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COMPARATIVE STUDY BETWEEN THE RECTANGULAR AND CIRCULAR MICROSTRIP PATCH ANTENNAS

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Résumé

Une conception précise des antennes patch de forme rectangulaire et circulaire imprimées sur un substrat diélectrique est appliquée en utilisant la méthode des moments, l'approche adoptée dans cet article pour la détermination du tenseur spectral de Green est très efficace. Une étude comparative entre ces deux formes rectangulaire et circulaire est faite et comparée avec d'autres résultats disponibles dans la littérature. L'influence des différents paramètres de l'antenne sur la directivité est présentée.

***Mots clés**: Antenne, patch, rectangulaire, circulaire, méthode des moments.*

Abstract

An accurate design of both rectangular and circular patch antenna on a micro strip substrate is applied by using the moments method, an efficient technique to dyadic Green's function is presented, in order to show the feasibility of the method some numerical data are generated, the effects of different parameters on the directivity are discussed, comparative study between this two different shapes is done and compared with other computed results.

***Key Words**: Antenna, patch, rectangular, circular, moment's method.*

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ملخص

في هذا العمل قمنا بتقديم دراسة دقيقة لهوائي مستطيل ودائري ذو الشريط المصغر مطبوع على طبقة عازلة وذلك بتطبيق طريقة العزوم، الدراسة المتبعة في هذا العمل لإيجـاد الـشعاع المركـب الطيفي لغرين Green جد فعالة وقد تم أيضا في هذا العمل القيام بمقارنة نموذجيـة بـين هـذين الهوائيين المستطيل والدائري وكذا دراسة تـأثير مختلـف الوسـائط علـى وجهـة الحقـل الكهرومغناطيسي.

***الكلمات المفتاحية:** هوائي، الشريط المصغر، مستطيل، دائري، طريقة العزوم.*

Over the past twenty five years microstrip resonators have been widely used in the range of microwave frequencies. In general these structures are poor radiators, but by proper design the radiation performance can be improved and these structures can be used as antenna elements [1-3]. In recent years microstrip patch antennas became one of the most popular antenna types for use in aerospace vehicles, telemetry and satellite communication. A microstrip antenna is fabricated by printing a specifically shaped metallic patch on a grounded dielectric slab. At low frequencies, the analysis for this structure can be done by using either the transmission model or a cavity model [1]. However, for high frequency operation in the millimeter-wave range, the thin substrate approximation of low frequencies is not valid and a rigorous analysis is necessary for greater accuracy in the design of the element.

An accurate design of both a rectangular and circular patch on a thick substrate can be done by using the method of moments [1-5], which has proven to be a very useful and precise tool to analyse and design microstrip structures. An integral equation can be formulated by using the Green’s function on a thick dielectric substrate to determine the electric field at any point. The solution of such an integral equation is finally obtained by the moment method with the given set of boundary conditions. From this analysis, the current distribution on the patch is determined [1]; the choice of entire domain defined in the field of the patch was illustrated to develop the unknown currents on the patch. Since the effect of different parameters on the directivity of a microstrip antenna has not yet been treated, a number of results pertaining to this case are presented in this study; also it is important to compare the directivity between the rectangular and circular patch.

THÉORIE

The patch is assumed to be located on a grounded dielectric slab of infinite extent, and the ground plane is assumed to be perfect electric conductor, the rectangular (or circular) patch is embedded in a single substrate containing isotropic materials and has a uniform thickness of *h*, all the dielectric materials are assumed to be nonmagnetic with permeability μ_0. To simplify the analysis, the antenna feed will not be considered. The boundary condition on the patch is given by [3]

$$\overline{\mathbf{E}}_{scat} + \overline{\mathbf{E}}_{inc} = 0 \tag{1}$$

$\overline{\mathbf{E}}_{inc}$ Tangential components of incident electric field

$\overline{\mathbf{E}}_{scat}$ Tangential components of scattered electric field

From Maxwell's equations in the Fourier transform domain, the transverse fields in the (TM, TE) representation can be written in terms of the longitudinal components $\widetilde{E}_z$ and $\widetilde{H}_z$ as [3] [7]

$$\widetilde{\mathbf{E}}_s(k_\rho, z) = \begin{bmatrix} \widetilde{\mathbf{E}}_s^{TM}(k_\rho, z) \\ \widetilde{\mathbf{E}}_s^{TE}(k_\rho, z) \end{bmatrix} = \mathbf{A}(k_\rho)e^{jk_z z} + \mathbf{B}(k_\rho)e^{-jk_z z} \tag{2}$$

$$\widetilde{\mathbf{H}}_s(k_\rho, z) = \begin{bmatrix} \widetilde{\mathbf{H}}_s^{TM}(k_\rho, z) \\ \widetilde{\mathbf{H}}_s^{TE}(k_\rho, z) \end{bmatrix} = \overline{\mathbf{g}}(k_\rho)\left[\mathbf{A}(\mathbf{k}_\rho)e^{jk_z z} - \mathbf{B}(k_\rho)e^{-jk_z z}\right] \tag{3}$$

A and **B** are two unknowns vectors to be determined.
Where

$$\overline{\mathbf{g}} = \begin{bmatrix} \dfrac{\omega \varepsilon_0 \varepsilon_r}{k_{z1}} & 0 \\ 0 & \dfrac{k_{z1}}{\omega \mu_0} \end{bmatrix} \tag{4}$$

k_z and k_{z1} are, respectively, propagation constants in the free space and the substrate. By eliminating the unknowns **A** and **B**, in the equations (2) and (3) we obtain the following matrix which combine the tangential field components on both sides of the considered layer as input and output quantities.

$$\begin{bmatrix} \widetilde{\mathbf{E}}_2(k_\rho, z) \\ \widetilde{\mathbf{H}}_2(k_\rho, z) \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{I}}\cos(k_{z1}h) & i\overline{\mathbf{g}}^{-1}\sin(k_{z1}h) \\ i\overline{\mathbf{g}}\sin(k_{z1}h) & \bar{\mathbf{I}}\cos(k_{z1}h) \end{bmatrix} \times \begin{bmatrix} \widetilde{\mathbf{E}}_1(k_\rho, z) \\ \widetilde{\mathbf{H}}_1(k_\rho, z) \end{bmatrix} \tag{5}$$

Where $k_z = k_0 \cos(k_{z1} h)$, $k_{z1}^2 = k_1^2 - k_\rho^2$, $k_1 = k_0\sqrt{\varepsilon_r}$ and $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$

In the spectral domain the relationship between the patch current and the electric field on the patch is given by

$$\widetilde{\mathbf{E}}_s(\mathbf{k}_s) = \overline{\mathbf{G}}(\mathbf{k}_s) \cdot \widetilde{\mathbf{J}}(\mathbf{k}_s) \tag{6}$$

Where $\overline{\mathbf{G}}$ is the spectral dyadic Green's function given by equation (7) and $\widetilde{\mathbf{J}}(\mathbf{k}_s)$ is the current on the patch which related to the vector Fourier transform of $\mathbf{J}(\mathbf{r_s})$ in the case of a rectangular patch as [6] and to the vector Hankel transform in the case of a circular patch as [7]

$$\overline{\mathbf{G}} = \begin{bmatrix} \mathbf{G}^{TM} & 0 \\ 0 & \mathbf{G}^{TE} \end{bmatrix} \tag{7}$$

With

$$\mathbf{G}^{TM} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\cos(k_{z1} h)}{\left(1 - j\,\varepsilon_r\, k_z \cot(k_{z1} h)/k_{z1}\right)} \tag{8.a}$$

$$\mathbf{G}^{TE} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{\cos(k_{z1} h)\left(1 - j\, k_{z1} \cot(k_{z1} h)/k_z\right)} \tag{8.b}$$

A rectangular patch with length *a* and width *b* is printed on the grounded substrate. Which has a uniform thickness of *h*, the surface current on the patch can be expanded into a series of known basis functions J_{xn} and J_{ym}

$$\mathbf{J}(\mathbf{r}_s) = \sum_{n=1}^{N} a_n \begin{bmatrix} J_{xn}(\mathbf{r}_s) \\ 0 \end{bmatrix} + \sum_{m=1}^{M} b_m \begin{bmatrix} 0 \\ J_{ym}(\mathbf{r}_s) \end{bmatrix} \tag{9}$$

Where a_n and b_m are the unknown coefficients to be determined in the x and y direction respectively. For the resonant patch, entire domain expansion currents lead to fast convergence and can be related to a cavity model type of interpretation [6]. In the case of a rectangular micro strip patch a Fourier transform of J_{xn} and J_{ym} were used, starting with the sinusoid basic function without edge condition definite on the whole domain, given by Newman and Forrai to develop the currents [5] those currents associated with the complete orthogonal modes of the magnetic cavity, both x and y directed currents were used, with the following forms [5] [8].

$$J_{xn}(\mathbf{r_s}) = \left[\sin\left(\frac{n_1 \pi}{a}\left(x + \frac{a}{2}\right)\right)\right] \times \cos\left(\frac{n_2 \pi}{b}\left(y + \frac{b}{2}\right)\right) \tag{10.a}$$

$$J_{ym}(\mathbf{r_s}) = \left[\cos\left(\frac{m_1 \pi}{a}\left(x + \frac{a}{2}\right)\right)\right] \times \sin\left(\frac{m_2 \pi}{b}\left(y + \frac{b}{2}\right)\right) \tag{10.b}$$

In the case of a circular microstrip patch antenna a Hankel transform of $\mathbf{k}_{nm}(\rho)$ and $\mathbf{f}_{np}(\rho)$ were used [7]

$$\mathbf{k}_n(\rho)=\begin{cases}\sum_{m=1}^{\infty} a_{nm}\ \mathbf{k}_{nm}(\rho)+\sum_{p=1}^{\infty} b_{np}\ \mathbf{f}_{np}(\rho) \\ 0 \qquad \rho > a\end{cases} \tag{11}$$

where $\mathbf{k}_{nm}(\rho)$ and $\mathbf{f}_{np}(\rho)$ is given by

$$\mathbf{k}_{nm}(\rho)=\begin{cases}\begin{bmatrix} \dot{J}_n(\boldsymbol{\beta}_{\mathbf{nm}}\rho) \\ \dfrac{in}{\boldsymbol{\beta}_{nm}\rho} \mathrm{J}_n(\boldsymbol{\beta}_{\mathbf{nm}}\rho)\end{bmatrix} & \rho \le a \\ 0 & \rho \ge a\end{cases} \tag{12.a}$$

$$\mathbf{f}_{np}(\rho)=\begin{cases}\begin{bmatrix} \dfrac{-in}{\boldsymbol{\alpha}_{np}\rho} \mathrm{J}_n(\boldsymbol{\alpha}_{np}\rho) \\ \dot{J}_n(\boldsymbol{\alpha}_{np}\rho)\end{bmatrix} & \rho \le a \\ 0 & \rho \ge a\end{cases} \tag{12.b}$$

Where

$\dot{J}_n(\boldsymbol{\beta}_{nm}\, a)=0$ pour $m=1,2,3,\ldots M$.

$\mathrm{J}_n(\boldsymbol{\alpha}_{np}\, a)=0$ pour $\mathrm{p}=1,2,3,\ldots P$.

The integral equation describing the field **E** in the rectangular or circular patch can be discretized into the following matrix

$$\begin{bmatrix} (\overline{Z}_1)_{N\times N} & (\overline{Z}_2)_{N\times M} \\ (\overline{Z}_3)_{M\times N} & (\overline{Z}_4)_{M\times M}\end{bmatrix} \cdot \begin{bmatrix} (a)_{N\times 1} \\ (b)_{M\times 1}\end{bmatrix} = 0 \tag{13}$$

In the rectangular case the element of the submatrices is given by

$$(\overline{Z}_1)=\iint_{-\infty}^{\infty} d\mathbf{k}_s \frac{1}{k_s^2}\left[k_x^2 G^e + k_y^2 G^h\right]\times \tilde{J}_{xk}(-\mathbf{k}_s)\tilde{J}_{xn}(\mathbf{k}_s) \tag{14.a}$$

$$(\overline{Z}_2)=\iint_{-\infty}^{\infty} d\mathbf{k}_s \frac{k_x k_y}{k_s^2}\left[G^e - G^h\right]\times \tilde{J}_{xk}(-\mathbf{k}_s)\tilde{J}_{ym}(\mathbf{k}_s) \tag{14.b}$$

$$(\overline{Z}_3)=\iint_{-\infty}^{\infty} d\mathbf{k}_s \frac{k_x k_y}{k_s^2}\left[G^e - G^h\right]\times \tilde{J}_{yl}(-\mathbf{k}_s)\tilde{J}_{xn}(\mathbf{k}_s) \tag{14.c}$$

$$(\overline{Z}_4)=\iint_{-\infty}^{\infty} d\mathbf{k}_s \frac{1}{k_s^2}\left[k_y^2 G^e + k_x^2 G^h\right]\times \tilde{J}_{yl}(-\mathbf{k}_s)\tilde{J}_{ym}(\mathbf{k}_s) \tag{14.d}$$

In the circular case the element of the submatrices is given by :

$$(\overline{\mathrm{Z}}_1) = \int_0^{\infty} d\,\mathrm{k}_\rho\, \mathrm{k}_\rho\, \tilde{\mathrm{K}}_{nj}^{\mathrm{T}}(\mathrm{k}_\rho)\overline{\mathrm{G}}(\mathrm{k}_\rho)\tilde{\mathrm{K}}_{nm}(\mathrm{k}_\rho) \tag{15.a}$$

$$(\overline{\mathrm{Z}}_2) = \int_0^{\infty} d\mathrm{k}_\rho\, \mathrm{k}_\rho\, \tilde{\mathrm{K}}_{nj}^{\mathrm{T}}(\mathrm{k}_\rho)\overline{\mathrm{G}}(\mathrm{k}_\rho)\tilde{\mathrm{F}}_{np}(\mathrm{k}_\rho) \tag{15.b}$$

$$(\overline{\mathrm{Z}}_3) = \int_0^{\infty} d\mathrm{k}_\rho\, \mathrm{k}_\rho\, \tilde{\mathrm{F}}_{nk}^{\mathrm{T}}(\mathrm{k}_\rho)\overline{\mathrm{G}}(\mathrm{k}_\rho)\tilde{\mathrm{K}}_{nm}(\mathrm{k}_\rho) \tag{15.c}$$

$$(\overline{\mathrm{Z}}_4) = \int_0^{\infty} d\mathrm{k}_\rho\, \mathrm{k}_\rho\, \tilde{\mathrm{F}}_{nk}^{\mathrm{T}}(\mathrm{k}_\rho)\overline{\mathrm{G}}(\mathrm{k}_\rho)\tilde{\mathrm{F}}_{np}(\mathrm{k}_\rho) \tag{15.d}$$

In (14), $\tilde{J}$ is the scalar Fourier transforms of J and in (15), $\tilde{\mathrm{F}}$, $\tilde{\mathrm{K}}$ are the scalar Hankel transforms of F and K respectively. K_x and K_y k_ρ are the spectral variables. Since the resonant frequencies are defined to be the frequencies at which the field and the current can sustain themselves without a driving source. Therefore, for the existence of nontrivial solutions, the determinant of the [**Z**] matrix must be zero. This condition is satisfied by a complex frequency $f = f_r + if_i$ that gives the resonant frequency f_r and the half power bandwidth $2f_i/f_r$ of the microstrip antenna.

NUMERICAL RESULTS AND DISCUSSION

A rigorous analysis is presented to obtain the complex resonant frequency of a rectangular patch antenna; the Galerkin procedure of the moment method with entire domain sinusoidal basis functions without edge condition is investigated. Patch dimensions of the rectangular antenna is 1.5 cm × 1 cm, the substrate has a relative permittivity of ε_r = 2.35, the TM01 mode is considered, the real and imaginary parts of the normalized complex resonant frequency for this structure are displayed as a function of thickness h, the normalization is with respect to f_0 of the magnetic wall cavity. It is found that length of the integration path required to reach numerical convergence at 60 k_0, a comparative study show a precise agreement between our results and those available in the literature.

In Table 1, comparisons for the calculated and measured data presented by [6] and the calculated results from our model are shown. The perfectly conducting patches of different sizes without dielectric substrates (air) are considered. It is important to note that the normalization is with respect to f_0 of the magnetic wall cavity, the mode studied in this letter is the dominant mode TM01.

A rigorous analysis is presented to obtain the directivity of a rectangular and circular patch antenna; the directivity can be found by the numerical integration of the far field power pattern. The Galerkin procedure of the moment method is investigated; the TM01 mode is considered for the case of a rectangular patch and the TM11 mode is considered for a circular one, the directivity for these rectangular and circular structures are displayed as a function of the substrate relative permittivity in Figure 3 and as a function of frequency in Figure 4.

Figure 3 shows the effect of the relative substrate permittivity in the directivity for a patch antenna operate at 2.4 Ghz with a substrate thickness h=0.159 cm, we note that an increase in the relative permittivity of substrate cause a decrease in the directivity, however in Figure 3 it is found that an increase in the frequency causes an increase in the directivity, we note also from this figure that in the case of a circular we are slightly increase in the directivity compared to a rectangular patch, we note also from these two figures that the directivity is more important for a circular patch antenna compared to a rectangular one.

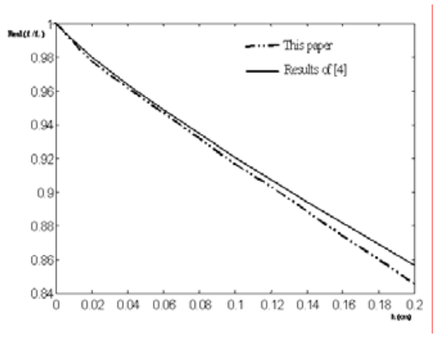

Figure 1: Real part of normalized resonant frequency versus the substrate thickness for *TM01* mode without edge condition.

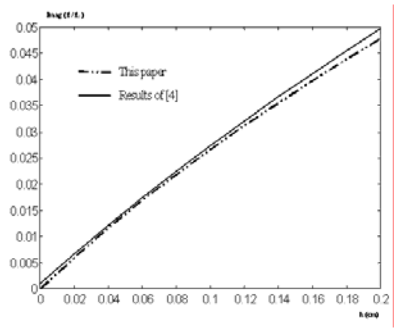

Figure 2: Imaginary part of normalized resonant frequency versus the substrate thickness without edge condition.

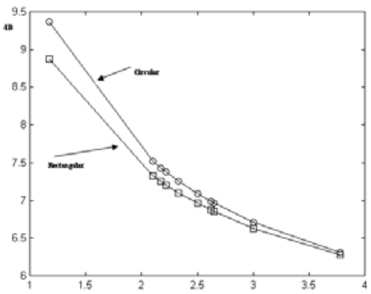

Figure 3: Directivity for a rectangular and circular patch antenna versus the substrate relative permittivity h=0.159cm *freq*=2.4 Ghz

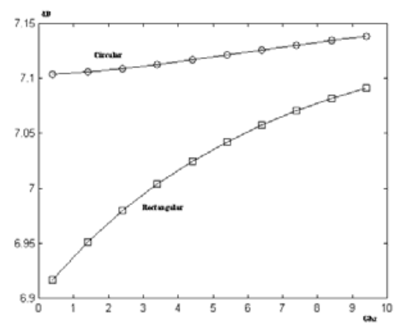

Figure 4: Directivity for a rectangular and circular patch antenna versus the frequency. h=0.159cm ε_r = 2.5

CONCLUSION

The moment method technique has been developed for both a rectangular and circular micro strip patch antenna, the directivity can be found by the numerical integration of the far field power pattern, an efficient technique to dyadic Green's function is presented; comparative study between these two different shapes is done and compared with earlier calculated results available in the literature. The effects of different parameters on the directivity are investigated. Accuracy of the computed technique shows a very precise agreement between theory and those available in the literature.

Table 1: Measured and calculated resonant frequencies of rectangular microstrip antennas.

a (cm)	*b* (cm)	*h* (cm)	Measured[6] (f/f_0)	James (f/f_0)	Hammerstad (f/f_0)	[6] (f/f_0)	Our results (f/f_0)
5.70	3.80	0.317	0.893	0.889	0.920	0.919	0.882
4.55	3.05	0.317	0.897	0.866	0.900	0.903	0.863
2.95	1.95	0.317	0.841	0.816	0.861	0.859	0.816
1.95	1.30	0.317	0.773	0.754	0.810	0.805	0.765
1.70	1.10	0.317	0.761	0.724	0.785	0.773	0.740
1.40	0.90	0.317	0.705	0.683	0.750	0.722	0.710
1.20	0.80	0.317	0.673	0.662	0.733	0.684	0.693
1.05	0.70	0.317	0.651	0.633	0.710	0.620	0.672
1.70	1.10	0.152	0.881	0.835	0.878	0.876	0.834
1.70	1.10	0.317	0.761	0.724	0.785	0.773	0.739

REFERENCES

[1]- A. Boufrioua, A. Benghalia, ‘‘Analyse d’une antenne microbande circulaire par l’application de la méthode intégrale,’’ OHD’01, 16ème colloque international optique et hertzienne et diélectrique, France,.pp. 193-196, 3-4-5, Sep 2001.

[2]- O. Lafond, M. Himdi, and J. P. Daniel, ‘‘Aperture coupled microstrip patch antenna with thick ground plane in millimetre waves,’’ Electron. Lett., vol. 35, no. 17, pp. 1394-1395, 1999.

[3]- D. M. Pozar, ‘‘Radiation and scattering from a microstrip patch on a uniaxial substrate,’’ *IEEE Trans. Antennas Propagat.,* vol. AP-35, pp. 613-621, June 1987.

[4]- F. Bouttout, F. Benabdelaziz, T. Fortaki and D. Khedrouche, ‘‘Resonant frequency and bandwidth of a superstrate-loaded rectangular patch on a uniaxial anisotropic substrate,’’ communications in numerical methods in Engineering (John Wiley & sons)., vol. 16, issue7, pp. 459-473. Jul. 2000.

[5]- Jeen-Sheen Row and Kin-Lu Wong ‘‘Resonance in a superstrate-loaded rectangular microstrip structure,’’ *IEEE Trans. Antennas Propagat.,* vol. AP-29, pp. 1349-1355, Aug. 1993.

[6]- Chew, W.C.& Liu, Q. ‘‘Resonance frequency of a rectangular microstrip patch,’’ IEEE Transactions on Antennas and Propagation, 36, PP.1045-1056, 1988.

[7]- W. C. Chew and J. A. Kong, ‘‘Analysis of a circular microstrip disk antenna with a thick dielectric substrate,’’ *IEEE Trans. Antennas Propagat.,* vol. AP-29, pp. 68-76, Jan. 1981.

[8]- Kin-Lu Wong & Jeen-Sheen Row. ‘‘Resonance of a rectangular microstrip patch on a uniaxial substrate,’’ IEEE Transactions on Microwave Theory and Techniques, Vol 41 No 4, PP.698-701, Apr 1993.