DYNAMIC ANALYSIS OF DISCRETE MECHANICAL STRUCTURES USING THE BEAM FINITE ELEMENT METHOD UNDER DIFFERENT LIMIT CONDITIONS

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Résumé

Les structures discrètes sont d'une grande importance dans les domaines d'application de la mécanique, de l'aéronautique et du génie civil. Ils sont composés d'éléments barres, poutres ou treillis assemblés entre eux par soudure ou rivetage ans deux ou trois dimension. Ces structures peuvent être de grandes dimensions et sont caractérisées par de rigidités de contraintes élevées et d'importantes qualités de résistance avec un poids minimum, ce qui leur permet d'être utilisées dans de nombreux systèmes mécaniques, des charpentes métalliques et des structures aérospatiales. Cependant, au cours de leur durée de vie, ces types de structures sont soumises à des forces extérieures considérables ou à des fortes amplitudes de vibration que peuvent leur causer de larges déformations ou même provoquer leur totale destruction. Par conséquent, le concept de l'analyse statique et dynamique de ces structures est recommandé, et en raison de leur forme et leur complexité, la méthode des éléments finis est utilisée. Il est noté que chaque élément discret de la structure est modélisé comme un élément de poutre continu où ses matrices de masse et de rigidité ont été déterminés utilisant la méthode de l'énergie de déformation et après assemblage de tous les éléments, les matrices globales de toute la structure peuvent être obtenue.

En fait, la méthode des éléments finis est utilisée pour résoudre les problèmes en divisant le corps déformable sous forme de sous-domaine d'assemblage compliqué ou en construisant des éléments simples, et les solutions approchées du système sont trouvées sous la forme d'une combinaison de fonctions de forme et des supports compacts. Les forces d'excitation sont basées sur des forces périodiques, aléatoires ou impulsives. Dans notre cas, nous présentons une solution numérique pour décrire le comportement dynamique des structures discrètes qui ont des applications de grande importance dans de nombreux secteurs de l'ingénierie et de l'industrie. Aujourd'hui, cette méthode est un outil puissant, disponible pour des coûts raisonnables avec un temps réduit d'exécution utilisée pour l'analyse de ces structures. Dans notre travail, nous avons développé un programme de langage Pascal, pour calculer les déplacements, les réactions aux nœuds, les forces axiales dans les éléments et les modes propres des structures sous différentes conditions limites. Des exemples de validation du programme développé ont été réalisés, sous différentes charges et conditions aux limites dans les cas statiques et dynamiques, et de bons résultats ont été obtenues comparés à ceux obtenus utilisant les logiciels "SAP2000" et "Robot2009".

***Mots clés** : Structures discrètes, poutre éléments finis, masse de rigidité et de matrice, déplacements nodaux, charges appliquées, énergie de déformation, modes de vibration.*

Abstract

The discrete structures are of great importance in the application fields of mechanical, aeronautical and civil engineering. They are composed of truss, beam or lattice elements assembled together by welding or riveting in two or three dimension. Such structures can be of big dimensions and characterized by high rigidities of stress and important qualities of resistance with a minimum weight, which allow them to be used in many mechanical systems, metal frames and aerospace structures. However, during their lifetime, these types of structures are subjected to considerable external forces or strong vibration amplitudes that may cause them to large deformations or even their destruction. Consequently, the static and dynamic concept analysis of these structures is recommended, and due to their complexity form, the finite element method has been used. It is knotted that each discrete element of the structure is modeled as a continuum beam element where its mass and rigidity matrices has been determinated using the energy method and after the assembly the hall matrices of the hall structure can be obtained.

In fact, the finite element method is used to solve problems by dividing the deformable body in a complicated assembling sub domain form or by constructing single elements and the approximate solutions in the form of a combination of shape functions and compact supports. The excitation forces are based on periodic, random or impulsive ones. In our case, we present a numerical solution to describe the dynamic behavior of discrete structures which have applications of big importance in many sectors of engineering and industry. Nowadays, this method is a powerful tool, available for reasonable costs with a reduced time of execution used for the analysis of these structures. In our work, we have developed a Pascal language program, to calculate the displacements, reactions in nodes, the axial strengths in elements and the clean fashions of the structure under different limit conditions. Examples of verification of the developed language have been made, under different loads and boundary conditions in static and dynamic cases, and good results have been obtained compared with those obtained using "SAP2000" and Robot2009 software programs.

***Key words**: Discrete structures, beam finite element, rigidity and mass matrix, nodal displacements, thrust loads, energy deformation, vibration modes.*

ملخص

هياكل منفصلة ذات أهمية كبيرة في ميادين التطبيق الميكانيكية والطيران والهندسة المدنية. وتتألف من عناصر تروس أو الحزم أو شعرية تجميعها معا باللحام أو التثبيت في البعد اثنين أو ثلاثة. يمكن أن تكون ذات أبعاد كبيرة مثل هذه الهياكل وتتسم بالتصلب عالية من الإجهاد والصفات الهامة لمقاومة ذات وزن الحد أدنى، التي تسمح لهم باستخدامها في العديد من النظم الميكانيكية وإطارات معدنية وهياكل الفضاء الجوي. ومع ذلك، خلال حياتهم، تتعرض هذه الأنواع من هياكل لقوى خارجية كبيرة أو ستريك الاهتزاز القوية التي قد تؤدي إلى تشوهات كبيرة أو حتى تدميرها. ونتيجة لذلك، يوصي بتحليل مفهوم ثابت وحيوي لهذه الهياكل، ونظرا لتعقيد النموذج الخاص بهم، وقد استخدم طريقة العناصر المحدودة. هو حياكة هو على غرار كل عنصر من عناصر منفصلة للهيكل كعنصر شعاع متوالية حيث تم تصميمه مصفوفات الكتلة والصلابة باستخدام الأسلوب الطاقة وبعد الجمعية العامة يمكن الحصول على المصفوفات قاعة هيكل القاعة.

وفي الواقع، يستخدم طريقة العناصر المحدودة حل المشاكل بتقسيم الجسم تشوه في نموذج مجال فرعي تجميع معقدة أو عن طريق بناء عناصر مفردة والحلول التقريبية في شكل مزيج من وظائف الشكل وضغط يدعم. تتمركز القوات الإثارة في الدوري، وعشوائية أو متهورة منها. وفي حالتنا، نحن نقدم حل عددي لوصف السلوك الديناميكي لهياكل منفصلة لها تطبيقات ذات أهمية كبيرة في العديد من القطاعات للهندسة والصناعة. في الوقت الحاضر، هذا الأسلوب أداة قوية، متاح لتكاليف معقولة مع انخفاض وقت التنفيذ المستخدمة في تحليل هذه الهياكل. في عملنا، قمنا بتطوير برنامج لغة باسكال، لحساب عمليات النزوح، ردود فعل في العقد، ومواطن قوة محورية في العناصر والموضات نظيفة للهيكل تحت ظروف مختلفة الحد. أمثلة على تحقق اللغة المتقدمة قد بذلت تحت الأحمال المختلفة وشروط الحدود في الحالات الثابتة والدينامية، وقد تم الحصول على نتائج جيدة بالمقارنة مع تلك التي تم الحصول عليها باستخدام برامج الحاسب الآلي " SAP2000" و "Robot2009".

كلمات مفتاحية : مقاطع مغلفة، المطاوعة، مرونة لدونة، سلوك، خضوع ، خرسانة – فولاد .

The discrete structures are composed of bar, beam elements riveted or welded to each other at points called "nodes", and subjected to external forces or moments. Under the effect of these forces, the structure may be deformed and the internal stresses in each element may occur. These structures are characterized by a finite number of unknown displacements and the forces at the nodes parameters. This method is used also to analyze continuum tree dimensional bodies or cylindrical bodies using plate, triangular or shell elements. In fact, this method is based on the discretization of the structure or continuous body into infinitesimal elements limited by nodes and then by assembling them in order to obtain the overall and the entire structure [1]. Thus, the shape of the structure or the body is obtained in respecting its conditions with the initial limits and applied efforts. In general, the behavior of all the assembled basic members describes all of the behavior of the whole structure or body. The present work consists on the use of this method for static and dynamic analysis of structures under the influence of outside excitation with different boundary conditions. The stiffness and mass matrix of each element is computed, and then assembled to find the overall stiffness and mass matrix of the structure. In fact, the finite element method is known as a very powerful technique used to analysis discrete or continuum structures, in the field of engineering. It is now used in many sectors of the industry, mechanical, civil, aerospace and robotics. This work is devoted to the use of this method for static and dynamic analysis of structures in porches (beam element) due to excitement outside with different boundary conditions. Understanding this method gives necessity in the development of certain scientific knowledge as the theory of elasticity, mechanical environment continues, the strength of materials, structural dynamics, and applied mathematics. If the structure has a complex system of behavior and continues defined by the infinite number of parameters [2], it becomes very difficult to analyze or find the analytical solution. However, the finite element method grows the ability to find the most perfect solution while replacing the continuous system by a discrete system, characterized by a finite number of parameters [11].

In this context, a program of computation based on Pascal language has been developed. The displacements, forces and reactions at the nodes as well as the axial strengths in each element and the clean fashions of the all structure have been determinate under different applied loads and boundary conditions in static and dynamic cases. The obtained results are compared to those found using existing programs such as "SAP2000" and "Robot2009".A good comparison has been observed.

1. PROBLEM AND METHODOLOGY

The finite element method is based on the geometric shape of the structure (element bar, beam, plate and hull). The discretization of these structures provides an elementary matrix dislodgment depending on the strain energy, to provide an overall stiffness matrix.

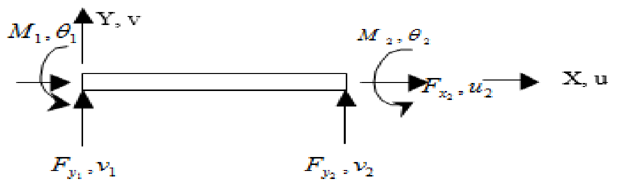


Figure 1 : Element beam in local coordinates

After the implementation of boundary conditions and loads, we calculate the unknown in all nodes displacements and the stresses (axial strength) in all elements.

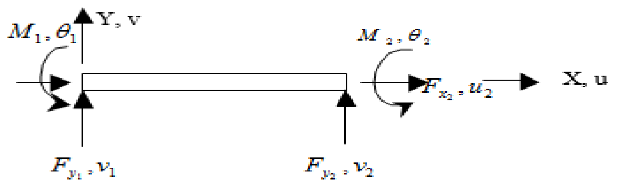
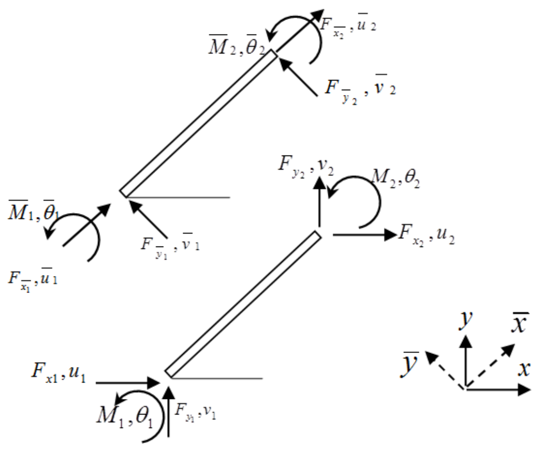


Figure 1 : Element beam in local coordinates

Forces are applied to the structure to determine the mass matrix. The dynamic analysis of bending selects the best condition to limit the structure [4].



2. MATHEMATICAL FORMULATION

2.1. Function of displacement

For an element of beam, the equation of balance is given by the following formula [5] :

$$\frac{\partial^4 v}{\partial x^4} = o \tag{1}$$

The behavior of the beam is described by a polynomial function of the 3 degree. It is the function of displacement and the solution of this equation [6] :

$$\begin{aligned} V(x) &= a_1 + a_2 x + a_3 x^2 + a_4 x^3 \\ \theta(x) &= \frac{\partial v}{\partial x} = a_2 + 2a_3 x + 3a_4 x^2 \end{aligned} \tag{2}$$

The coefficients of displacement, the displacement vector coefficient can be obtained by using the boundary conditions for the nodes. Displacement nodes equations can be written in matrix form as follow :

$$\begin{Bmatrix} v(x) \\ \theta(x) \end{Bmatrix} = \begin{bmatrix} 1 & x & x^2 & x^3 \\ 0 & 1 & 2x & 3x^2 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} \quad (3)$$

Using the equations (2) and (3), we can writethe matrix form :

$$\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & l & l^2 & l^3 \\ 0 & 1 & 2l & 3l^2 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} \quad (4)$$

Or:

$$\{q(x)\} = [A]\{a\} \quad (5)$$

The inverse shape of this equation gives expressions of constants a_i. After substitution of the coefficients in the equation (3) and rearrangement, we can present the final shape of the displacement function :

$$V(x) = v_1 + x\theta_1 - \frac{3x^2}{L^2}v_1 - \frac{2x^2}{L}\theta_1 + \frac{3x^2}{L^2}v_2 - \frac{x^2}{L}\theta_2 + \frac{3x^3}{L^3}v_1 + \frac{x^3}{L^2}\theta_1 - \frac{2x^3}{L^3}v_2 + \frac{x^3}{L^2}\theta_2 \quad (6)$$

Then:

$$V(x) = f_1(x)v_1 + f_2(x)\theta_1 + f_3(x)v_2 + f_4(x)\theta_2 \quad (7)$$

Where $f_i(x)$ are the shape functions which can be written as follows :

$$\begin{cases} f_1(x) = 1 - 3\left(\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^3 \\ f_2(x) = x - 2\left(\frac{x^2}{L}\right) + \left(\frac{x^3}{L^2}\right) \\ f_3(x) = 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3 \\ f_4(x) = -\left(\frac{x}{L}\right) + \left(\frac{x^3}{L^2}\right) \end{cases} \quad (8)$$

2.2. Vibration of bending of an element beam

The kinetic energy of an element beam in vibration of bending is defined as follows [7] :

$$T = \frac{\rho A}{2}\int_0^l \dot{v}(x)^2 \, dx \quad (9)$$

The expression of the potential energy of an element beam is written as follow:

$$U = \frac{EI}{2}\int_0^L v''(x)^2 \, dx \quad (10)$$

Where $v''(x)$ indicates the derivative assists by report of x. By replacing the function of x displacement (6) in the expressions (7) and (8), and after drifting by the method of Lagrange, we get :

$$[m]\{\ddot{q}\} + [k]\{q\} = \{F(t)\} \quad (11)$$

2.3. Rigidity and mass matrixVibration of bending of an element beam

The equation of movement can be gotten by the use of the Lagrange's equation, written as follows :

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) + \frac{\partial U}{\partial q_i} = F_I \quad (12)$$

Using the indicial nation (11) can be written as :

$$m_{ij}\ \ddot{q}_i + k_{ij}\ q_i = F_i \quad (13)$$

Which translates from the first law of Newton and q_i is the degree of freedom at the two nodes (u_i, v_i and θ_i). By substituting the equation of motion (6) for the beam element in the equation of the kinetic and potential energy (9 and 10), and while calculating strengths from the equation (12), we get :

$$\begin{aligned} F_{x_1} &= \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{u}_1}\right) + \frac{\partial \gamma}{\gamma u_1} \\ &= \frac{d}{dt}(\rho A)\int_0^l \{f_1(x)\dot{u}_1 + f_2(x)\dot{u}_2\}\{f_1(x)\}dx \\ &\quad + EA\int_0^l \{f_1(x)u_1 + f_2'(x)u_2\}\{f_1(x)\}dx \\ &= \left[\rho A\int_0^l f_1(x)f_1(x)dx \quad \rho A\int_0^l f_1(x)\ f_2(x)\ dx\right]\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} \\ &\quad + \left[EA\int_0^l f_1'(x)f_1'(x)dx \quad EA\int_0^l f_1'(x)f_2'(x)dx\right]\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \\ &= \begin{bmatrix} m_{11} & m_{12} \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \end{aligned} \quad (14)$$

In the same way, we can calculate :

$$F_{x_2} = \begin{bmatrix} m_{21} & m_{22} \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_{21} & k_{22} \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad (15)$$

As a result and in a general form the mass and rigidity elements are given by :

$$m_{ij} = \rho A \int_0^l f_i(x)\ f_j(x)\ dx \qquad (16)$$

$$k_{ij} = EI \int_0^l f_i''(x)\ f_j''(x)\ dx \qquad (17)$$

Using the function forms (8), the resulting equation of motion of one element of the structure is given by :

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ M_1 \\ F_{x2} \\ F_{y2} \\ M_2 \end{Bmatrix} = \frac{EI}{L}\begin{bmatrix} \frac{12}{L^2}\mu^2 & & & & & \\ \frac{12}{L^2}\lambda\mu & +\frac{12}{L^2}\lambda^2 & & & & \\ -\frac{6}{L}\mu & \frac{6}{L}\lambda & 4 & & & \\ -\frac{12}{L^2}\mu^2 & \frac{12}{L^2}\lambda\mu & \frac{6}{L}\mu & +\frac{12}{L^2}\mu^2 & & \\ \frac{12}{L^2}\lambda\mu & -\frac{12}{L^2}\lambda^2 & -\frac{6}{L}\lambda & -\frac{12}{L}\lambda\mu & +\frac{12}{L^2}\lambda^2 & \\ -\frac{6}{L}\mu & \frac{6}{L}\lambda & 2 & \frac{6}{L}\mu & -\frac{6}{L}\lambda & 4 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix} \qquad (18)$$

$$-\omega^2\frac{\rho AL}{420}\begin{bmatrix} 156\,\mu^2 & & & & & \\ -156\,\lambda\,\mu & 156\,\lambda^2 & & & & \\ -22\,L\,\mu & 22\,L\,\lambda & 4\,L^2 & & & \\ 54\,\mu^2 & -54\,\lambda\,\mu & -13\,L\,\mu & 156\,\mu^2 & & \\ -54\,\lambda\,\mu & 54\,\lambda^2 & 13\,L\,\lambda & -156\,\lambda\,\mu & 156\,\lambda^2 & \\ 13\,L\,\mu & -13\,L\,\lambda & -3\,L^2 & 22\,L\,\mu & -22\,L\,\lambda & 4\,L^2 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix}$$

2.4. Free vibration

The equation of motion for the free vibration beam is given by :

$$[m]\{\ddot{q}\} + [k]\{q\} = 0 \qquad (19)$$

The shape displacement is considered sinusoidal of the form :

$$q(x) = a \sin \omega t \qquad (20)$$

By substituting the equation (20) in the equation (19) we can write :

$$\left[[k] - \omega^2 [m]\right] q(x) = \{0\} \qquad (21)$$

To determine the clean fashions of the structure to solve the system of equation [9] :

$$\left[[k][m]^{-1} - \omega^2 [m][m]^{-1}\right] q(x) = 0 \qquad (22)$$

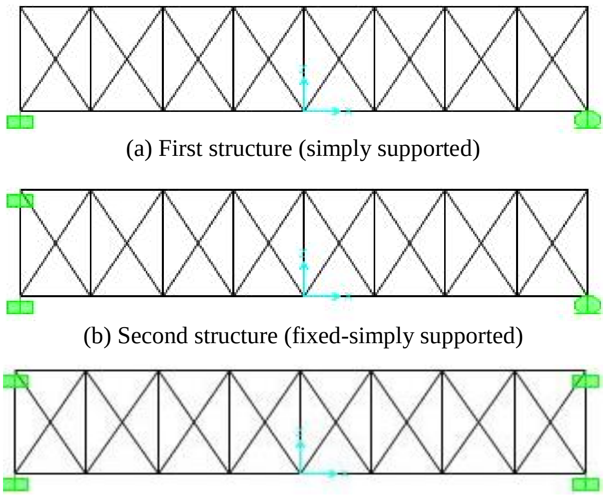
Then :

$$\det\left[A - \lambda I\right] = 0 \qquad (23)$$

Solutions of λ_i are the clean values of the matrix $[A]$, as every clean value corresponding to a clean vector (or eigen values or eigen vectors).

3. STRUCTURE CHARACTERISTICS

The considered structures are composed by metallic beam elements welded together at the nodes, with different limit conditions at their extremities (fig.3). Their geometrical features have a square section S=0.0244 m^2, a length lattice L=2.85 m and a height H=4 m. Their mechanical properties are given by the Young module E=20010^9 N/m^2 and the density ρ=7849 kg.m^{-3}. After assembly, the hall mass and rigidity matrices of the structure can be determinate using numerical programs.



(a) First structure (simply supported)

(b) Second structure (fixed-simply supported)

(c) Third structure (fixed-fixed)

Figure 3 : Beam structures under different limit conditions

4. RESULTS AND DISCUSSION

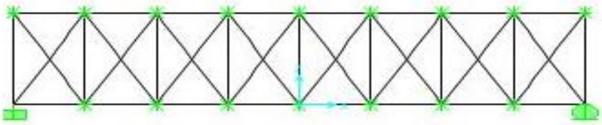
Using our code, we have performed the calculation of normal modes directly from input data of the mechanical problem as the mass matrix of the whole structure in the frequencies making clean. This code is elaborate for advanced computing vibration modes studies [10]. Here, we consider a model without damping [11], preload without structure, and with a number of degrees of reasonable freedom. The most ergonomic solution is to use our computer code which is based on the solution of the equation of clean fashion obtained for the hall assembled structure.

5.1. Vibration modes

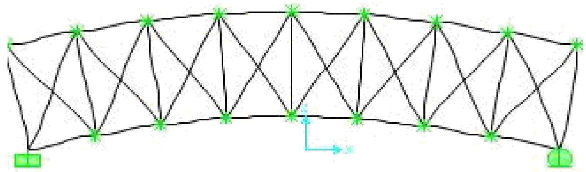
In the vibration analysis of the three considered structures, we have determined the first four vibration modes with different boundary conditions and with the same load of three different structures as shown on figures 4, 5 and 6. In this study, we calculate the natural modes of vibration of a mechanical structure without external excitation and their frequencies on table 1.

Table 1 : Frequencyfor the three structures

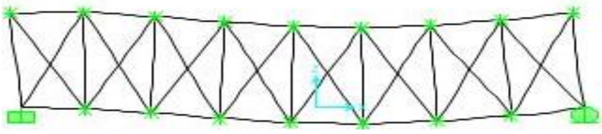
Structure	One	Two	Three
Frequency(HZ) of the first mode	23,83	170,47	23,83
Frequency(HZ) of the second mode	91,51	352,54	133,83
Frequency(HZ) of the third mode	165,36	375,52	185,77
Frequency(HZ) of the fourth mode	302,19	552,45	326,51



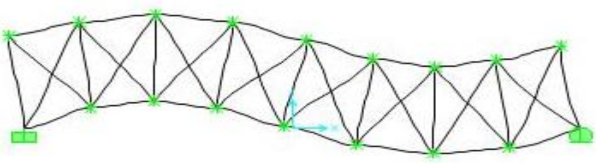
(a) First mode



(b) Second mode

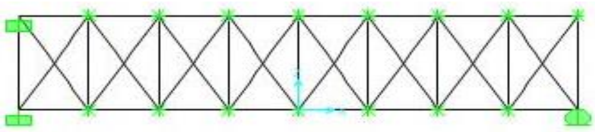


(c) Third mode

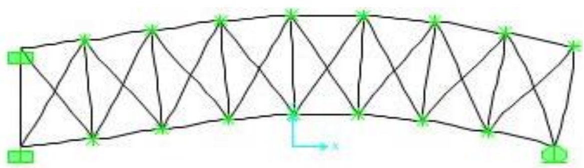


(d) Fourth mode

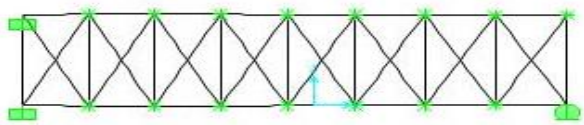
Figure 4 : Vibration modes of the first structure



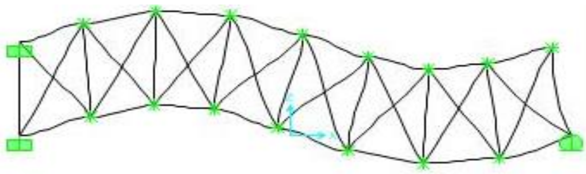
(a) First mode



(b) Second mode

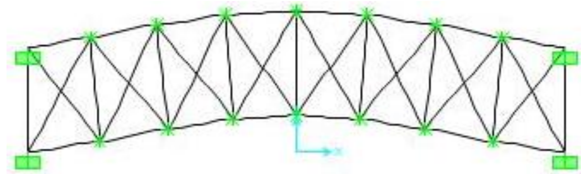


(c) Third mode

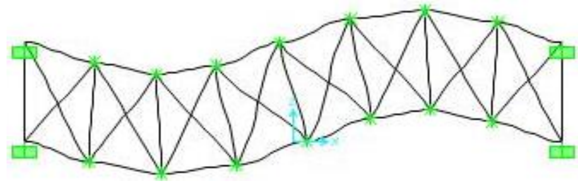


(d) Fourth mode

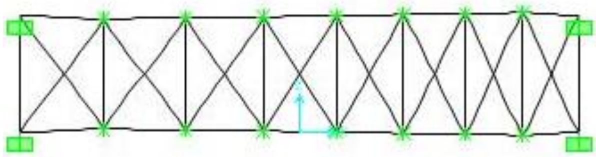
Figure 5 : Vibration modes of the second structure



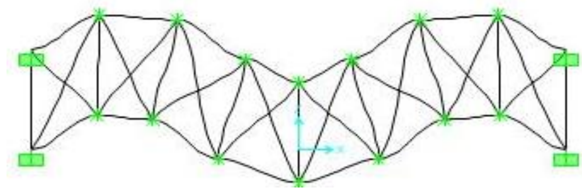
(a) First mode



(b) Second mode



(c) Third mode



(d) Fourth mode

Figure 6 : Vibration modes of the third structure

5.2. Displacements

Figures 7, 8 and 9 present the variation of the displacements under the axis *X(u)*, the axis *Y(v) and* the rotation angle θ respectively. According to these results, it has been noted that the maximal displacements under the axis *X(u)* and *Y(v)* are obtained in the second and the seventeen nodes. However, the maximal displacements under the rotation angle θ is obtained in the third node. The comparison between the numerical results developed using our developed program and the SAP2000 program present a good agreements. This confirms our numerical method.

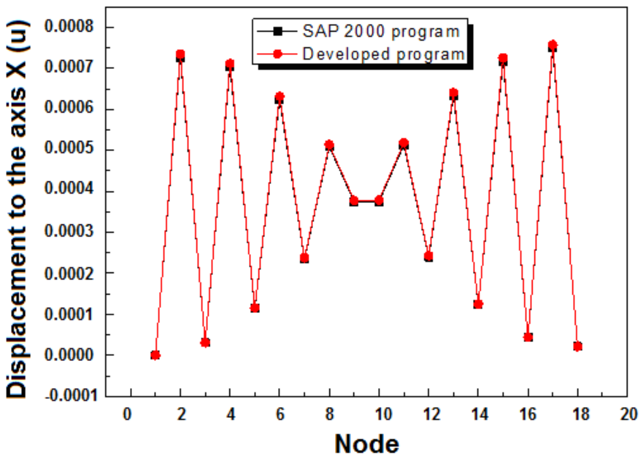


Figure 7 : Comparison of the displacements under the axis X(u)

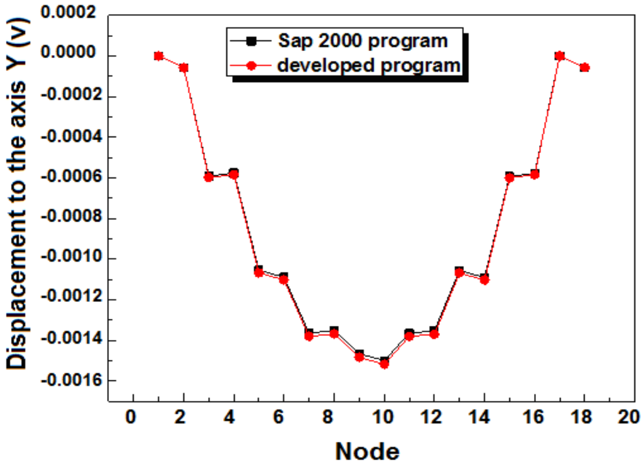


Figure 8 : Comparison of the displacements under the axis Y(v)

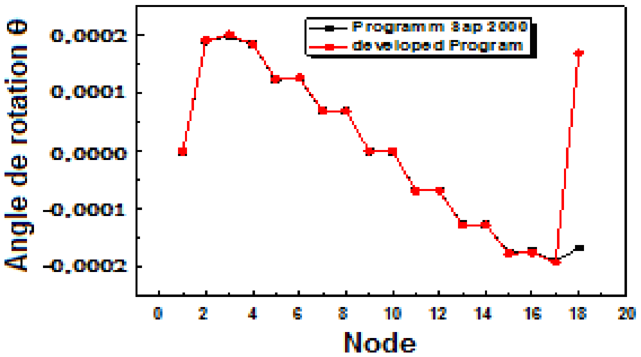


Figure 9 : Displacements in term of the rotation angle θ

5.3. Reactions

Figure 10 presents the variation of the reaction over the nodes. According to these results, it has been noted that the maximal value of the reaction is obtained in the first node. However, the minimal value of the reaction is obtained in the twenties-two node. The comparison between the numerical results developed using our developed program and the SAP 2000 program present a good agreements. This confirms our numerical method.

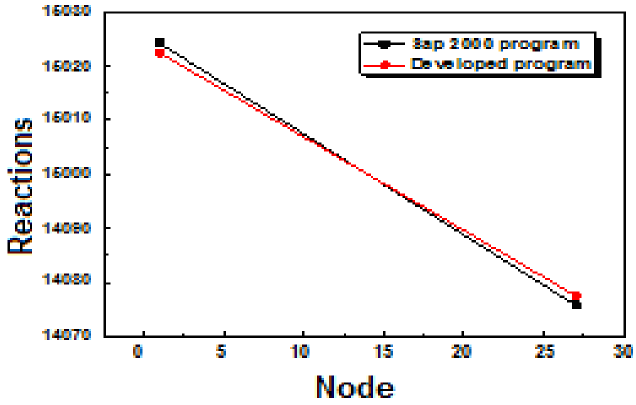


Figure 10 : Variation of the reaction

5.4. Reactions

Figure11 presents the variation of the axial strengths over the elements. According to these results, it has been noted that the maximal value of the axial strength is obtained in the twenties-two element. However, the minimal value of the axial strength is obtained in the twenties element.

The comparison between the numerical results developed using our developed program and the SAP 2000 program present a good agreements. This confirms our numerical method.

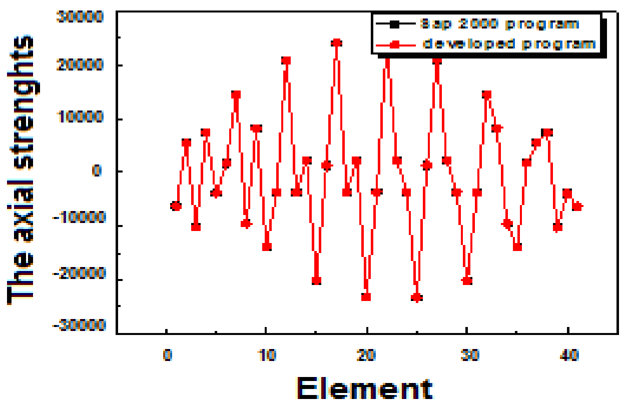


Figure 11 : Variation of the axial strength

CONCLUSION

Dynamic analysis of a structure by the finite element method has been done to obtain the specific modes of the structure. The results have been presented the effect of a change of load applied boundary conditions. Our developed code program allows us to choose the appropriate structure. Through these results, the displacement of each node, reactions, axial forces in elements in static analysis on one side and the other side own frequencies in the modal analysis. The comparison between the numerical results developed using our developed program and the SAP2000 program present a good agreements which confirms our numerical method.

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