EFFECT OF GEOMETRICAL SINGULARITY ON BUCKLING BEHAVIOR OF RECTANGULAR LAMINATED PLATES

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Résumé

Le présent travail concerne l'analyse de l'instabilité par flambage élastique des plaques stratifiées munies d'une singularité géométrique. Le flambage des plaques stratifiées en matériaux composite est un phénomène très complexe, pour l'analyse du flambage des plaques minces stratifiées, nous avons employé un élément de quatre nœuds 32 degré de liberté, la formulation a été basée sur la théorie de Kirchhoff étendue au plaque stratifiées en adoptant l'approche mono couche équivalente.

Nous présentons en suite la formulation du problème d'instabilité en utilisant le principe de la variation seconde de l'énergie potentielle pour la construction des matrices de rigidité. Une série d'exemples a été testé au flambage des plaque mince isotropes et stratifiées, les résultats obtenus et comparés a ceux disponible dans la littérature, ont montré la rapidité de convergence et la bonne performance de l'élément. Une étude paramétrique a été entreprise pour mettre en évidence l'effet de certains paramètres sur le comportement de flambage des plaques minces munies d'ouvertures carré isotrope et stratifiées ont montre que la charge critique de flambage augmente avec l'augmentation de l'ouverture pour certaines condition aux limites.

Mots clés *: Stratifié, Composite, Flambage, Instabilité, Plaque, Singularité géométrique, Elément fini*

Abstract

In this paper, we present an analysis of a buckling behaviour of rectangular and square laminated plates with central cutouts. The laminates have in general an anisotropic behaviour, significant transverse shearing strains and a coupling between extention and bending strains. We used a four nodes shell finite element with 32 degrees of freedom. The element is based upon the Kirchhoff theory extended to the laminated structure with adoption of the equivalent mono-layer approach.

A parametric study was undertaken to show the effect of certain parameters on the buckling behaviour of thin laminated plates containing square cental cutouts. The results show that the critical buckling load increases with the increase of the cutout dimension for certain boundrey conditions

Keys words *: Buckling, Laminated, orthotropic, finite element*

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ملخص

يتمثل هذا العمل في تحليل عدم الاستقرار مرونة انبعاج الصفائح المصففة ذات التشويه الهندسي. الانبعاج الصفائح ذات المواد المركبة و هي ظاهرة معقدة جدا. لتحميل الانبعاج الصفائح الرقيقة المصففة قمنا باستخدام عنصر متكون من أربع عقد و 32 درجة متحررة، العلاقة المعتمدة على نظرية **(كرشوف)** و التي تتمثل في أحادي الطبق المكافئة لطبقات الصفائح المصففة. ثم قمنا بتقديم مشاكل صياغة عدم الاستقرار باستعمالنا مبدأ تغيير الثاني للطاقة الكامنة في بناء صلابة المصفوفات.

قمنا بتجارب عدة عينات لظاهرة الانبعاج الصفائح الرقيقة المتجانسة و غير المتجانسة (مركبة) و النتائج المتحصل عليها مقارنة مع النتائج المرجعية المتوفرة أثبتت تقارب النتائج و نجاعة العنصر المستخدم. ثم ثقب الصفحة و المركبة عدة مرات وبينت النتائج أن الحمولة الحدية لانبعاج تزداد مع زيادة مساحة ثقب مربع الشكل و ذلك مع بعض الشروط الحدية.

الكلمات المفتاحية: *انبعاج , الصفائح المصففة , شويه التالهندسي, عدم الاستقرار*

The purpose of the mechanical analysis of the structures is to determine stresses, deformation or displacements which will be compared with acceptable values, based on the properties of materials. And this is according to service needs or simply for esthetics reasons. The analysis of the structures to instability behavior is less frequent in spite of the importance of the phenomenon underlined by the rupture in service of many monumental structures. The finite element method allowed advance up to the point of sophistication in the analysis of the structures of complex geometries and under the action of any type of loads, which was not the case with the analytical methods. Thin laminated structures made of composite materials are widely used nowdays. These structures are used in vast fields, particularly in aeronautics, automotive industry, shipbuilding and civil construction as alternative to traditional materials such as steel and concrete. Indeed, This large utilisation is due to advantages of composite materials such as light weight, corrosion resistance and ability to vary their properties over wide range of values. Although composite materials have existed for many years, there is still much about them that needs to be understood before they will be accepted as building materials in civil engineering structures. When thin structures are subjected to loading of mechanical or thermal nature, their cross sections undergo compressive stresses as well as tensile stresses. The compressive stresses can have increasingly large values so that buckling takes place. These thin structures become unstable for loads or relatively weak variations in temperature, and buckle in the elastic region. Consequently, buckling presents a very great consideration when designing this type of structures. In laminated structures, the existance of cutouts is very frequent. They are commonly used as access ports for mechanical and electrical systems, or simply to reduce weight. That is the reason to study the behaviour of this type of structures.

Very main efforts are provided through these last decades with an aim of studying the bending or the buckling behavior of thin plates and shells. For this end, various means were used, namely the analytical methods undertaken by S. Timoshenko and W. Kriger [1] [7] and numerical methods, especially the finite element method which was the subject of many investigations to develop increasingly effective and reliable elements. P. G. Bergan et al. [2] described a quadrilateral finite element for thin and moderately thick plates. Their formulation was not based on the traditional variation principle but is rather based on a free formulation which satisfies the mathematical convergence requirements. The transverse displacement is expanded in a set of fundamental rigid-body and constant curvature mode plus a set of higher order modes. By using this formulation, the authors avoided the many difficulties encountered with the elements based on Resister theory. Very good results have been obtained for the thin and thick plates of various geometries.

Reinhard and al [6] described a quadrilateral finite element of a lower order for the inflection of the thin and thick plates, by using bilinear approximations for displacement and rotations out of plan.

The authors used eight modes of deformations in order to improve the results. Although the element is of a lower order, the obtained results obtained are excellent.

Calvin D. Austin [5] undertook a comparative study on the buckling of the laminated thin plates in FRP. The author carried out the calculation of the critical loads buckling of a number of laminated plates for a number of parameters by the means of commercial software ANSYS and confront the results obtained to those obtained analytically [7]. The objective of the work was to test the performance of the software in the analysis of the buckling of the laminated thin plates. The author noted that the ± 45 degrees orientation of the layers was optimal for the cases of the simply supported plates but it was not the case for the other boundary conditions.

F. Auricchio, proposed a new finite element for the analysis of laminated composite plates. The element is based on first-order shear deformation theory and is obtained through a mixed-enhanced approach. To improve the in-plane deformation, the author adopted the variational formulation that includes as variables the transverse shear , and the enhanced incompatible modes.

In this work which is a contribution to the analysis of the laminated thin plates, an approach of quadrilateral finite element for bending and mechanical or thermal buckling is established. The proposed element is a combination of a membrane Isoperimetric quadrilateral element and a rectangular Hermit plate element of first order, transformed to be adapted to the case of general inflection of thin plates.

The formulation is based on the minimal potential energy principle adopting Kirchhoff-Love theory. Almost elements adopt the Reissner theory, especially during the analysis of the composite material structures owing to the fact that the effect of shearing is of great importance in these cases. However in the present study, transverse shearing was neglected. That is justified by the fact of supposing that the Kirchhoff theory is checked for all the layers which are rather thin, identical and having transverse rigidity modulus of the same order of magnitude and obviously this assumption is supposed to be checked for the whole plate [9].

More, especially during the analysis of buckling, the inflection is supposed to be weak. Indeed the comparison between the results and those obtained analytically or using other type of element taking in account transverse shearing, showed the efficiency of this approach.

1. FINITE ELEMENT FORMULATION

The proposed element is a combination of an isoperimetric membrane quadrilateral element and of a first order Hermit rectangular plate element of high degree of accuracy. The element has 4 nodes of 8 degrees of freedom each.

The Cartesian and intrinsic co-ordinates are shown on the figure 1. The components of the mid -plane displacements are noted u and v and w.

1.1. Kinematics relations

The kinematics relations strains displacements are given by [4]

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + z k_x \\ \varepsilon_y &= \varepsilon_y^0 + z k_y \\ \varepsilon_{xy} &= \varepsilon_{xy}^0 + z k_{xy} \end{aligned} \qquad (1)$$

Where:

$$\begin{aligned} \varepsilon_x^0 &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \varepsilon_y^0 &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \varepsilon_{xy}^0 &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \end{aligned} \qquad (2)$$

and

$$\begin{aligned} k_x &= -\frac{\partial^2 w}{\partial x^2} \\ k_y &= -\frac{\partial^2 w}{\partial y^2} \\ k_{xy} &= -2\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \qquad (3)$$

1.2. Behavior Law

The forces and the moments resultants are related to mid-surface strains and to the curvatures by [4] :

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & B_{11} & B_{12} & B_{13} \\ A_{21} & A_{22} & A_{23} & B_{21} & B_{22} & B_{23} \\ A_{31} & A_{32} & A_{33} & B_{31} & B_{32} & B_{33} \\ B_{11} & B_{12} & B_{13} & D_{11} & D_{12} & D_{13} \\ B_{21} & B_{22} & B_{23} & D_{21} & D_{22} & D_{23} \\ B_{31} & B_{32} & B_{33} & D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \end{Bmatrix} \qquad (4)$$

Denoting by σ_i the in plane stresses, then :

$$N_i = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_i \, dz \qquad (5)$$

$$M_i = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_i z \, dz$$

Extensional, coupling and bending stiffness of the laminate are defined by :

$$\begin{aligned} A_{ij} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij} \, dz \\ B_{ij} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij} z \, dz \\ D_{ij} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij} z^2 \, dz \end{aligned} \qquad (6)$$

With $\overline{Q}_{ij}$ are the stiffness coefficients for principal material directions.

The strain potential energy of the element is given by [4]:

$$U = \frac{1}{2}\int\int \Big(\{\varepsilon_l^0\}^t [A] \{\varepsilon_l^0\} + \{\varepsilon_l^0\}^t [B] \{k\} + \{k\}^t [B] \{\varepsilon_l^0\} + \{k\}^t [D] \{k\} + \{\varepsilon_{nl}^0\}^T (N) \Big) dxdy \qquad (7)$$

Where :

$$\{\varepsilon_L^0\}^t = \left\{ \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\}$$

$$\{\varepsilon_{nL}^0\}^t = \left\{ \left(\frac{\partial w}{\partial x}\right)^2, \left(\frac{\partial w}{\partial y}\right)^2, 2\frac{\partial w}{\partial x}.\frac{\partial w}{\partial y} \right\} \qquad (8)$$

$$\{k\}^t = \left\{ -\frac{\partial^2 w}{\partial x^2}, -\frac{\partial^2 w}{\partial y^2}, -2\frac{\partial^2 w}{\partial x \partial y} \right\}$$

$$\{N\}^T = \{N_x, N_y, N_{xy}\} \qquad (9)$$

As the element is a combination of an isoperimetric membrane element and a high precision plate element of Hermit type, the interpolation functions of the co-ordinates and displacements through the element are given by:

- the real co-ordinates are connected to the co-ordinates of the reference element by:

$$x(\xi,\eta) = \sum N_i(\xi,\eta).x_i \,,\; y(\xi,\eta) = \sum N_i(\xi,\eta).y_i \quad (i = 1,2,3,4) \qquad (10)$$

In the same way, the in plane displacements are given by:

$$u(\xi,\eta) = \sum N_i(\xi,\eta).u_i \,,\; v(\xi,\eta) = \sum N_i(\xi,\eta).v_i \quad (i = 1,2,3,4) \qquad (11)$$

Where:

$$N_i(\xi,\eta) = \frac{1}{4}(1 + \xi\xi_i)(1 + \eta\eta_i) \qquad (12)$$

Displacements out of plane of the element of reference is expressed as the products of one dimensional first order Hermit interpolation polynomials [4]

$$w = H_{00} w_i + H_{10}\frac{\partial w}{\partial \xi} + H_{01}\frac{\partial w}{\partial \eta} + H_{11}\frac{\partial^2 w}{\partial \xi \partial \eta} \qquad (13)$$

Where:

$$\begin{aligned} H_{00} &= \frac{1}{16}(\xi+\xi_0)^2(\xi\xi_0 - 2)x(\eta+\eta_0)^2(\eta\eta_0 - 2) \\ H_{10} &= -\frac{1}{16}\xi_0(\xi+\xi_0)^2(\xi\xi_0 - 1)x(\eta+\eta_0)^2(\eta\eta_0 - 2) \\ H_{01} &= \frac{1}{16}(\xi+\xi_0)^2(\xi\xi_0 - 2)x\eta_0(\eta+\eta_0)^2(\eta\eta_0 - 1) \\ H_{11} &= \frac{1}{16}\xi_0(\xi+\xi_0)^2(\xi\xi_0 - 1)x\eta_0(\eta+\eta_0)^2(\eta\eta_0 - 1) \end{aligned} \qquad (14)$$

With:

$$\begin{aligned} \frac{\partial w}{\partial \xi} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \xi} \\ \frac{\partial w}{\partial \eta} &= \frac{\partial w}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \eta} \\ \frac{\partial^2 w}{\partial \xi \partial \eta} &= \frac{\partial^2 w}{\partial x^2}\frac{\partial x}{\partial \xi}\frac{\partial x}{\partial \eta} + \frac{\partial^2 w}{\partial x \partial y}\left(\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} + \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi}\right) + \\ & \frac{\partial^2 w}{\partial y^2}\frac{\partial y}{\partial \xi}\frac{\partial y}{\partial \eta} + \frac{\partial w}{\partial x}\frac{\partial^2 x}{\partial \xi \partial \eta} + \frac{\partial w}{\partial y}\frac{\partial^2 y}{\partial \xi \partial \eta} \end{aligned} \qquad (15)$$

Then, after transformation, the interpolation functions of the real element are written :

$$w=L_w w+L_{\theta x}\frac{\partial w}{\partial x}+L_{\theta y}\frac{\partial y}{\partial y}+L_{\theta xy}\frac{\partial^2 w}{\partial x\partial y}+L_{\theta xx}\frac{\partial^2 w}{\partial x^2}+L_{\theta yy}\frac{\partial^2 w}{\partial y^2} \quad (16)$$

where;

$$\begin{aligned} L_w &= H_{00} \\ L_{\theta x} &= H_{10}\frac{\partial x}{\partial \xi}+H_{01}\frac{\partial x}{\partial \eta}+H_{11}\frac{\partial^2 x}{\partial \xi \partial \eta} \\ L_{\theta y} &= H_{10}\frac{\partial y}{\partial \xi}+H_{01}\frac{\partial y}{\partial \eta}+H_{11}\frac{\partial^2 y}{\partial \xi \partial \eta} \\ L_{\theta xy} &= H_{11}\left(\frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta}+\frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi}\right) \\ L_{\theta xx} &= H_{11}\frac{\partial x}{\partial \xi}\frac{\partial x}{\partial \eta} \\ L_{\theta yy} &= H_{11}\frac{\partial y}{\partial \xi}\frac{\partial y}{\partial \eta} \end{aligned} \quad (17)$$

The displacements state leads to a 32 degrees of freedom element with 8 degrees of freedom by node and the resulting displacement vector is:

$$\{q\}=\left\{u, v, w, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial^2 w}{\partial x \partial y}, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}\right\} \quad (i=1,2,3,4) \quad (18)$$

$\frac{\partial^2 w}{\partial x^2}$ et $\frac{\partial^2 w}{\partial y^2}$ are nonessential variables.

By subsisting the polynomials of interpolation in the equation of energy, we obtain:

$$\begin{aligned} U &= \frac{1}{2}\int\int \{q\}^t \{[S_\varepsilon]^t[A][S_\varepsilon]+[S_\varepsilon]^t[B][S_k]+[S_k]^t[B][S_\varepsilon] \\ &+ [S_k]^t[D][S_k]\ \}\{q\}|J|d\xi d\eta \\ &+ \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\{q\}^T\{[G]^T[N_0][G]\}\{q\}|J|d\xi\xi\,\eta \end{aligned} \quad (19)$$

Where:

$$\begin{aligned} \{\varepsilon_1^0\} &= [S_\varepsilon]\{q\} \\ \{k\} &= [S_k]\{q\} \\ [G]\{q\} &= \begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{bmatrix} \\ [N_0] &= \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix} \end{aligned} \quad (20)$$

in which:

{ q}is the resulting displacement vector of the element which is a 32x1 vector; $[S_\varepsilon]$, $[S_k]$ are 3x32matrices of which relate the linear membrane strain and curvature of the element to vector {q} respectively;

[G] is a 2x32matrix which relates the vector [W / W, W / y]to the displacement vector {q};

|J | is determinant of the Jacobean matrix.

1.4. Buckling Analysis

In almost buckling problems of plates, the determination in advance of the distribution of the stresses through the plate is not necessary. However in the general case, when the stresses are not uniformly distributed through the plate, in particular when the plate contains cutouts or undergoes a non uniform variation of temperature, it will be necessary to determine the distribution of the membrane efforts as first stage in this analysis.

With:

$$\{N_0\} = ([A][S_\varepsilon] + [B][S_k])\{q\} \quad (23)$$

By setting the second variation of the strain energy to zero, the standard eigenvalue problem is obtained :

$$[K_G]\{X\}+ \lambda[K_\sigma]\ \{X\} =0 \quad (24)$$

Where $[K_g]$ is the global geometry matrix which is the assembly of the element geometry matrix $[K_g^e]$,

$$[K_\sigma^e] = \int_{-1}^{1}\int_{-1}^{1}[G]^t[N_0][G]|J|d\xi d\eta \quad (25)$$

2. NUMERICAL RESULTS AND DISCUTION

2.1. problem presentation

Table 1 : Geometrical characteristics

a (cm)	a/b	h
20	1.0	1,05
30	1,5	1,05

Mechanical proprieties of lamina :
$E1 = 123 \times 10^5$ N/cm^2 $E2 = 8{,}2 \times 10^5$ N/cm^2
$G12 = 4{,}1 \times 10^5$ N/cm^2
$\nu 12 = 0{,}5$

The plate is formed by 6 laminas with sequence [90/-90/0] s

Boundaries conditions

We consider 2 boundries conditions

- Simply supported plates on 4 edges (4SA)
- Clamped plates on 4 edges (4C)

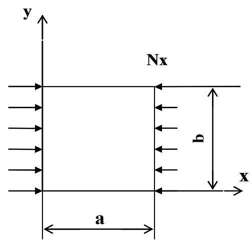


Figure 1 : Uniaxial Compression

Table 2 : Critical load N_{cr} for a simply supported laminated [90/-90/0] s

a/b	Meshes	4x2	4x4	5x5	8x8	10x10	Ncr analytica [5]	(m,n)
1	N_{cr}	23.492	23.60966	23.808	23.8846	23.885	23.885	(2.1)
1.5	N/cm	21.660	21.689	23.294	23.857	23.885	23.885	(3.1)

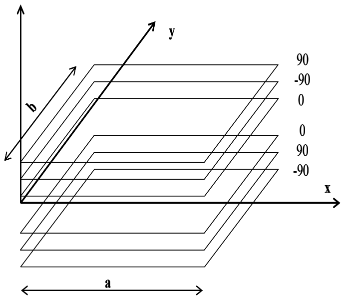
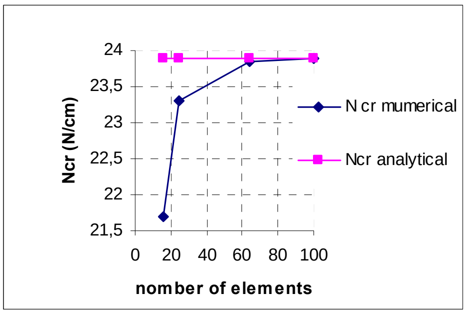
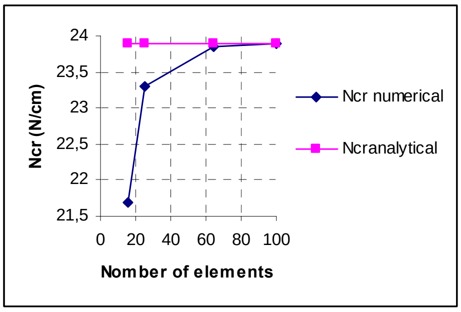


Figure 2 : The plate laminated with an Orientation (90, -90,0) s



for a/b=1



for a/b=1,5

Figure 3 : Variation of uniaxial buckling load for simply supported plates

3. BUCKLING BEHAVIOR OF PLATES WITH CUTOUTS

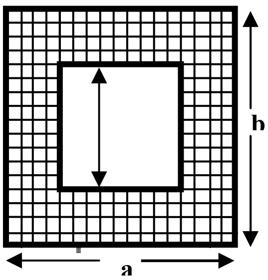


Figure 4 : Finite element mesh

4. NUMERICAL RESULTS AND DISCUSSION

The plates are subdivided into 2x2, 4x4, 6x6 and 10x10 elements as for the first study. The computed values of the critical loads for various parameters by the present element the results presented on the table 2 and over of figures 3 and 4 show the performance of the element where compared to results obtained analytical by Whitney.

In this chapter, there is an analysis of some cases of the small plates of the singularities centered during the analysis, certain watches results that the presence of opening under certain conditions of support increases the critical load of buckling compared to that relating to the corresponding blank flanges. The results as showed as the position of the opening can have a direct influence on the value of the critical load in certain measurements.

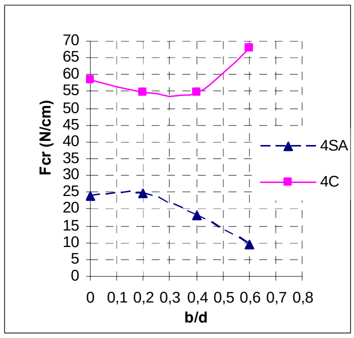


Figure 5 : The variation Fcr in function b/d for plate Laminated a/b=1

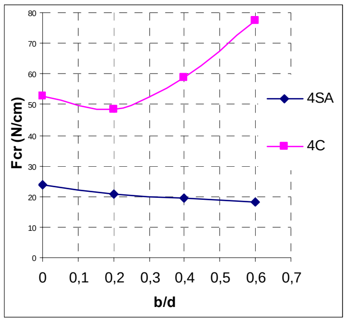


Figure 6 : The variation Fcr in function b/d for plate Laminated a/b=1,5

In figures 5 and 6 there is an analysis of some cases of the small plates of the singularities centered During the analysis, certain watches results that the presence of opening under certain conditions of support increases the critical load of buckling compared to that relating to the corresponding blank flanges. The results as showed as the position of the opening can have a direct influence on the value of the critical load in certain measurements.

CONCLUSION

Buckling laminated plates is a very complex phenomenon because of the specificity of this type of the materials. Indeed, the laminates have in general an anisotropic behavior, significant shearing transverse deformations in the direction of the thickness and a coupling extension bending.

For buckling analysis of the laminated thin plates, a four nodes finite element of 32 degrees of freedom was developed the formulation was based on the theory of kirchhoff extension to the plate laminated adopting the equivalent mono-laver Approach. For the construction rigidity and geometrical matrices, the Principe of minimum potential energy was used. The developed element was tested to buckling of laminated thin plates.

The obtained results when compared to those available in literature, showed the rapidity of convergence and the good performance of the element. In continuation, we showed the effect of square opening centers on the plates square or rectangular solicited by a uniaxial pressing, the critical load of buckling decrease. But For the case of the laminated plates, the effect of the dimension of the opening depends on the type of boundary conditions. The critical load of buckling believes with the increase in the dimension of the opening, although it keeps the same pace for the case of the simply supported plates.

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