controller is constructed according to the condition $\dot{s} = 0$. and, after algebraic calculations the final form of sliding mode controller that satisfies the sliding condition in the sliding equation (12) is represented in the following form, [11]:

$$u = \beta^{-1}[\ddot{r} - \alpha - c_1(\dot{q} - \dot{r}) - c_0(q - r) - K.\mathrm{sgn}\ (s)] \qquad (13)$$

Where: K is a third order diagonal square gain matrix to be determined,

Sgn (s): represents the signum function vector

α, β are the parameters defined in equation (6), and C_1,C_0 are constant diagonal matrices, we assume that β^{-1} exists. We can easily see that:

$$\beta^{-1} = \frac{1}{q_4}\begin{bmatrix} q_4^2 + q_1^2 & q_3 q_4 + q_2 q_1 & q_3 q_1 - q_2 q_4 \\ -q_3 q_4 + q_2 q_1 & q_4^2 + q_2^2 & q_4 q_1 + q_3 q_2 \\ q_3 q_1 + q_2 q_4 & -q_4 q_1 + q_3 q_2 & q_4^2 + q_3^2 \end{bmatrix}$$

2.2. First case of sliding surface selection (for c_0=0)

It is clearly seen from the system (11) describing the sliding surface, that if the contact c_0 is equal to zero, so we obtain a linear sliding surface as follows:

$$s(t) = \dot{e} + c_1 e = 0 \tag{14}$$

2.2.1. Control Law Design

The sliding mode control law divided into two main parts [4]:

$$u(t)=u_q(t)+u_d(t) \tag{15}$$

The first component of the proposed controller is $u_q(t)=[u_{q1}, u_{q2}, u_{q3}]$ which will make the sliding surface s(t) invariant and it is calculated by setting $\dot{s}$ to zero and considering s(t) to be zero. Second component $u_d(t)=[u_{d1},u_{d2},u_{d3}]^t$ is an extra control effort which forces the quaternion and angular velocity component to reach on sliding surface in finite time in spite of disturbances and it is computed according to constant reaching law [10, 12] as:

$$u_d = - K\ \mathrm{sgn}(s) \tag{16}$$

Where k is a positive definite gain vector which will be derived in the next section

From equation (13) we can determinate $u_q(t)$ as:

$$u_q(t)= \beta^{-1}\left[\ddot{r} - \alpha - c_1(\dot{q} - \dot{r})\right] \tag{17}$$

The control law in equation (13), and for this case (c_0=0) has two design parameters (**c_1**, **k**) that should be selected to provide stability and better performance. The sliding surface slope, **c_1** is selected such that the system during the sliding mode is stable. In this section, c_1 selection is depending on the body angular velocity constraint. Most of the studies are getting the suitable values of them by try and error with many runs of the algorithm. And this process repeated every time the initial error is changed. For spacecraft attitude control these parameters should be known in real time. The proposed algorithm suggests a geometric rule to compute the discontinuous feedback gain matrix for fast reaching sliding mode control.

2.2.2. Feedback Gain Vector (K) Selection

In this section we propose a novel fast reaching sliding mode**,** the use of a large enough discontinuous signal, k is necessary to complete the reach ability condition despite perturbations, but as small as possible in order to limit the chattering **[5]**.

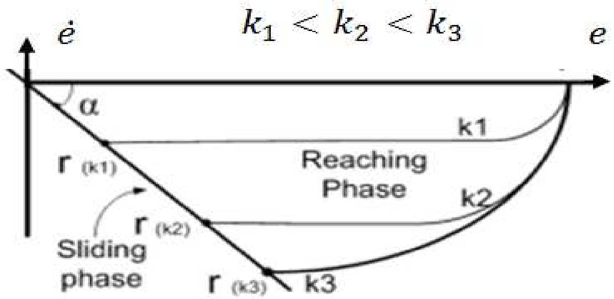


Figure 1: Error phase-plane at different values of k

For small value of k, the state trajectories take more time and long path to reach and vice-versa. Therefore for a pre-specified sliding surface slope there is a certain value of k which reduces the reaching phase, consequently and decreases the reaching time.

By substituting the error Eq. (7) into the control signal u in Eq. (13) and after algebric calculations, we obtain :

$$\ddot{e} - c_1\dot{e} = k\ \mathrm{sgn}(s) \tag{18}$$

The objective is to get the value of k which makes the smallest reaching time to the sliding surface, for this we can see from figure.1 that for a predetermined value of c_1 there is a value of the gain vector k which makes fast reaching time, and reduces the reaching phase.

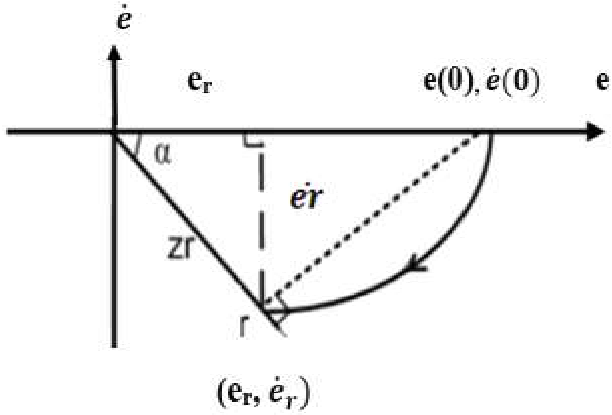


Figure 2 : Selection of reaching point position

We notice that, k is mainly depending on the position of reaching point (e_r, $\dot{e}_r$) and on the sliding line slope c_1, which is taken equal to 1. So, to decrease the reaching time to the sliding surface, the reaching point "r" is selected as the intersection between the perpendicular line from the initial point and the sliding surface, as shown in figure 2.

From figure 2, we can distinguish some equalities:

$$Zr = e(0)\cos(\alpha);\ \text{with: } \alpha = \mathrm{artg}(c) \tag{19}$$

$$er = zr\cos(\alpha) = e(0)\cos^2(\alpha) \tag{20}$$

$$\dot{e}_r = zr\sin(\alpha) = e(0)\cos(\alpha)\sin(\alpha) = \frac{e(0)}{2}\sin(2\alpha) \tag{21}$$

If we derivate equation (20), we obtain:

$$\ddot{e}_r = e(0)(\cos^2(\alpha)-\sin^2(\alpha)) = e(0)\cos(\alpha) \tag{22}$$

To get k, in eq.18, we put: $\ddot{e} = \ddot{e}_r$, and $\dot{e} = \dot{e}_r$

2.3. Second case sliding surface selection (for $c_0 \neq 0$)

The sliding surface is defined in eq.11, in this case the control signal will be the same of eq.13 and the fast reaching is not discussed in this case **[9].**

3. SIMULATION RESULTS

The attitude control problem of a rigid body spacecraft system is simulated and results are presented in this section to illustrate the performances of the sliding mode control law proposed in this paper. Consider a rigid spacecraft with the nominal inertia matrix I = [14.28 0 0; 0 15.74 0; 0 0 12.5] kg.m^2. The initial attitude orientation of the unit-quaternion is q (0) = [0.1603, -0.1431, 0.06252, 0.9746]^T (which is equivalent to initial Euler angles E(0) = [20 -15 10] deg) , and the initial value of the angular velocity is ω(0) = [0, 0, 0]T rad/s. The disturbances are assumed to be bounded by d_{max}= [0.0001 ; 0.0001 ; 0.0001] Nm

The first case algorithm computes the suitable feedback gain, k = [.03992 .03564 .01563] required to steer the satellite from initial attitude condition to desired attitude. With steady state error =[8.038(10)-5 7.27(10)-5 9.174(10)-5] and slope, c=[1 1 1]. By using the function that replacing signum function, the control signals have no chattering.

To validate the developed control law, the results are compared with a first case of sliding surface and the second case. A simulator of sliding mode controller is so built as:

1. First case:

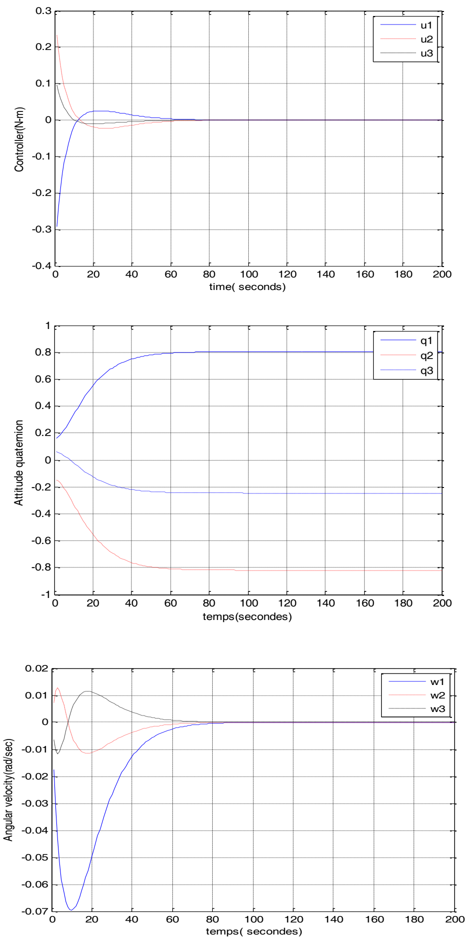


Figure 3 : Simulation results for $c_0 = 0$

The response of the output attitude quaternion follows asymptotically the final attitude quaternion. The angular velocity responses are obtained from the equations describing the behavior of the quaternion. The convergence of speed Angular to zero is evident. Overall, the performance of the asymptotic tracking is reached.

Second case:

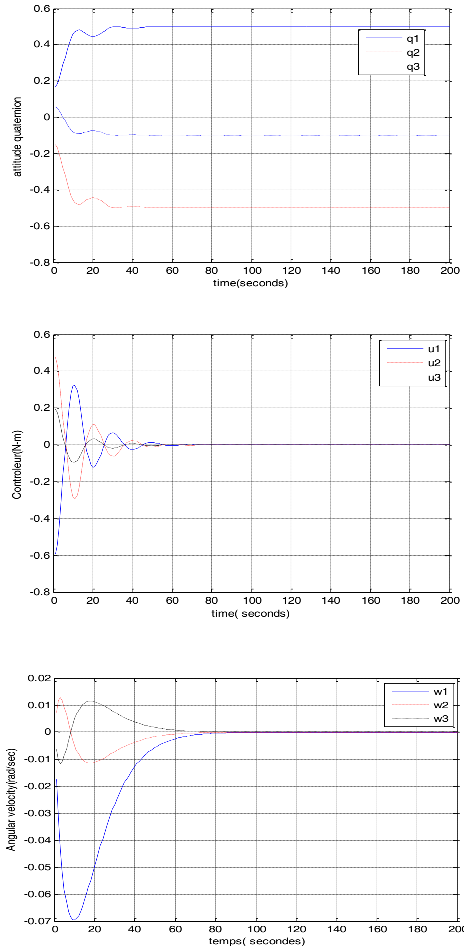
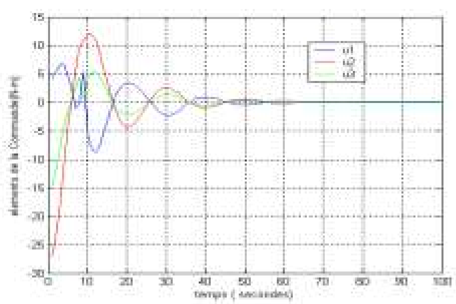


Figure 4 : Simulation results for $c_0 \neq 0$

In this case also, the performance of the asymptotic tracking is reached. The retching is faster, with almost 20 seconds, so a nonlinear sliding surface performance's is butter than the linear sliding function.

Results comparison with a feedback controller:



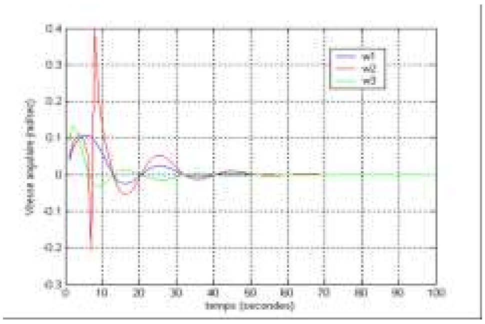


Figure 5 : Simulation results with feedback controller

Comparing the feedback controller's results with this of the proposed approach before, we can see that we reach our objectives with a smaller value of command (25 times smaller) and that is very important in for actioners and energy consumption, and we note a faster retching of final point.

CONCLUSIONS

In this paper a nonlinear controller for attitude maneuvers of a satellite subject to sliding surface selection is developed. The quaternion vector is adopted as the attitude parameters for global representation without singularities or trigonometric functions calculation. The main contribution of this work is the process of selection for the switching function parameters. The proposed algorithm is able to choose the sliding surface and to tune the discontinues feedback gains according to how far is the initial states of the system from the desired states such that the reaching time and tracking error in the approaching phase can be significantly reduced. In addition the computation of controller parameters is simple and its tuning is straightforward. Moreover, the proposed control strategy can guarantee no chattering.

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