PREDICTION OF FATIGUE LIFE OF AUTOMOTIVE ALUMINUM LOWER SUSPENSION ARM

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Résumé

L'objectif de cette étude est d'évaluer le potentiel de l'utilisation d'alliage léger dans des pièces d'automobile en étudiant leur résistance à la fatigue en utilisant divers paramètres tels que l'effet de la suspension, la nature de l'excitation, la géométrie et le poids de la pièce. La pièce à l'étude est le bras de suspension inférieur fait d'alliage 7075-T6 d'aluminium. Nous avons utilisé l'approche de l'énergie qui nous permet de comparer deux tenseurs du même ordre, soit le cas uniaxial et multiaxial. Dans les deux cas, la densité d'énergie de tension est un tenseur d'ordre nul. L'excitation aléatoire en termes de déplacement est obtenue analytiquement à partir de la densité spectrale de puissance PSD. La force d'excitation est obtenue par la normalisation du spectre du déplacement. Pour éviter l'utilisation de la méthode de Newton-Raphson, pendant l'étape partielle du calcul de la résistance à la fatigue dans tous les éléments de la maille, nous avons crée une interface Matlab pour identifier les éléments critiques. Le signal de l'énergie de déformation SENER de l'élément critique est corrigé pour enlever les anomalies en utilisant un algorithme d'interface de WAFO Matlab. Des cycles de Rainflow sont extraits en utilisant la formulation de Markov afin de calculer le nombre de répétitions du signal à la rupture, qui est calculé à partir de la loi de miner. Les résultats prouvent que le signal de chargement étudié doit être répété $8.86*10^{11}$ fois avant la rupture de la pièce fait en alliage d'aluminium 7075-T6 d'épaisseur de 25 millimètres. L'optimisation de la forme de la pièce peut la rendre encore plus légère et sa fréquence plus loin de la gamme de fréquence de la densité spectrale de puissance PSD décrivant le profil de la piste. En effet, pendant un processus d'optimisation nous avons gagné entre 5 et 11% du poids sans affecter d'une manière significative la résistance à la fatigue de la pièce et sa fréquence naturelle. Le rapport de rejet que nous avons développé dans cette étude, est directement lié à la notion de la fatigue, en prenant en considération la variation de la densité d'énergie de déformation SENER, en simplifiant le calcul du temps par l'adaptation de l'aspect aléatoire afin d'éviter l'extraction des cycles "rainflows" et le calcul de résistance à la fatigue dans tous les éléments.

***Mots clés** : Fatigue, aluminium, automobile, suspension, optimisation, rupture*

Abstract

The objective of this study is to evaluate the potential of light alloy use in automobile parts by studying their fatigue life using various parameters such as the effect of the suspension, the nature of the excitation, the geometry and the weight of the part. The part under study is the lower suspension arm made of 7075-T6 aluminium alloy. The energy approach enables us to compare two of the same order of tensors, the multiaxial and the uniaxial cases. In both cases, the strain energy density is a zero tensor order. The random displacement excitation is obtained analytically from the power spectral density PSD. The force excitation is obtained by a simple normalisation of spectrum displacement. To avoid the use of the Newton-Raphson method, during the partial fatigue life calculation step in all the elements of the mesh, we create a Matlab interface to identify the critical elements. The strain energy SENER signal of the critical element is corrected to remove the anomalies by a WAFO Matlab interface algorithm. Rainflow cycles are extracted using the Markov formulation in order to calculate the number of signal repetitions to failure, wich is calculated from the Miner law. The results show that the studied loading signal must be repeated $8.86*10^{11}$ times before a 25 mm thick aluminium 7075-T6 alloy part ruptures.

***Keys words** : Fatigue, aluminium, automotive, suspension, optimisation, failure*

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ملخص

تهدف هذه الدراسة إلى تقييم قدرة استخدام الخليط المرن في قطع السيارة من خلال دراسة مقاومتهم للتعب باستخدام مختلف مؤشرات مثل أثرالمعاليق السفلية المصنوعة الخليط 7075-T6 للألمنيوم.

لقد استعملنا مقاربة الطاقة التي تسمح لنا مقاربة وزوجين من Tenseurs نفس الفرع أي الحالة أحادية المحورفي كلتا الحالتين الطاقة للضغط هي من النوع صفري و تم الحصول تحليليا على الاستثارة العشوائية في حلة تنقل وتم الحصول على قوة PSD .

لتجنب استعمال طريقة نيوطن-رافسون حين مرحلة حساب المقاومة المال في كل اجزاء، لقد انجزن حسابية Matlab التعرف على الاجزاء الحسسية.

النتائج المحصلة عليها تبين ان اشارة الحمولة المدروسة يجب ان تعاد $8.86*10^{11}$ مرة قبل انقطاع القطعة المصنوعة الخليط 7075-T6 للألمنيوم ببعد 25 مم.

الكلمات المفتاحية: *الملل، الاليمنيوم، الالية، المعاليق، انقطاع*

Weight reduction not only improves the slip angle between the tires and the road, the reaction to turns and the stability of the vehicle, but it also makes driving more effective and safe over long distances and yields lower gasoline consumption [1, 2, 3]. In this framework, the research goals are to study the dynamic and vibratory effects on certain aluminium alloy elements, in particular the fatigue life of the lower suspension arm. Almost all the machine elements that undergo repeated runs in time can become subject to fatigue phenomena. Fatigue causes cracking, which develops gradually under the action of the random loading repetition. These random loadings can lead to rupture by fatigue during the application of stress levels lower than the tensile strength or even yield stress levels. Rupture occurs when a crack reaches a critical length l_c and the factor of intensity of the constraint K [4] reaches a critical value K_c. The complexity of the problem of fatigue led many researchers to approach the subject using several methods. Andre Bazergui [5] based their work on the curves of fatigue *SN* (stress *S* depending on the number of cycles to failure *N*) and two empirical models of fatigue to prevent rupture. They report rotational bending as a cheaper standardized test. To improve the ability of predicting the fatigue life of a part, they practised on standard test-tubes before installing the operational part on the apparatus. In practice, stress variation is often periodic, but it is not always sinusoidal and the average value of the stress is not null because of the static contribution from the weight of the part and the premature tightening.

Cervello [6] analyzed and studied the design of railway wheels with weak noise. A numerical procedure was used for the calculation of the loss factor. This procedure checked on plates by means of experimental modal analyses. It allowed a better treatment of smoothness and the choice of a commercially available arrangement with feasible technology. A. Elmarakbi [7] studied the validity of the multiaxial criterion of fatigue failure based on the density of the energy equivalent to the uniaxial case. Three dimensional finite element analysis on the notch axis of SAE which is used as a test component to evaluate the criterion of multiaxial damage caused by fatigue. A. Elmarakbi [7] equalized the energy density of the multiaxial case obtained from finite element commercial software. The uniaxial case was calculated by exact integral, since it is two of the same order of tensors : zero order. W. V. March [8] based his study on the same criterion as A. Elmarakbi the density of the energy of cracking. W. V. March performed fatigue and tension-torsion tests and compared the results with the energy density criterion. The model aimed to predict the fatigue life of rubber particles using two approaches. One focused on crack initiation, giving the history of some parameters such as the stress state and the deformations. Other approaches were based on ideas of the breaking process and were interested in predicting the propagation of particular cracks, giving the history of the released energy rate. The crack propagation approach was developed by Rivlin and Thomas according to reference [8] who applied Griffith's criterion to rubber. The difficulty in applying the crack propagation approach to rubber is that it requires advance knowledge of the initial specimen and the state of the crack which causes the final rupture. Compared to other part endurance prediction models that were based on the Paris law, A. De-Andrés [9] used a cohesive law of the element. In this approach, the creation of new surfaces is the final result of a process of gradual loss of elasticity when separation increases.

In the present paper, the prediction of the part's fatigue life is made by the development of a multiaxial elasto-plastic numerical model. Knowing the mechanical properties of alloy 7075-T6, the fatigue rupture prediction will be simulated with the commercial finite element software Abaqus. The lower vehicle suspension arm of a quarter vehicle suspension system is studied in a critical case, where the part is embedded in the suspension join. The stress state gives values of shorter and safer fatigue life by underestimating their true value. The treatment and the analysis of the numerical results include :

- development of a dynamic vehicle suspension system model
- development of a numerical model using the commercial finite element software Abaqus
- analysis of the results of the strain density energy time evolution of the multiaxial case in all mesh elements
- filtering the results to extract the critical elements in the elasto-plastic case
- correction of the critical element signal of the strain density energy: multiaxial case
- extraction of the rainflow cycles by the Markov algorithm
- calculation of the partial and total fatigue life of the part from the uniaxial criterion equivalent to the multiaxial case.

1. VEHICLE SUSPENSION MODEL

The vehicle part studied is the lower suspension arm of the suspension system. It is necessary to evaluate the dynamic behaviour of the suspension system and the roadway profile model. H. Rahnejat [10] studied the dynamics of the Macpherson suspension system in a quarter vehicle. In fact, H. Rahnejat proposes a simple model where the mass and inertia are represented by parameters m and I respectively. In the present study a simple model is developed as illustrated in Figure 1. In this model, the partial stability of the vehicle is ensured by a control suspension system. This system has for an action chain a tire of stiffness constant k_p and for a return chain in negative feedback a spring of stiffness k_s assembled with a shock absorber C_s. The excitation force F_2 of the road irregularity, is balanced through the tire by the F_1 negative feedback. The F_1 negative feedback brings back the suspension arm to its place of equilibrium linearly. For cases of small disturbances and the domination of the elastic behaviour of the material, it produces a state of the low stress. We can then write the following relation :

$$\frac{x_1}{L_1} = \frac{x_2}{L_2} \Rightarrow x_1 = \frac{L_1}{L_2} x_2 \qquad (1)$$

Equation 1 lets us know x_2, the deflection, compared to the equilibrium position and caused by the road profile and then we can deduce the displacement x_1. Unsprung mass, such as the mass of the spring, the tire and the shock absorber is negligible compared to the dynamic stress brought into play. The forces applied to the lower suspension arm by a quarter vehicle are : $F_2 = k_p x_2$ exerted on the tire by x_2 and $F_1 = K_s x_1 + C_s \dot{x}_1$ negative feedback force exerted by the spring and the shock absorber.

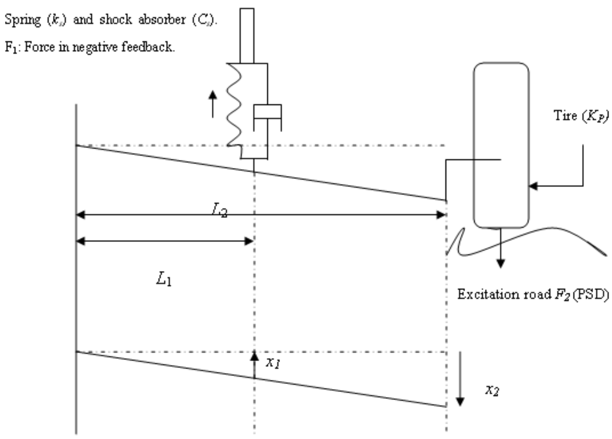


Figure 1: Vehicle suspension control and the lower suspension arm.

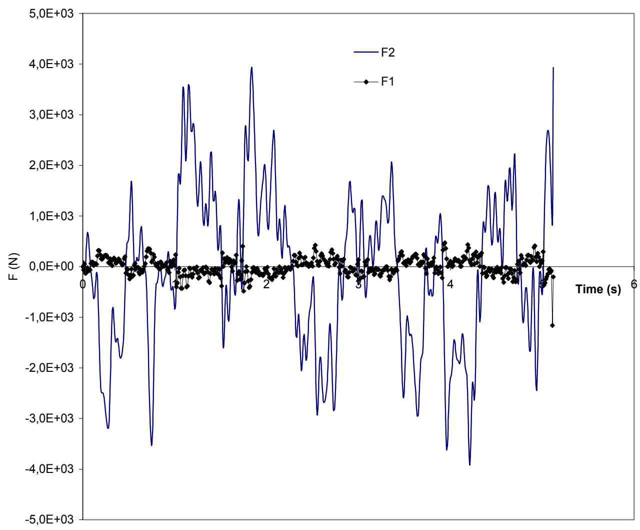
Knowing the PSD experimental values, we can deduce x_2 and consequently F_2. Therefore equation 1 enables us to have the x_1 spectrum. Once x_1 is known, the complete determination of the F_1 values is done. The road profile spectrum is a function of the vehicle speed, which is obtained from the power spectral density PSD. Normalisation of the road profile spectrum is necessary to keep the same frequency band and to be able to transform it into a force excitation. The factor, which must be multiplied by the values of the power spectral density, depends on the numerical values of the suspended mass, the tire stiffness constant and the shock absorber coefficient, and the damping ratio of both the shock absorber and the unsprung mass. In this study, the numerical value of this factor is estimated at 0.025. $\dot{x}_1$ is numerically given by :

$$\dot{x}_1 = \frac{x_1(t+\Delta t) - x_1(t)}{\Delta t}$$

The excitation caused by the road profile has a random nature. In this study, the road profile is estimated from the power spectral density. The road profile is defined by function *X(t)* and by assuming the road surface is a random, stationary, Gaussian and centered process i.e. all its statistical properties are invariable in any change of the argument *t*, that the law of distribution of variable *X(t)* is a Gaussian law, and that the average of *X(t)* for any *t* pertaining to [0, T], is null. The stationary random process and the Gaussian law *X(t)* can be regarded as a periodic function in time *t,* of amplitude *α*, circular frequency *ω* and phase ϕ [11]. The road spectrum thus obtained corresponds to a vertical random displacement. However, the present study takes into account the direct excitation by a random dynamic force. Thus it is necessary to transform the random displacement into a random force while keeping the same pulsation or band frequency. Therefore the minor road spectrum is normalised. The values of this spectrum are multiplied by a common factor estimated from the statistical data. For a minor road, the speed of the vehicle is lower than 75 km/h. For 75 km/h (21 m/s), a 960 kg vehicle has a momentum equal to 20160 kg.m/s. Thus in 0.5 second, we estimate the maximum force excitation to be 40320 N. This force is distributed equally on the four quarters of the vehicle's suspension system. If undamped, the vehicle weight opposes about 3750N in the return chain to the maximum 4000N random excitation value. The maximum value of the force of excitation through a tire of stiffness 160000 N/m is about 4000 N. Figure 2, shows the comparison between the damped and undamped spectrums.

Elasto-plastic numerical model development:

Aluminium alloys have an elasto-plastic behaviour with a plastic contribution of about 36% in certain cases. M. Gbadebo [12, 13] proposed an analytical solution for the elasto-plastic case, where the matrix of the total tensor deflection increments is the sum of the elastic and plastic contributions. They extrapolated the nonlinear uniaxial behaviour of the material in the multiaxial case to calculate the strain energy density. This approach is the same as that used in the commercial finite element software Abaqus.



(b) Damped mode: road excitation F_2 and feedback chain F_1

Figure 2 : Different force spectrums in the vehicule control suspension system.

This model takes into account the two cases of nonlinearity: non-geometric linearity (variation of the geometrical configuration of the system in time) and material nonlinearity which expresses the elasto-plastic coupling. The numerical diagram of integration depends on the stability of the diagram, the computing time and the precision of the method. In this study the implicit integration scheme is used [14, 15]. The loading and boundary conditions specify the places of excitation and the degrees of freedom imposed on the system. The mesh takes into account the state of the stress and the point sources, such as the places of loadings and the boundary conditions. The equivalent in uniaxial mode is calculated, since they both have the same tensor order (order zero). Since it is a fatigue and not a static case. The critical element is filtered through all the elements of the mesh, giving the maximum sum of the positive variations. The critical element loading signal is corrected. The rainflow cycles are extracted using the Markov method. Given the non linearity of the Mansson-Coffin equation, the partial fatigue life is calculated by the Newton method. The number of random cycle repetitions leading to the part's rupture is deducted at the final stage using the Miner law. The multiaxial case can be studied by calculating the fatigue life of a part in the uniaxial mode by a quantity of strain energy density equivalent to multiaxial case one. In this paper we use the uniaxial criterion equivalent to the multiaxial case as suggested by Elmarakbi [7].

Since the aluminium alloys present a more important plastic then elastic behaviour, the nonlinear profile is given by the Ramberg-Osgood as:

$$\varepsilon = \frac{\sigma}{E} + (\frac{\sigma}{k'})^{1/n'} \qquad (2)$$

where $K' = \frac{\sigma'_f}{\varepsilon'^{n'}_f}$ is the coefficient of the endurance limit, $n' = \frac{b}{c}$ the cyclic hardness exponent, σ'_f the coefficient of the fatigue strength, c the exponent of the fatigue ductility, b the fatigue strength exponent and ε'_f the coefficient of the fatigue ductility. The strain energy density, relative to the multiaxial case, can be expressed as :

$$U_s = U_\sigma, \text{ or } \int_0^{e_{ij}} S_{ij} de_{ij} = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij} \qquad (3)$$

In the present study the three-dimensional elastic finite element analysis of the model was used. The value of the strain energy density obtained from the analysis in three-dimensional MEF is equalized to the strain density energy of the uniaxial case, which makes up the plastic and elastic deformation energy. The mathematical formulation of s the total uniaxial deformation energy per unit of volume, obtained by an exact integration is represented as:

$$U_a = U_{ae} + U_{ap} = \int_0^{\varepsilon_a} \sigma_a d\varepsilon_a \qquad (4)$$

We know that:

$$\varepsilon_a = \frac{\sigma_a}{E} + (\frac{\sigma_a}{K'})^{\frac{1}{n'}} \qquad (5)$$

thus:

$$\frac{d\varepsilon_a}{d\sigma_a} = \frac{1}{E} + \frac{\sigma_a^{(\frac{1-n'}{n'})}}{n' k'^{\frac{1}{n'}}} \qquad (6)$$

Equation 4 becomes:

$$U_a = \int \sigma_a \left(\frac{1}{E} + \frac{\sigma_a^{(\frac{1-n'}{n'})}}{n' K'^{\frac{1}{n'}}} \right) d\sigma_a \qquad (7)$$

and finally, we obtain:

$$U_a = \frac{\sigma_a^2}{2E} + \frac{\sigma_a}{n'+1} \left(\frac{\sigma_a}{K'} \right)^{\frac{1}{n'}} \qquad (8)$$

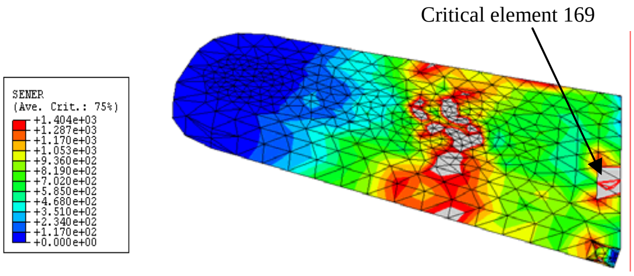
The resolution of (8 gives the necessary constraint to produce the same energy density as in the uniaxial case. Replacing the stress value in (5, gives the corresponding deformation. Knowing the deformation, fatigue life can be predicted by using the Manson-Coffin relation as in (9 :

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E} (2N)^b + \varepsilon'_f (2N)^c \qquad (9)$$

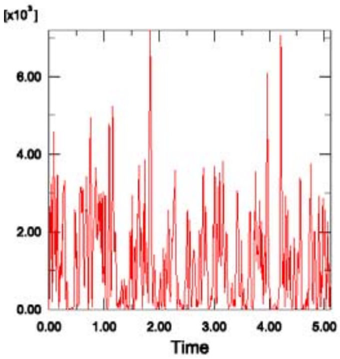
where $\Delta\varepsilon = \varepsilon_{\max} - \varepsilon_{\min}$ is the interval of deformation, σ'_f the coefficient of the fatigue strength, N the number of cycles to failure, E the Young modulus, C the exponent of the fatigue ductility, b the exponent of the fatigue strength and ε'_f the coefficient of the fatigue ductility.

2. ELASTO-PLASTIC NUMERICAL MODEL : UNDAMPED RIGID CASE

According to H. Rahnejat [10] the suspension arm cannot support a negative feedback force of the quarter vehicle which is superior to 3750N, corresponding to the rigid case. This part is subjected to the random excitation of the road described by a PSD and to the quarter weight of the vehicle which opposes it. The right end of the part is linked to a join suspension in order to allow a rotation around the y-axis. The suspension arm is 2.5 cm thick, 50 cm long and 30 cm wide. The random minor road spectrum is normalised and multiplied by a common factor estimated from the momentum exchanged between the road and the tire of a quarter vehicle going with a speed of 75 km/h. The maximum force of excitation through the tire is estimated at 4000N. The concentrated forces are replaced by uniform surface pressures. Moreover, because of the bending in a stress plane state, the mesh must be refined in the thickness to represent the state of the stress plane better. Vertical partitions are created in the thickness. The linear elements of reduced integration tolerate distortion, therefore we use a refined mesh of these elements in any simulation where the distortion levels can be very high. Thus the linear tetrahedral elements are chosen in an implicit integration stable scheme. The need for determining the critical element and its coordinates led us to choose a strategy which allows the isolation of the element having the maximum positive variation. Contrary to the static case, the rough maximum value of the strain energy density does not necessarily correspond to the critical element (Figure 3). This allows the application of the Newton-Raphson algorithm to only one element instead of applying it to all the mesh points in the structure. In order to extract the number of cycles to rupture, we use the nonlinear Mansson-Coffin equation. This filter generalizes the case where the excitation is multipoint and shifted in time, giving a tangle of the mesh material element signals. Indeed, it is the variation of the strain energy density which is involved in rupture by fatigue in the case of the dynamic stress and not the absolute value of the strain energy density as in the static case.



(a) Instantaneous strain energy density in the 25mm specimen



(b) Strain energy density time evolution of critical element no. 169

Figure 3 : Numerical simulation results

Anomalies which may be present in the strain energy density signal of the critical element can be avoided if the rainflow cycles are counted using the Markov method. Successive equality of two values of strain energy density at consecutive moments could be a problem.

When the loadings are composed of various cycles of various amplitudes and various average values, it is necessary to measure the total damage produced by these cycles. According to reference [16], Fatemi and Yang present a complete review of the laws of damage calculation which were developed from the linear damage rule suggested by Palmgren in 1924. The mathematical formulation under which it is currently known was proposed by Miner in 1945 and it is expressed as

(10 :

$$D = \sum_{i=1}^{n} \frac{n_i}{N_i} \qquad (10)$$

Random stress history is described as a sequence of blocks of constant amplitude. Each block i is composed of N_i cycles of amplitudes. Partial fatigue life N_i corresponding to this stress amplitude is determined from the Wöhler curve or from the strain energy density approach. Failure is predicted when damage D is equal to 1.

It is thus necessary to find the number of times the random loading is repeated so that D is equal to the unit. It is necessary to find the B_f number which one must multiply by D to reach rupture. B_f is calculated using

(11 :

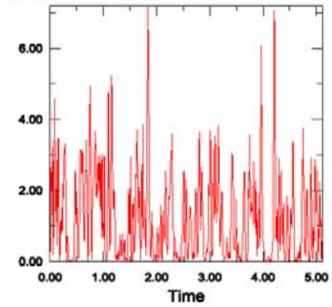
$$B_f = \frac{1}{\sum_i \frac{n_i}{N_i}}, \text{ car } B_f D = B_f \left(\sum_i \frac{n_i}{N_i}\right) = 1 \qquad (11)$$

In this study the number of cycles to rupture is calculated from the density of uniaxial strain energy equivalent to the multiaxial case.

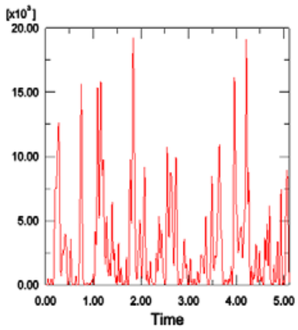
The signal of studied loading must be repeated $8{,}86 \times 10^{11}$ times before rupture occurs. The critical element maximum stress is about 31 MPa and the corresponding maximum deformation is about 4.3×10^{-4}. These slightly low values can be further attenuated to make the part safer by reinforcing it at the critical points. Optimization of the part shape will make it possible to increase the number of repetitions needed for rupture, will make the vehicle safer and will decrease its weight more in non critical places. The final approach adopted in this study is one that gives a machine element with a weight and an optimum geometry giving natural frequencies more different from the frequency of the power spectral density of road profile.

3. ELASTO-PLASTIC NUMERICAL MODEL : DAMPED CASE

The damping of the suspension largely decreases the applied force in the chain of return in the middle section of the suspension. The weight of the vehicle is considered at 3750N, not exceeding the absolute value of F_{1max} = 1200N in the case of damping by a spring and a shock absorber as shown in the Figure 4-b. This force is opposed in the return chain by the random excitation of the road deadened by tire stiffness K_p = 160000 N/m, and maximum value F_{2max} = 4000N. The return chain in negative feedback to F_2 is useful to stabilize the vehicle, but will not compensate for F_2 in terms of stress since the difference between F_1 and F_2 is higher in the case of damping than in the rigid case. The critical element filtered in both cases is element 169 located at the level of embedding. Consequently, the fatigue life of the part in the damping case will be lower than in the rigid case.



(a) 25mm rigid case



(b) 25mm damping case

Figure 4 : Critical element strain energy density time evolution

CONCLUSION

The principal objective of this research was to develop a hybrid model in order to study the potential of an aluminium alloy used in the lower suspension arm of a vehicle. We worked out an analytical and numerical model to simulate possible dynamic behaviour of a suspension system as well as the state of the stress and the strain energy density in the lower arm of the vehicle suspension. We presented a detailed description of the mechanical and fatigue properties of aluminium alloys. Moreover, several fatigue criterion were presented and analyzed.

We presented studies of the spectral aspects of fatigue and the dynamic behaviour of a vehicle suspension system. We concluded that the multiaxial criterion of the uniaxial case strain energy density equivalent to the multiaxial one is a more rational approach. Moreover, the strain energy density criterion, independent of the average value of a loading, is more practical than the Morrow model which requires corrections due to the effect of the average value of the loading.

We developed the equations of motion which describe the suspension control consisting of a forward path represented by a tire of stiffness K_p and negligible damping coefficient. We considered the maximum force of impact between a road and the tire of a vehicle measured in a 0.5 second fraction at 10 KN. The maximum value of the force transmitted by the tire is about 4 KN.

We established a model of the negative feedback and forward path of the vehicle's lower suspension arm, which is subjected to very important dynamic stresses. Numerical calculations give a force of maximum negative feedback of 1200N in the damping case and 3750N in the rigid case, corresponding to the quarter vehicle weight.

To filter the critical element and to extract the fatigue life, we generated a Matlab interface to locate automatically the critical elements without applying the Newton-Raphson algorithm in every element of the mesh. This filter produces the case where the multipoint excitation shifted in time giving a tangle of the signals of the constraints of the material mesh elements.

The elastoplastic nonlinear case was described using the Ramberg-Osgood uniaxial relation binding stress to deformation. The commercial finite element software Abaqus version 6.4 extrapolates this relationship in the multiaxial case. The sensitivity of certain types of mesh elements makes the execution of the stable implicit integration scheme difficult.

The maximum stress of the critical element in a 2.5cm thick lower suspension arm is about 31MPa and a corresponding maximum deformation is about $4.3*10^{-4}$.

The results of the numerical simulations show that the signal of studied loading must repeat $8.86*10^{11}$ times before rupture occurs. In future, a complete model should be developed to study the various cases, most importantly, the effect of the "Jounce bumper" rubber support.

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