IMPEDANCE REPRESENTATION OF TWO PHASE COMPOSITE MATERIALS AND EXPERIMENTAL MODELLING

Résumé

Cet article présente une méthode non destructive, basée sur les impédances électriques équivalentes, permettant de caractériser des hétérostructures ou des matériaux composites. Les hétérostructures sont constituées d'un matériau diélectrique hôte (une matrice tridimensionnel homogène) et d'inclusions diélectriques ou conductrices. Deux exemples d'hétérostructures utilisées en génie civil sont particulièrement considérés, à savoir des matériaux creux et des matériaux comportant des tubes métalliques simulant, par exemple, des briques creuses et des piliers (ou pylônes) respectivement. Des lois de mélanges (c'est à dire la permittivité et la resistivité (conductivité) effectives) sont également déduites en fonction de la concentration en inclusions ainsi que de la permittivité et de la résistivité de chacun des constituants de l'hétérostructure. Pour illustrer cette méthode, une validation expérimentale est effectuée sur des échantillons contenant des inclusions identiques alignées, en forme de tubes carré, encastrées dans une matrice polymère et remplies soit d'air soit d'eau. Les résultats prédits par la méthode proposée concordent de façon raisonnable avec ceux obtenus expérimentalement dans une large gamme de concentrations volumiques d'inclusions.

Mots clés : *Méthode non destructive, Impédance, Matrice, Inclusion, Permittivité, Résistivité, Lois de mélange.*

Abstract

This paper presents a nondestructive method enabling to characterize heterostructures or composite materials based on the equivalent electrical impedance. The heterostructures consist of a host dielectric material (a three-dimensional homogeneous matrix) and dielectric or conducting inclusions. We especially consider two examples of heterostructures used in building namely hollow materials and materials containing metallic tubes simulating for instance hollow bricks and pillars (or pylons) respectively. The mixture laws (i.e., the effective permittivity and resistivity) are also deduced in function of the concentration of inclusions and both the permittivity and resistivity of each component of the heterostructure. To illustrate this method, experimental validation is achieved on samples containing identical aligned inclusions, in the form of square section tubes, embedded in a polymer matrix and which are filled either by air or water. The results predicted by the proposed method are found in reasonable accordance with those obtained experimentally over a wide range of volume fraction of inclusions.

Keywords: *Non destructive method, Impédance, Matrix, Inclusion, Permittivity, Resistivity, Mixture laws.*

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ملخص

The characterization of composite materials or heterostructures is of a great interest for different applications (building, mechanics, electrical engineering, biology…). In building engineering, this characterization is needed to evaluate the cavities or water contents of a given medium, and the presence of cracks resulting from different stresses (chemical aggression, heating, earthquakes, bad proportioning …). Among the methods used to evaluate the water content in a given medium, one can cite the thermal method [1], the nuclear methods [2], the method based on the measurement of the resistivity called "electrical method" [3-5] and the methods based on the measurement of the dielectric constant called "dielectric methods" [6-8]. These latter consist either to measure the capacitance of a given sample of the considered material or heterostructure (capacitive method) or to analyse the propagation velocity of an electromagnetic wave in the medium (reflectometry method known as T.D.R. method); this propagation velocity varies with the permittivity of the medium.

During the last decade, nondestructive methods using ultra wideband radar have been developed to visualize variations of cracks, voids, and rebars within the concrete and to determine the structural properties [9-10]. These methods enable, through the measurement of dielectric property, to evaluate the performance of cement concrete or new construction materials, such as polymer concrete developed to reduce the high cost of rehabilitation of structural concrete [11].

This paper is aimed at the characterization of heterostructures consisting of a three-dimensional homogeneous matrix (host material) and dielectric or conducting inclusions using a method based on the determination of the equivalent impedance. It is organized as follows. In the first section, we present the basic principle of the equivalent impedance method and we compute the equivalent impedance of typical structures that we further used, namely sandwich structures the constituents of which are dielectrics, conductors or both (dielectrics and conductors) and we deduce the corresponding mixture laws (i.e., the effective permittivity and resistivity). In the second section, we compute the equivalent impedance of two typical heterostructures: hollow material which can simulate for instance hollow bricks and a material containing long conductors which can represent for example pillars (pylons). Finally, we present an experimental validation of our method by considering samples containing identical aligned inclusions, in the form of square section tubes, embedded in a polymer matrix and which are filled either by air or water.

1. BASIC PRINCIPLE OF THE EQUIVALENT IMPEDANCE METHOD

The method of equivalent impedance is often used to estimate the quality of electrical insulation of a given material or component in electrical engineering. It consists in the representation of the dielectric material by a capacitance in series or in parallel with a resistance (Figure 1), the resistance representing the dielectric losses. Thus the equivalent impedance Z will be expressed by the following relationships:
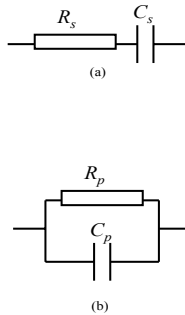
Figure 1: Scheme of electrical equivalent circuit representing a material: **(a)** series representation and **(b)** parallel representation.

- Series representation:

$$Z_s = R_s - j\frac{1}{C_s\omega} = R_s + jX_s \qquad \textbf{(1)}$$

with $X_s = -\frac{1}{C_s\omega}$ and $j^2 = -1$

ω is the angular frequency ; $\omega = 2\pi f$ where f is the frequency of the electrical signal used for the characterization.

For a perfect conductor: $Z_s = R_s$

and

For a perfect insulating: $Z_s = -j\frac{1}{C_s\omega}$

- Parallel representation:

$$\frac{1}{Z_p} = \frac{1}{R_p} + jC_p\omega$$

or

$$Z_p = \left[\frac{R_p}{1-\left(R_pC_p\omega\right)^2}\right] + j\left[\frac{-R_p^2C_p\omega}{1-\left(R_pC_p\omega\right)^2}\right] = R_p + jX_p$$

$$\textbf{(2)}$$

with

$$R_p = \left[\frac{R_p}{1-\left(R_pC_p\omega\right)^2}\right]$$

and

$$X_p = \left[\frac{-R_p^2C_p\omega}{1-\left(R_pC_p\omega\right)^2}\right]$$

It appears from eqs(1) and (2) that Z_s and Z_p also depend on the frequency. X_s and X_p are the reactances.

For a perfect conductor : $Z_p = R_p$

and

For a perfect insulating : $Z_p = -j\frac{1}{C_p\omega}$

Each combination of dielectric and conductor phases can be represented by series or parallel R-C circuits. Before analysing the case of some heterostructures that one can meet in the building applications such as hollow bricks or pylons (pillars), let consider the elementary heterostructures namely sandwich configurations. For that, we analyse the general case where the heterostructure (sandwich) consists of a layer having an impedance Z_i within two layers of a given material of impedances Z_{1m} and Z_{2m} respectively (Figure 2). We assume that: (i) each layer has plane interfaces, (ii) each component is described by its series or parallel impedance, (iii) each component has identical

surface area, and (iv) the layers are independent of each other, i.e., no coupling exists between the components. By taking the equivalent series model for both materials, we have:
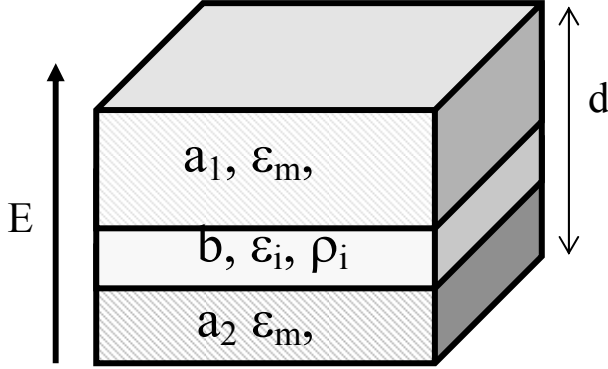

Figure 2: Scheme of a sandwich structure.

$$Z_{1m} = R_{1m} - \frac{1}{jC_{1m}\omega} \quad \text{where } R_{1m} = \rho_m \frac{a_1}{S} \quad \text{and}$$

$$C_{1m} = \varepsilon_m \frac{S}{a_1} \quad \text{(3)}$$

$$Z_{2m} = R_{2m} - \frac{1}{jC_{2m}\omega} \quad \text{where } R_{2m} = \rho_m \frac{a_2}{S} \quad \text{and}$$

$$C_{2m} = \varepsilon_m \frac{S}{a_2} \quad \text{(4)}$$

$$Z_i = R_i - \frac{1}{jC_i\omega} \quad \text{where } R_i = \rho_i \frac{b}{S} \quad \text{and}$$

$$C_i = \varepsilon_i \frac{S}{b} \quad \text{(5)}$$

ρ_m and ρ_i are respectively the resistivities of the material and the inclusion; and ε_m and ε_i are the electric permittivities of the material and the inclusion respectively. Since the impedances Z_{1m}, Z_{2m} and Z_i are in series, the equivalent impedance of the structure Z_e is equal to

$$Z_e = Z_{1m} + Z_{2m} + Z_i \quad \text{(6)}$$

By substituting eqs (3), (4) and (5) in (6), it yields

$$Z_e = \left(\frac{d}{S}\right)\left\{ \begin{array}{l} [\rho_m(1-\beta) + \rho_i\beta] - j\frac{1}{\omega\varepsilon_i\varepsilon_m} \\ [\varepsilon_i(1-\beta) + \varepsilon_m\beta] \end{array} \right\} \quad \text{(7)}$$

where $\quad d = a_1 + a_2 + b$ and $\beta = \frac{b}{d}$;

β is an dimensionless parameter; it also represents the volume concentration of the inclusion in the host matrix. On the other hand, in the limit of **>> , where** is the wavelength of the electromagnetic wave **and the** typical scale of the inhomogeneities, the wave cannot resolve the individual scattering centers. In this so-called quasi-static approximation, the material medium appears homogeneous to the probing wave and is characterized by an effective permittivity [12]. Thus, by taking the series equivalent model, it yields

$$Z_e = R_e - j\frac{1}{C_e\omega} = \rho_e\frac{d}{S} - j\frac{1}{\omega\varepsilon_e}\frac{d}{S} = \left(\frac{d}{S}\right)(\rho_e - j\frac{1}{\omega\varepsilon_e})$$
(8)

with $\quad R_e = \rho_e\frac{d}{S} \quad$ and $\quad C_e = \varepsilon_e\frac{S}{d}$

ρ_e and ε_e are the resistivity and the permittivity of the homogenized heterostructure.
Thus

$$Z_e = \left(\frac{d}{S}\right)(\rho_e - j\frac{1}{\omega\varepsilon_e}) \quad \text{(9)}$$

By identifying the real and imaginary parts in eqs (7) and (9), we get the mixture laws

$$\rho_e = \rho_m(1-\beta) + \rho_i\beta \quad \text{(10)}$$

or

$$\frac{1}{\sigma_e} = \frac{1}{\sigma_i\sigma_m}[\sigma_i(1-\beta) + \sigma_m\beta] \quad \text{(11)}$$

(σ_e, σ_m and σ_i are the conductivities of the homogenized heterostructure, the host material and the inclusion respectively)
and

$$\frac{1}{\varepsilon_e} = \frac{1}{\varepsilon_i\varepsilon_m}[\varepsilon_i(1-\beta) + \varepsilon_m\beta] \quad \text{(12)}$$

or

$$\varepsilon_e = \frac{\varepsilon_i\varepsilon_m}{\varepsilon_i(1-\beta) + \varepsilon_m\beta} \quad \text{(13)}$$

Equations (10) and (13) represent the mixture laws. These general equations can be applied to different particular cases as indicated in Table 1. Note that, in the same way, one can compute the equivalent impedance as well as the equivalent resistivity and permittivity of a given heterostructure using the parallel equivalent circuit to represent each constituent. In that case, we use the following relationships for Z_{1m}, Z_{2m}, Z_i and Z_e

Table 1: Mixture laws for different sandwich structures.

Sandwich structure type	Mixture laws
dielectric with losses within a dielectric with losses	$\rho_e = \rho_m(1-\beta) + \rho_i\beta$ $\varepsilon_e = \dfrac{\varepsilon_i\varepsilon_m}{\varepsilon_i(1-\beta) + \varepsilon_m\beta}$
conductor - conductor - conductor	$\rho_e = \rho_m(1-\beta) + \rho_i\beta$
dielectric-dielectric-dielectric	$\varepsilon_e = \dfrac{\varepsilon_i\varepsilon_m}{\varepsilon_i(1-\beta) + \varepsilon_m\beta}$
dielectric-conductor-dielectric	$\rho_e = \rho_i\beta$ $\varepsilon_e = \dfrac{\varepsilon_m}{(1-\beta)}$
conductor-dielectric-conductor	$\rho_e = \rho_m(1-\beta)$ $\varepsilon_e = \varepsilon_i$
dielectric without losses within a dielectric with losses	$\rho_e = \rho_m(1-\beta)$ $\varepsilon_e = \dfrac{\varepsilon_i\varepsilon_m}{\varepsilon_i(1-\beta) + \varepsilon_m\beta}$
dielectric with losses within a dielectric without losses	$\rho_e = \rho_i\beta$ $\varepsilon_e = \dfrac{\varepsilon_i\varepsilon_m}{\varepsilon_i(1-\beta) + \varepsilon_m\beta}$
conductor within a dielectric with losses	$\rho_e = \rho_m(1-\beta) + \rho_i\beta$ $\varepsilon_e = \dfrac{\varepsilon_m}{(1-\beta)}$
dielectric with losses within a conductor	$\rho_e = \rho_m(1-\beta) + \rho_i\beta$ $\varepsilon_e = \dfrac{\varepsilon_i}{\beta}$

$$Z_{1m,p} = \left[\frac{R_{1m,p}}{1-(R_{1m,p}C_{1m,p}\omega)^2}\right] + j\left[\frac{-R_{1m,p}^2 C_{1m,p}\omega}{1-(R_{1m,p}C_{1m,p}\omega)^2}\right] = R_{1m,p} + jX_{1m,p} \quad (14)$$

$$Z_{2m,p} = \left[\frac{R_{2m,p}}{1-(R_{2m,p}C_{2m,p}\omega)^2}\right] + j\left[\frac{-R_{2m,p}^2 C_{2m,p}\omega}{1-(R_{2m,p}C_{2m,p}\omega)^2}\right]$$
$$= R_{2m,p} + jX_{2m,p} \quad (15)$$

$$Z_{i,p} = \left[\frac{R_{i,p}}{1-(R_{i,p}C_{i,p}\omega)^2}\right] + j\left[\frac{-R_{i,p}^2 C_{i,p}\omega}{1-(R_{i,p}C_{i,p}\omega)^2}\right] =$$
$$R_{i,p} + jX_{i,p} \quad (16)$$

We calculate the equivalent impedance Z_e of $Z_{1m,p}$, $Z_{2m,p}$ and $Z_{i,p}$ such as

$$Z_{e,p} = Z_{1m,p} + Z_{2m,p} + Z_{i,p} \qquad \textbf{(17)}$$

And we identify the real and imaginary parts of $Z_{e,p}$ to those of the impedance of the homogenized heterostructure

$$Z_{e,p} = \left[\frac{R_{e,p}}{1-(R_{e,p}C_{e,p}\omega)^2}\right] + j\left[\frac{-R_{e,p}^2 C_{e,p}\omega}{1-(R_{e,p}C_{e,p}\omega)^2}\right] = R_{e,p} + jX_{e,p}$$

$$\textbf{(18)}$$

By introducing the relationships of the resistance and capacitance for each impedance as in eqs. (3), (4), (5) and (8), one computes the resistivity and the permittivity of the homogenized heterostructure $\rho_{e,p}$ and $\rho_{e,p}$ resp**ectively.**

2. HETEROSTRUCTURE WITH LONG INCLUSIONS PERIODICALLY DISTRIBUTED

Different heterostructures can be find in practice. These consist of inclusions embedded in a host matrix. The inclusion can be either dielectric (holes, cavities, water, …) or conductor (metallic) and the host material can be also either dielectric or conductor. The inclusions can be randomly or periodically distributed. In the following, we assume that the heterostructures can be divided into cells, the inclusions having the same length as the host material (**Figure 3**).
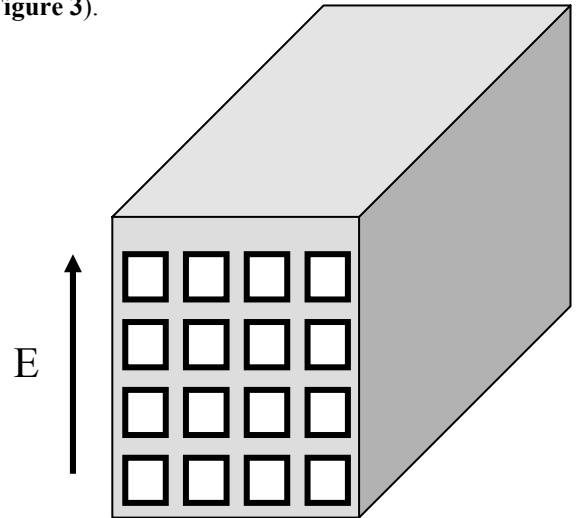

Figure 3: Geometry of the heterostructure sample with the electric field perpendicular to the croos section of the inclusions.

This problem is scale invariant, i.e. if the entire system is shrunk or dilated uniformly, the effective permittivity does not change [13]. Thus, the study can be reduced to that of a unit cell (Figure 4). Thus, according to Figure 4, one can subdivide the unit cell into three regions: region (0) containing the inclusion and regions (1) and (2) consisting only of the host matrix. Let S_1 and S_2 the sections of regions (1) and (2) respectively, and S_0 that of the central region (0); and Z_{S1m}, Z_{S2m} and Z_0 the impedances of regions (1), (2) and (0) respectively. These three impedances are in parallel. Then, the equivalent impedance of unit cell $Z_{e,c}$ will be

$$\frac{1}{Z_{e,c}} = \frac{1}{Z_{S1m}} + \frac{1}{Z_{S2m}} + \frac{1}{Z_0} \qquad \textbf{(19)}$$

The values of these impedances depend on the nature and the geometry of each constituent of heterostructure. In the following, we specifically consider heterostructures in which the inclusions are holes or conductors distributed periodically.
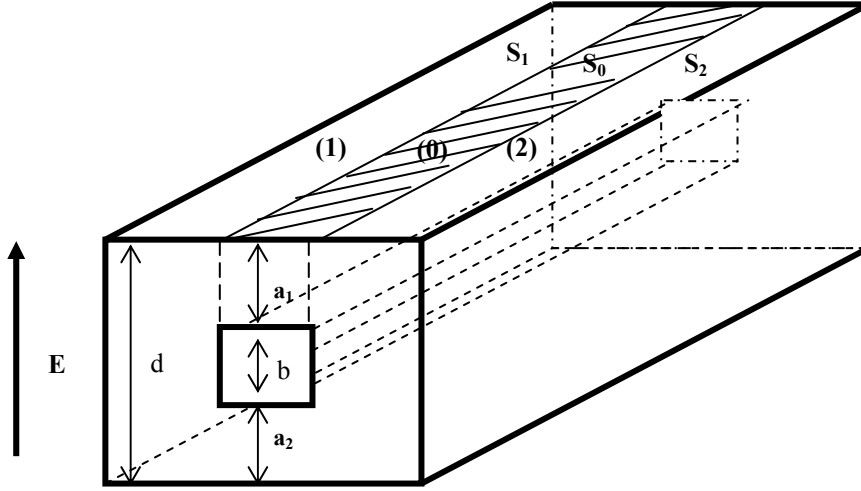
$$Z_{S1m} = \frac{1}{jC_{S1m}\omega} \qquad \text{with}$$

$$C_{S1m} = \varepsilon_m \frac{S_1}{d} \qquad \textbf{(20)}$$

and

$$Z_{S2m} = \frac{1}{jC_{S2m}\omega} \qquad \text{with}$$

$$C_{S2m} = \varepsilon_m \frac{S_1}{d} \qquad \textbf{(21)}$$

The central region (0) is a sandwich. Thus to calculate the impedance Z_0 of the region (0), one can use the relationship established in section 1.

$$Z_0 = \left(\frac{d}{S_0}\right)\left\{-j\frac{1}{\omega\varepsilon_i\varepsilon_m}[\varepsilon_i(1-\beta)+\varepsilon_m\beta]\right\} \quad \textbf{(22)}$$

Figure 4: Schematic diagram of the unit cell of a two-component periodic heterostructure (three-dimensional) investigated in the computation. The isolated inclusion is periodically arranged in a three-dimensional parallelepipedic structure.

2.1 Heterostruture with holes

Such a heterostructure can be a hollow brick for instance. The host material can be a perfect dielectric or not (i.e., without or with losses).

2.1.1 Host material being a dielectric without losses

The host material being a dielectric without losses, then according to the above (see section 1) the impedances for both regions (1) and (2) will be

By substituting eqs.(20), (21) and (22) in eq.(23), it yields

$$\frac{1}{Z_{e,c}} = j\omega\left\{\varepsilon_m\left(\frac{S_1+S_2}{d}\right)+\left(\frac{\varepsilon_i\varepsilon_m}{\varepsilon_i(1-\beta)+\varepsilon_m\beta}\right)\left(\frac{S_0}{d}\right)\right\}$$

$$\textbf{(23)}$$

On the other hand, the equivalent impedance of homogenized heterostructure is

$$\frac{1}{Z_{e,c}} = jC_{e,c}\omega = j\omega\varepsilon_{e,c}\frac{S}{d} \qquad \textbf{(24)}$$

where $S = S_0 + S_1 + S_2$

$\varepsilon_{e,h}$ **is the** dielectric constant of homogenized heterostructure.

Thus eqs.(23) and (24) give the following mixture laws

$$\varepsilon_{e,c} = \varepsilon_m \left(1 - \frac{S_0}{S}\right) + \left(\frac{\varepsilon_i \varepsilon_m}{\varepsilon_i(1-\beta) + \varepsilon_m \beta}\right)\left(\frac{S_0}{S}\right) \quad (25)$$

or

$$\varepsilon_{e,c} = \varepsilon_m \left\{\left(1 - \frac{c}{\beta}\right) + \left(\frac{\varepsilon_i}{\varepsilon_i(1-\beta) + \varepsilon_m \beta}\right)\left(\frac{c}{\beta}\right)\right\} \quad (26)$$

where c is the volume concentration of inclusions

$$c = \beta \frac{S_0}{S} \quad (27)$$

The dielectric constant of holes (cavities) is equal to the unity. Thus, equation (26) becomes

$$\varepsilon_{e,c} = \varepsilon_m \left\{\left(1 - \frac{c}{\beta}\right) + \left(\frac{1}{(1-\beta) + \varepsilon_m \beta}\right)\left(\frac{c}{\beta}\right)\right\} \quad (26.a)$$

2.1.2 Host material being a dielectric with losses

In that case, regions (1) and (2) will be represented respectively by the impedances Z_{S1m} and Z_{S2m} such as

$$Z_{S1m} = R_{S1m} + \frac{1}{jC_{S1m}\omega} = \rho_m \frac{d}{S_1} + \frac{1}{j\omega\varepsilon_m}\frac{d}{S_1} \quad (28)$$

$$Z_{S2m} = R_{S2m} + \frac{1}{jC_{S2m}\omega} = \rho_m \frac{d}{S_2} + \frac{1}{j\omega\varepsilon_m}\frac{d}{S_2} \quad (29)$$

and according to the relationships established in section 1

$$Z_0 = \left(\frac{d}{S_0}\right)\left\{\rho_m(1-\beta) + \frac{1}{j\omega\varepsilon_m\varepsilon_i}\left[\varepsilon_i(1-\beta) + \varepsilon_m\beta\right]\right\} \quad (30)$$

Then the equivalent impedance of the unit cell will be

$$\frac{1}{Z_{e,c}} = \frac{S_1}{d}\left(\frac{j\omega\varepsilon_m}{1+j\omega\varepsilon_m\rho_m}\right) + \frac{S_2}{d}\left(\frac{j\omega\varepsilon_m}{1+j\omega\varepsilon_m\rho_m}\right) + \quad \mathbf{31}$$

$$\frac{S_0}{d}\left\{\frac{j\omega\varepsilon_m\varepsilon_i}{\left[\varepsilon_i(1-\beta) + \varepsilon_m\beta\right] + j\omega\varepsilon_m\varepsilon_i\rho_m(1-\beta)}\right\}$$

$$\frac{1}{Z_{e,c}} = \frac{S}{d}\left\{\left(1 - \frac{S_0}{S}\right)\left(\frac{j\omega\varepsilon_m}{1+j\omega\varepsilon_m\rho_m}\right) + \frac{S_0}{S}\left[\frac{j\omega\varepsilon_m\varepsilon_i}{\left[\varepsilon_i(1-\beta) + \varepsilon_m\beta\right] + j\omega\varepsilon_m\varepsilon_i\rho_m(1-\beta)}\right]\right\} \quad (32)$$

By taking the parallel equivalent circuit to represent the homogenized unit cell, it yields

$$\frac{1}{Z_{e,c}} = \frac{1}{R_{e,c}} + j\omega C_{e,c} \quad (33)$$

where $R_{e,c} = \rho_{e,c}\frac{d}{S}$ and $C_{e,c} = \varepsilon_{e,c}\frac{S}{d}$

and by identifying eqs. (32) and (33), we deduce the mixture laws

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_m}\left\{\left(1 - \frac{S_0}{S}\right)\left(\frac{(\omega\varepsilon_m\rho_m)^2}{1+(\omega\varepsilon_m\rho_m)^2}\right) + \frac{S_0}{S}\left[\frac{(\omega\varepsilon_m\varepsilon_i\rho_m)^2(1-\beta)}{\left[\varepsilon_i(1-\beta) + \varepsilon_m\beta\right]^2 + (\omega\varepsilon_m\varepsilon_i\rho_m)^2(1-\beta)^2}\right]\right\} \quad (34)$$

and

$$\varepsilon_{e,c} = \varepsilon_m\left\{\left(1 - \frac{S_0}{S}\right)\left(\frac{1}{1+(\omega\varepsilon_m\rho_m)^2}\right) + \frac{S_0}{S}\left[\frac{\varepsilon_i\left[\varepsilon_i(1-\beta) + \varepsilon_m\beta\right]}{\left[\varepsilon_i(1-\beta) + \varepsilon_m\beta\right]^2 + (\omega\varepsilon_m\varepsilon_i\rho_m)^2(1-\beta)^2}\right]\right\} \quad (35)$$

Or by introducing the volume concentration

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_m}\left\{\left(1 - \frac{c}{\beta}\right)\left(\frac{(\omega\varepsilon_m\rho_m)^2}{1+(\omega\varepsilon_m\rho_m)^2}\right) + \frac{c}{\beta}\left[\frac{(\omega\varepsilon_m\varepsilon_i\rho_m)^2(1-\beta)}{\left[\varepsilon_i(1-\beta) + \varepsilon_m\beta\right]^2 + (\omega\varepsilon_m\varepsilon_i\rho_m)^2(1-\beta)^2}\right]\right\} \quad (36)$$

and

$$\varepsilon_{e,c} = \varepsilon_m\left\{\left(1 - \frac{c}{\beta}\right)\left(\frac{1}{1+(\omega\varepsilon_m\rho_m)^2}\right) + \frac{c}{\beta}\left[\frac{\varepsilon_i\left[\varepsilon_i(1-\beta) + \varepsilon_m\beta\right]}{\left[\varepsilon_i(1-\beta) + \varepsilon_m\beta\right]^2 + (\omega\varepsilon_m\varepsilon_i\rho_m)^2(1-\beta)^2}\right]\right\} \quad (37)$$

The term $(\omega\varepsilon_m\rho_m)$ represents the dielectric losses factor tanδ ($\tan\delta = \omega\varepsilon_i\rho$) of the material. If it is very smaller

than 1 ($\omega \varepsilon_m \rho_m \ll 1$), eq.(37) will be reduced to eq.(26). On the other hand, if the cross section of both host material and inclusions are square, the ration (c/β) will be equal to *β*. Then, equation (26) becomes

$$\varepsilon_{e,c} = \varepsilon_m \left\{ (1-\beta) + \left(\frac{\varepsilon_i \beta}{\varepsilon_i (1-\beta) + \varepsilon_m \beta} \right) \right\}$$
(26.b)

This relationship can be used to determine the effective permittivity $\varepsilon_{e,c}$ of a given heterostructure in function of the volume concentration of inclusions, knowing the dielectric constants of both constituents. Generally the dielectric constant of materials used in building (clay, polymers…) varies between 3 and 9. The dielectric constant of air (holes) is equal to 1. Figure 5 gives the variation of $\varepsilon_{e,c}$ for three values of the relative permittivity of host materials (ε_m = 4, 6 and 8), the inclusions being air cavities. Such characteristics can be also used to compute the global concentration of inclusions knowing the effective permittivity of heterostructure and both dielectric constant of the constituents.
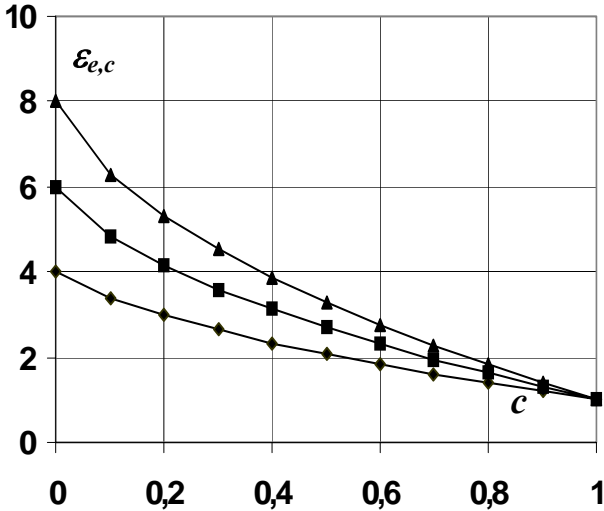

Figure 6: The computed effective relative permittivity as a function of the volume fraction of water for different values of the dielectric constant of the host material: (♦) ε_m= 4, (■) ε_m= 6 and (▲) ε_m= 8.

Figure 5: The computed effective relative permittivity as a function of the hole volume fraction for different values of the dielectric constant of the host material: (♦) ε_m= 4, (■) ε_m= 6 and (▲) ε_m= 8.

Figure 6 gives the variation of the effective permittivity of heterostructures in which the cavities are filled with water. One can make the same remarks as with air cavities.
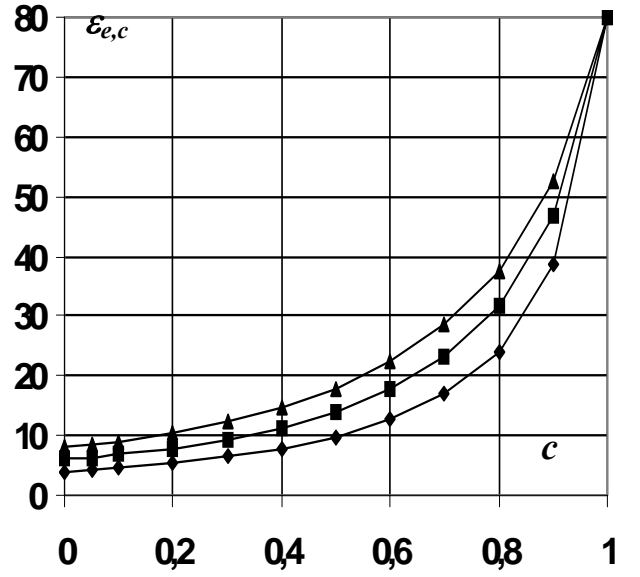
2.1.3 Host material being a conductor

In that case, regions (1) and (2) will be represented by two resistances

$$Z_{S1m} = R_{S1m} = \rho_m \frac{d}{S_1} \qquad \textbf{(38)}$$

$$Z_{S2m} = R_{S2m} = \rho_m \frac{d}{S_2} \qquad \textbf{(39)}$$

and according to the relationships established in section 1

$$Z_0 = \rho_m \left(\frac{d-b}{S_0} \right) + \frac{1}{j\omega\varepsilon_i} \frac{b}{S_0} \qquad \textbf{(40)}$$

Then the equivalent impedance of the unit cell will be

$$\frac{1}{Z_{e,c}} = \frac{1}{\rho_m} \frac{S_1}{d} + \frac{1}{\rho_m} \frac{S_2}{d} + \left\{ \frac{j\omega\varepsilon_i}{[j\omega\varepsilon_i \rho_m (1-\beta) + \beta]} \right\} \frac{S_0}{d}$$
(41)

$$\frac{1}{Z_{e,c}} = \frac{1}{\rho_m} \left(\frac{S}{d} \right) \left\{ \begin{array}{l} \left(1 - \frac{S_0}{S} \right) + \\ \left[\frac{(1-\beta)(\omega\varepsilon_i \rho_m)^2 + j\omega\varepsilon_i \rho_m \beta}{\beta^2 + (1-\beta)^2 (\omega\varepsilon_i \rho_m)^2} \right] \left(\frac{S_0}{S} \right) \end{array} \right\}$$
(42)

By taking the series equivalent circuit to represent the homogenized unit cell, it yields

$$\frac{1}{Z_{e,c}} = \frac{1}{R_{e,c}} + j\omega C_{e,c} \qquad \textbf{(43)}$$

where $R_{e,c} = \rho_{e,c} \dfrac{d}{S}$ and $C_{e,c} = \varepsilon_{e,c} \dfrac{S}{d}$

By identifying eqs. (42) and (43), we obtain the corresponding mixture laws

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_m}\left\{\left(1-\frac{S_0}{S}\right)+\left[\frac{(1-\beta)(\omega\varepsilon_i\rho_m)^2}{\beta^2+(1-\beta)^2(\omega\varepsilon_i\rho_m)^2}\right]\left(\frac{S_0}{S}\right)\right\}$$

$$\textbf{(44)}$$

and

$$\varepsilon_{e,c} = \varepsilon_i\left[\frac{\beta}{\beta^2+(1-\beta)^2(\omega\varepsilon_i\rho_m)^2}\right]\left(\frac{S_0}{S}\right) \qquad \textbf{(45)}$$

or by introducing the volume concentration c

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_m}\left\{\left(1-\frac{c}{\beta}\right)+\left[\frac{(1-\beta)(\omega\varepsilon_i\rho_m)^2}{\beta^2+(1-\beta)^2(\omega\varepsilon_i\rho_m)^2}\right]\left(\frac{c}{\beta}\right)\right\}$$

$$\textbf{(46)}$$

and

$$\varepsilon_{e,c} = \varepsilon_i\left[\frac{\beta}{\beta^2+(1-\beta)^2(\omega\varepsilon_i\rho_m)^2}\right]\left(\frac{c}{\beta}\right) \qquad \textbf{(47)}$$

In the same way, one can lay out the variation of the effective resistivity and permittivity of heterostructures. The values of these parameters are also function of the frequency of the applied electric field. This reminds another non-destructive method such as T.D.R method.

2.1.4 Host material and inclusions being dielectrics with losses

As in section 2.1.2, regions (1) and (2) will be represented respectively by the impedances Z_{S1m} and Z_{S2m} such as

$$Z_{S1m} = R_{S1m} + \frac{1}{jC_{S1m}\omega} = \rho_m\frac{d}{S_1}+\frac{1}{j\omega\varepsilon_m}\frac{d}{S_1}$$

$$\textbf{(48)}$$

$$Z_{S2m} = R_{S2m} + \frac{1}{jC_{S2m}\omega} = \rho_m\frac{d}{S_2}+\frac{1}{j\omega\varepsilon_m}\frac{d}{S_2}$$

$$\textbf{(49)}$$

and according to the relationships established in section 1

$$Z_0 = \left(\frac{d}{S_0}\right)\left\{[\rho_m(1-\beta+\rho_i\beta])+\frac{1}{j\omega\varepsilon_m\varepsilon_i}[\varepsilon_i(1-\beta)+\varepsilon_m\beta]\right\}$$

$$\textbf{(50)}$$

Then the equivalent impedance of the unit cell will be

$$\frac{1}{Z_{e,c}} = \frac{S_1}{d}\left(\frac{j\omega\varepsilon_m}{1+j\omega\varepsilon_m\rho_m}\right)+\frac{S_2}{d}\left(\frac{j\omega\varepsilon_m}{1+j\omega\varepsilon_m\rho_m}\right)+$$
$$\frac{S_0}{d}\left\{\frac{j\omega\varepsilon_m\varepsilon_i}{[\varepsilon_i(1-\beta)+\varepsilon_m\beta]+j\omega\varepsilon_m\varepsilon_i[\rho_m(1-\beta)+\rho_i\beta]}\right\}$$

$$\textbf{(51)}$$

$$\frac{1}{Z_{e,c}} = \frac{S}{d}\left\{\begin{array}{c}\left(1-\dfrac{S_0}{S}\right)\left(\dfrac{j\omega\varepsilon_m}{1+j\omega\varepsilon_m\rho_m}\right)+\dfrac{S_0}{S} \\ \left[\dfrac{j\omega\varepsilon_m\varepsilon_i}{[\varepsilon_i(1-\beta)+\varepsilon_m\beta]+j\omega\varepsilon_m\varepsilon_i[\rho_m(1-\beta)+\rho_i\beta]}\right]\end{array}\right\}$$

$$\textbf{(52)}$$

By taking the parallel equivalent circuit to represent the homogenized unit cell, it yields

$$\frac{1}{Z_{e,c}} = \frac{1}{R_{e,c}} + j\omega C_{e,c} \quad \textbf{(53)}$$

where $R_{e,c} = \rho_{e,c} \dfrac{d}{S}$ and $C_{e,c} = \varepsilon_{e,c} \dfrac{S}{d}$

and by identifying eqs. (52) and (53), we deduce the mixture laws

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_m}\left\{\begin{array}{c}\left(1-\dfrac{S_0}{S}\right)\left(\dfrac{(\omega\varepsilon_m\rho_m)^2}{1+(\omega\varepsilon_m\rho_m)^2}\right)+\dfrac{S_0}{S} \\ \left[\dfrac{(\omega\varepsilon_m\varepsilon_i)^2[\rho_m(1-\beta)+\rho_i\beta]}{[\varepsilon_i(1-\beta)+\varepsilon_m\beta]^2+(\omega\varepsilon_m\varepsilon_i)^2[\rho_m(1-\beta)+\rho_i\beta]^2}\right]\end{array}\right\}$$

$$\textbf{(54)}$$

and

$$\varepsilon_{e,c} = \varepsilon_m\left\{\begin{array}{c}\left(1-\dfrac{S_0}{S}\right)\left(\dfrac{1}{1+(\omega\varepsilon_m\rho_m)^2}\right)+\dfrac{S_0}{S} \\ \left[\dfrac{\varepsilon_i[\varepsilon_i(1-\beta)+\varepsilon_m\beta]}{[\varepsilon_i(1-\beta)+\varepsilon_m\beta]^2+(\omega\varepsilon_m\varepsilon_i)^2[\rho_m(1-\beta)+\rho_i\beta]^2}\right]\end{array}\right\}$$

$$\textbf{(55)}$$

Or by introducing the volume concentration

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_m}\left\{\begin{array}{l}\left(1-\dfrac{c}{\beta}\right)\left(\dfrac{(\omega\varepsilon_m\rho_m)^2}{1+(\omega\varepsilon_m\rho_m)^2}\right)+\dfrac{c}{\beta}\\[4mm]\left[\dfrac{(\omega\varepsilon_m\varepsilon_i)^2[\rho_m(1-\beta)+\rho_i\beta]}{[\varepsilon_i(1-\beta)+\varepsilon_m\beta]^2+(\omega\varepsilon_m\varepsilon_i)^2[\rho_m(1-\beta)+\rho_i\beta]^2}\right]\end{array}\right\}$$

(56)

and

$$\varepsilon_{e,c} = \varepsilon_m\left\{\begin{array}{l}\left(1-\dfrac{c}{\beta}\right)\left(\dfrac{1}{1+(\omega\varepsilon_m\rho_m)^2}\right)+\dfrac{c}{\beta}\\[4mm]\left[\dfrac{\varepsilon_i[\varepsilon_i(1-\beta)+\varepsilon_m\beta]}{[\varepsilon_i(1-\beta)+\varepsilon_m\beta]^2+(\omega\varepsilon_m\varepsilon_i)^2[\rho_m(1-\beta)+\rho_i\beta]^2}\right]\end{array}\right\}$$

(57)

2.2 Heterostruture with conductors as inclusions

Such a heterostructure can represents a pylon for instance. The host material can be a perfect dielectric or not (i.e., without or with losses).

2.2.1 Host material being a dielectric without losses

The host material being a dielectric without losses, then according to the above (see section 1) the impedances for both regions (1) and (2) (see figure 4) will be

$$Z_{S1m} = \frac{1}{jC_{S1m}\omega} \qquad \text{with} \qquad C_{S1m} = \varepsilon_m\frac{S_1}{d} \quad (58)$$

and

$$Z_{S2m} = \frac{1}{jC_{S2m}\omega} \qquad \text{with} \qquad C_{S2m} = \varepsilon_m\frac{S_1}{d} \quad (59)$$

In the same way as in section 1, the impedance of region (0) which is a sandwich structure is

$$Z_0 = \frac{d}{S_0}\left[\rho_i\beta + \frac{1}{j\omega\varepsilon_m}(1-\beta)\right] \quad (60)$$

Then the equivalent impedance of the unit cell will be

$$\frac{1}{Z_{e,c}} = j\omega\varepsilon_m\frac{S_1}{d} + j\omega\varepsilon_m\frac{S_2}{d} + \left[\frac{j\omega\varepsilon_m}{(1-\beta)+j\omega\varepsilon_m\rho_i\beta}\right]\frac{S_0}{d} \quad (61)$$

$$\frac{1}{Z_{e,c}} = j\omega\varepsilon_m\left(1-\frac{S_O}{S}\right)\frac{S}{d} + \left[\frac{j\omega\varepsilon_m}{(1-\beta)+j\omega\varepsilon_m\rho_i\beta}\right]\frac{S_0}{S}\left(\frac{S}{d}\right) \quad (62)$$

By taking the parallel equivalent circuit to represent the homogenized unit cell, it yields

$$\frac{1}{Z_{e,c}} = \frac{1}{R_{e,c}} + j\omega C_{e,c} \quad (63)$$

where $R_{e,c} = \rho_{e,c}\dfrac{d}{S}$ and $C_{e,c} = \varepsilon_{e,c}\dfrac{S}{d}$

and by identifying eqs. (62) and (63), we get the corresponding mixture laws

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_i}\left[\frac{(\omega\varepsilon_m\rho_i)^2\beta}{(1-\beta)^2+(\omega\varepsilon_m\rho_i\beta)^2}\right]\frac{S_0}{S} \quad (64)$$

and

$$\varepsilon_{e,c} = \varepsilon_m\left\{\left(1-\frac{S_O}{S}\right)+\left[\frac{(1-\beta)}{(1-\beta)^2+(\omega\varepsilon_m\rho_i\beta)^2}\right]\frac{S_0}{S}\right\} \quad (65)$$

or, by introducing the volume concentration c

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_i}\left[\frac{(\omega\varepsilon_m\rho_i)^2\beta}{(1-\beta)^2+(\omega\varepsilon_m\rho_i\beta)^2}\right]\frac{c}{\beta} \quad (66)$$

and

$$\varepsilon_{e,c} = \varepsilon_m\left\{\left(1-\frac{c}{\beta}\right)+\left[\frac{(1-\beta)}{(1-\beta)^2+(\omega\varepsilon_m\rho_i\beta)^2}\right]\frac{c}{\beta}\right\} \quad (67)$$

2.2.2 Host material being a dielectric with losses

In that case, regions (1) and (2) will be represented respectively by the impedances Z_{S1m} and Z_{S2m} each one consisting of a resistance in series (or in parallel) with a capacitance

$$Z_{S1m} = R_{S1m} + \frac{1}{jC_{S1m}\omega} = \rho_m\frac{d}{S_1} + \frac{1}{j\omega\varepsilon_m}\frac{d}{S_1} \quad (68)$$

$$Z_{S2m} = R_{S2m} + \frac{1}{jC_{S2m}\omega} = \rho_m\frac{d}{S_2} + \frac{1}{j\omega\varepsilon_m}\frac{d}{S_2} \quad (69)$$

and according to the relationships established in section 1, the impedance of region (0) is

$$Z_0 = \frac{d}{S_0}\left\{[\rho_m(1-\beta)+\rho_i\beta]+\frac{1}{j\omega\varepsilon_m}(1-\beta)\right\} \quad (70)$$

and, since Z_{S1m}, Z_{2Sm} and Z_0 are in parallel, the equivalent impedance will be

$$\frac{1}{Z_{e,c}} = \frac{S_1}{d}\left(\frac{j\omega\varepsilon_m}{1+j\omega\varepsilon_m\rho_m}\right)+\frac{S_2}{d}\left(\frac{j\omega\varepsilon_m}{1+j\omega\varepsilon_m\rho_m}\right)+\frac{S_0}{d}$$

$$\left\{\frac{j\omega\varepsilon_m}{(1-\beta)+j\omega\varepsilon_m[\rho_m(1-\beta)+\rho_i\beta]}\right\} \quad (71)$$

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$$\frac{1}{Z_{e,c}} = \frac{S}{d}\left\{\begin{array}{c}\left(1-\dfrac{S_0}{S}\right)\left(\dfrac{j\omega\varepsilon_m}{1+j\omega\varepsilon_m\rho_m}\right)+\dfrac{S_0}{S}\\[2mm]\left\{\dfrac{j\omega\varepsilon_m}{(1-\beta)+j\omega\varepsilon_m\left[\rho_m(1-\beta)+\rho_i\beta\right]}\right\}\end{array}\right\}$$

(72)

By taking the parallel equivalent circuit to represent the homogenized unit cell, it yields

$$\frac{1}{Z_{e,c}} = \frac{1}{R_{e,c}} + j\omega C_{e,c} \quad \textbf{(73)}$$

where $R_{e,c} = \rho_{e,c}\dfrac{d}{S}$ and $C_{e,c} = \varepsilon_{e,c}\dfrac{S}{d}$

and by identifying eqs. (72) and (73), we deduce the mixture laws

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_m}\left\{\begin{array}{c}\left(1-\dfrac{S_0}{S}\right)\left(\dfrac{(\omega\varepsilon_m\rho_m)^2}{1+(\omega\varepsilon_m\rho_m)^2}\right)+\dfrac{S_0}{S}\\[2mm]\left\{\dfrac{(\omega\varepsilon_m)^2\left[\rho_m(1-\beta)+\rho_i\beta\right]}{(1-\beta)^2+(\omega\varepsilon_m)^2\left[\rho_m(1-\beta)+\rho_i\beta\right]^2}\right\}\end{array}\right\}$$

(74)

and

$$\varepsilon_{e,c} = \varepsilon_m\left\{\begin{array}{c}\left(1-\dfrac{S_0}{S}\right)\left(\dfrac{1}{1+(\omega\varepsilon_m\rho_m)^2}\right)+\dfrac{S_0}{S}\\[2mm]\left\{\dfrac{(1-\beta)}{(1-\beta)^2+(\omega\varepsilon_m)^2\left[\rho_m(1-\beta)+\rho_i\beta\right]^2}\right\}\end{array}\right\}$$

(75)

Or by introducing the volume concentration c

$$\frac{1}{\rho_{e,c}} = \frac{1}{\rho_m}\left\{\begin{array}{c}\left(1-\dfrac{c}{\beta}\right)\left(\dfrac{(\omega\varepsilon_m\rho_m)^2}{1+(\omega\varepsilon_m\rho_m)^2}\right)+\dfrac{c}{\beta}\\[2mm]\left\{\dfrac{(\omega\varepsilon_m)^2\left[\rho_m(1-\beta)+\rho_i\beta\right]}{(1-\beta)^2+(\omega\varepsilon_m)^2\left[\rho_m(1-\beta)+\rho_i\beta\right]^2}\right\}\end{array}\right\}$$

(76)

and

$$\varepsilon_{e,c} = \varepsilon_m\left\{\begin{array}{c}\left(1-\dfrac{c}{\beta}\right)\left(\dfrac{1}{1+(\omega\varepsilon_m\rho_m)^2}\right)+\dfrac{c}{\beta}\\[2mm]\left\{\dfrac{(1-\beta)}{(1-\beta)^2+(\omega\varepsilon_m)^2\left[\rho_m(1-\beta)+\rho_i\beta\right]^2}\right\}\end{array}\right\}$$

(77)

These relationships can be very useful to appreciate the quality of pillars when the metal loses its properties due to an aggressive environment (water penetration and corrosion for instance).

Experimental validation

To validate our model, we consider an experimental model similar to that due to Beroual and Brosseau [13]. The heterostructure we consider is a "periodic" anisotropic two-component composite material. To provide the simplest geometry for subsequent analysis of the system, we have manufactured material samples consisting of a parallelepipeds of dimensions 40 mm x 40 mm x 80 mm in which 16 parallel non-overlapping identical cavities in the form of square section tubes have been periodically spaced in the plane transverse to the tubes. The orientation of the tube' axis is perpendicular to the direction of the electric field vector as indicated in Figure 3. The host matrix is made of poly(vinyl chloride) (PVC) the dielectric constant of which is $\varepsilon=6.5$ in the range of frequency we investigate (i.e., 20 Hz – 1 MHz) and the resistivity is $\rho=2.6\ 10^6\ \Omega.m$. The inclusions (tubes) contain air ($\varepsilon_i=1$) or water ($\varepsilon_i=80$ and $\rho=1.3\ 10^6\ \Omega.m$). To vary the volume fraction of inclusion, c, we modify the square section of tubes.

Figure 7 gives a schematic view of the measurement system. It consists of two parallelipipedic copper electrodes with an active rectangular section of dimensions 80 mm x 40 mm. A spring is placed on the upper electrode to assure a good contact between the electrodes and the sample; this enables to avoid the parasite capacitance induced by the presence of air interstices at the interfaces between the sample and the electrodes. The measurement of the capacitance and the resistance was performed using a Hewlett Packard HP4284 A (20 Hz-1 MHz) impedance analyzer at a frequency of 100 kHz and an amplitude of 4V/cm. The effective permittivity and the resistivity of heterostructure were then deduced. All measurements were performed at room temperature. The choice of this frequency (100 kHz) is justified by the fact that the measurements are more stable at this frequency.

Figures 8 and 9 give the measured and computed effective permittivity versus the volume fraction of inclusions containing voided materials (inclusions with air) and water. We observe a reasonable agreement between the experimental values and the predictions of the equivalent impedance method data.

The quantitative differences likely originates from some limitations of the experimental model such as the ill-contact between the electrode and the heterostructure and the fact that this experimental model is not rigorously periodic since it is of finite size. Thus it is important that the number of inclusions should be sufficiently large. Only in this case can we assume that this experimental model is quasi-periodic [13]. Despite these limitations, this work shows that knowing the effective permittivitty, one can deduce a global concentration of cavities or water in a given medium. Such an information is of major interest for numerous applications.
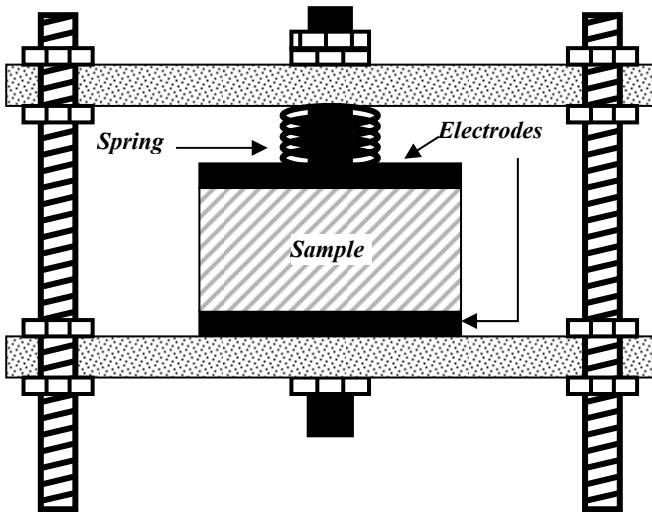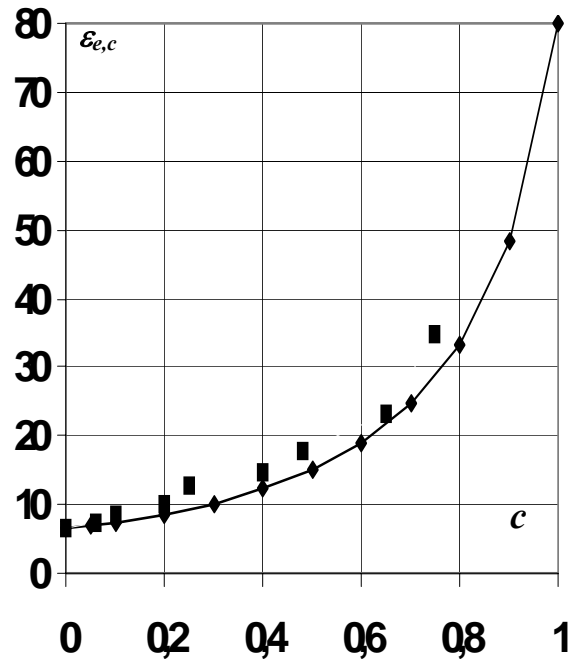
31

Figure 7: Scheme of the measuring set-up.

Figure 9: The effective relative permittivity as a function of the volume fraction of water: simulation ($\blacklozenge$) and experimental ($\blacksquare$).
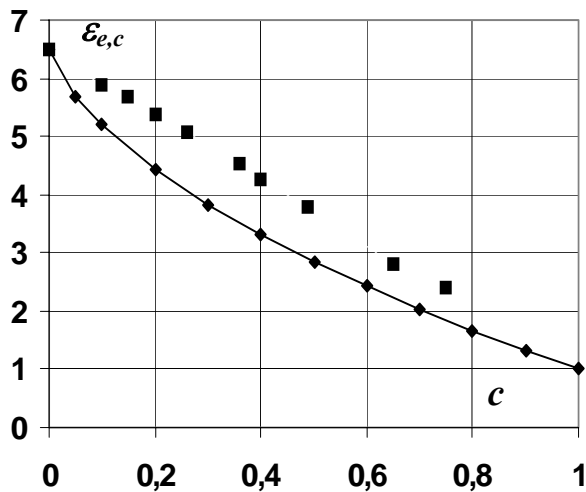
CONCLUSION

In this paper, we presented a non destructive method based on the computation of the equivalent impedance of composite materials and heterostructures as well as the mixture laws of such systems. We have shown that this method can be applied to building heterostructures such as hollow bricks or pillars. The good agreement observed between the experimental and computed results indicates that the equivalent impedance method is a valid descriptive tool to characterize different media of simple geometry and to predict the properties of a heterostructure.

Figure 8: The effective relative permittivity as a function of the hole volume fraction: simulation ($\blacklozenge$) and experimental ($\blacksquare$).

Note that in the investigated frequency range, we haven't observed a difference in the measurements of the conductivity of heterostructures.

CONCLUSION

In this paper, we presented a non destructive method based on the computation of the equivalent impedance of composite materials and heterostructures as well as the mixture laws of such systems. We have shown that this method can be applied to building heterostructures such as hollow bricks or pillars. The good agreement observed between the experimental and computed results indicates that the equivalent impedance method is a valid descriptive tool to characterize different media of simple geometry and to predict the properties of a heterostructure.

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