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IDENTIFYING DEFECT SIZE IN TWO DIMENSIONAL PLATES BASED ON BOUNDARY MEASUREMENTS USING REDUCED MODEL AND GENETIC ALGORITHM

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Abstract

In this study the proper orthogonal decomposition method is utilised as a model reduction technique in crack size estimation in a cracked plate under traction problem. The idea is to create a reduced model based on the results issued from finite element method, thus the crack size parameter is directly related to the boundary displacement obtained from the boundary nodes considered as sensor points. The inverse investigation is run using a genetic algorithm to minimization the error function expressed as the difference between data caused by the crack proposed by genetic algorithm in every individual and the one measured at the actual crack identity. The reduced model is validated by comparing the estimated structural response with the corresponding results from the finite element model. The effectiveness of the approach related to the used number of sensors is presented. Finally the stability of the method against uncertainty is tested by introducing different levels of white noise to the reference data.

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I. INTRODUCTION

Crack initiation and propagation is an omnipresent fact in all structures undergoing cyclic loads due to the fatigue phenomenon. In most cases, cracks are engaged in a predictable location. Thus maintenance measures give big importance to the crack size, trying to follow its state to prevent reaching the dangerous level.

There are several numerical methods for crack detection [1-3], which use different theoretical bases, thus many of these methods are dedicated to the invention using completely theoretical parameters, where the data are not accessible experimentally.

Inverse problems are defined as the problems where the output is known and the input or source of output remains to be determined. They are contrary to the direct problems, in which output or response are determined using information from input [4]. In the case of the Inverse Elastostatics Problem (IESP) of internal flaw detection, the location, the orientation and the size of the flaw are unknown but the displacements along the boundaries are known. In order to analyze this kind of problems, the boundary displacements, usually called "experimental data", are obtained under known boundary conditions and compared with the calculated ones.

Inverse crack or void identification problems can be stated as an optimization task. There are several optimization techniques summoned in [5]; the genetic algorithm (GA) is the most popular evolutionary algorithms with a diverse range of applications. It has been employed in inverse crack identification method in [6-8].

The most utilized methods in the calculation of the mechanical behavior of structures are boundary element (BEM) and the finite element (FEM) methods, mainly used to obtain the displacement field. The FEM and BEM was employed in inverse methods of structural analysis [9,10]. The weak point of FEM based inverse methods in general is the very high computational cost, model reduction can be used to solve the FEM difficulties.

The proper orthogonal decomposition (POD) is a model reduction techniques proceed by the approximation of the problem solution using the appropriate set of approximation functions [11], which contributes to the huge acceleration of the procedure since, once a trained model is built, it computes the system response in a times shorter by about five orders of magnitude compared to FEM as proved in [12], leading to a very quick alternative in inverse problems which provides simplicity and a considerably lower computational time.

We introduce an inverse problem approach based on boundary measurement [13,14]. The proposed identification procedure is essentially the same as the one used in a traditional approach, except that the simulations required by optimization algorithm, are done with reduced model instead of FEM one. It is capable of estimating the crack's size using GA that compares at every iteration the calculated boundary displacement data and one obtained from sensor points posed on the plate's borders.

II. POD-RBF AS A MODEL REDUCTION METHOD

POD is a powerful statistical method for data analysis employed as model order reduction technique in many fields [15,16].In our study the POD is used to build a reduced model of a two dimensional central cracked plate under traction effort, determining the boundary displacement field corresponding to different crack size applications, by exploiting the correlation of results based on the results of the finite element simulations of the system with different crack parameter sets, this known as the method of snapshots. The snapshot consists of the displacement vectors of the boundary nodes which are expected to be correlated. They are stored in matrix U.

$$U = \begin{bmatrix} u_1^1 & u_1^2 & \cdots & u_1^S \\ u_2^1 & u_2^2 & & u_2^S \\ \vdots & \vdots & \ddots & \vdots \\ u_N^1 & u_N^2 & \cdots & u_N^S \end{bmatrix} \quad (1)$$

Where N is the total number of nodes and S is the number of snapshot vectors U_i or FEM simulations results, each corresponds to a crack length value. Parameters matrix P stores the crack length values P_i. The main purpose of POD is to construct a set Φ of orthogonal vectors called POD basis vectors, resembling the snapshot matrix U in an optimal way by exploiting the expected correlation between the results vectors. Expressed by the linear relationship:

$$U = \Phi \cdot A \quad (2)$$

A is the matrix collecting the coefficients of the new basis combination; it is called the amplitude matrix. Referring to the orthogonality of Φ it can be computed from:

$$A = \Phi^T \cdot U \quad (3)$$

Optimal basis vectors are defined by the performance of the proper orthogonal decomposition (POD) [17,18]:

$$\Phi = U \cdot V \cdot \Lambda^{-1/2} \quad (4)$$

Matrix V stores the normalized eigenvectors of the covariance matrix C , and Λ is a diagonal matrix storing its eigenvalues:

$$C = U^T \cdot U \quad (5)$$

A high accuracy $\widehat{\Phi}$ low dimensional approximation is extracted from Φ constructed as a POD basis.This is accomplished by preserving only K ($K \ll S$) columns of Φ that correspond to the largest eigenvalues; consequently the amplitude matrix $\widehat{A}$ is specified by:

$$\widehat{A} = \widehat{\Phi}^T \cdot U \quad (6)$$

Since,

$$U = \widehat{\Phi} \cdot \widehat{A} \quad (7)$$

The use of RBF interpolation different sets of parameters can be generalized not already included in the initial selection$P$. The amplitudes matrix A is defined by the combination of interpolation functions of the parameter vector P gathered in the matrix G. The matrix B gathers the unknown coefficients of this combination:

$$A = B \cdot G \quad (8)$$

The interpolation functions are stated by [19]:

$$g_i = g_i(|P - P_i|) = \frac{1}{\sqrt{|P-P_i|^2 + c^2}} \quad (9)$$

$P_i$is the parameter corresponding to U_i (for i=1,2,…,S). The argument of the i-th RBF is the distance $|P - P_i|$between its current parameter P_i and the reference parameter$P$. c is the RBF smoothing factor. As the vector P is normalized, c is defined within this range [0,1]. In this paper c is equal to 0.6. After the coefficient matrix B is evaluated, a reduced model of (8) can be put in vector form:

$$a(P) = B \cdot g(P) \quad (10)$$

By defining the amplitude vector as a function of parameters, the (7) can be expressed as the approximation of the snapshot u corresponding to a new parameter vector P:

$$u(P) = \widehat{\Phi} \cdot a(P) \quad (11)$$

This reduced model is referred to as the trained POD-RBF network; it is completely able of reproducing unknown boundary displacement field of the structure corresponds to any set of crack parameter (length) P. It is noted that extrapolation outside the range of$P$ leads to poor precision of the model. Also if the knot points $P_i$are very close to each other; the matrix$G$could be singular, which can be avoided by reducing the$c$value.

III. IDENTIFICATION ALGORITHM

A. Genetic Algorithms

The genetic algorithm is an evolutionary optimization method; widely used for a variety of optimization problems in last decade [20].

The general idea of this method is inspired by natural evolution processes. The algorithm operates on a set of designs, called population, and the approach is to allow its individuals, i.e. designs, to reproduce and cross among themselves in order to obtain designs with better fitness. The fittest designs, i.e. those with low objective function values in the case of minimization, have good genetic characteristics and these are given higher probability of becoming chosen as parents to new designs, where the characteristics of the parents are combined. A population of N feasible random designs is initially generated where each design is represented by a binary string of 0's and 1's (binary encoding) or by its numerical value (real encoding). The objective function value for each design is calculated and used to compute the corresponding fitness value (a low objective function value imply a higher fitness value). Four basic operators are now used to generate the next generation of designs: reproduction, crossover and mutation and migration.

Reproduction is an operator that basically selects designs from the population at the current generation and transfers them into a mating pool, i.e. a new population of same size, N. More fit designs have higher probability of getting selected and the same design can be selected more than once.

The idea of crossover is to generate new designs by exchanging characteristics of designs from the mating pool. Starting and ending positions in two randomly selected

design strings from the mating pool are therefore selected using random numbers. The strings between these positions on the two design strings are then exchanged and the two new designs, i.e. progenies, replace their parents in the mating pool.
Mutation is used to generate new designs by a mutation of existing designs. This is accomplished by changing the digit, i.e. 0 to 1 or vice versa in binary encoding, at a random location in a number of randomly selected design strings from the mating pool. The process of operating and updating the population is continued until a stopping criterion is satisfied.

B. Structure of the identification algorithm

The main notion of the proposed identification paradigm is to use model reduction for the generation of the structural response instead of simulation methods. The major steps are depicted in Fig. 1 and detailed as follows:

1. Creation of a starting population of N individuals randomly. Each individual has one chromosome, corresponding to the crack (s) the length of the crack.
2. Evaluation of every individual; by introducing the parameters to the reduced model that produces a corresponding boundary displacement vector $u(P)$, and calculate the fitness value which is the error between the resulting vector and the measured vector of displacement cause by the real crack parameters $u(P_0)$ expressed as:

$$\begin{cases} F(P) = \frac{\|u(P_0) - u(P)\|^2}{\|u(P_0)\|^2} \\ F(P_{optimal}) = \min[F(P)] \end{cases} \quad (12)$$

3. Terminate the algorithm f the maximum number of generations or a defined fitness value is reached. Else continue.
4. Ranks the population according their fitness value. Then select a proportion for reproducing a new generation. The top ranked are favorable to be selected.
5. Performance of the crossover operation.
6. Mutation of an indicated percentage of the resulting individuals.
7. Replacement of the old population by new one and go to step 2.

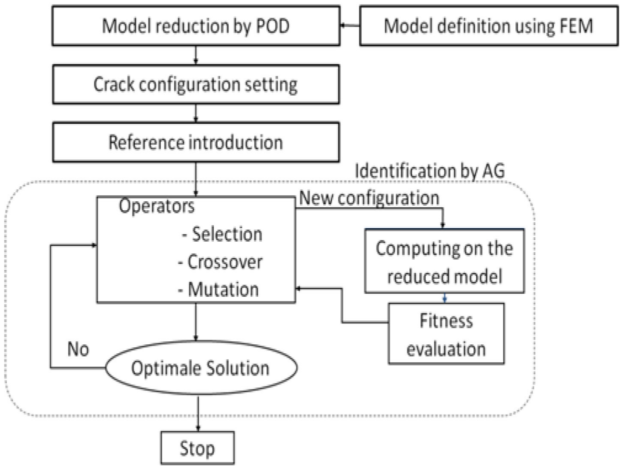

Fig. 1. Evolutionary Identification algorithm

IV. IMPLEMENTATION AND DISCUSSIONS

A. Problem description

A plane strain plate containing a single crack is considered, the square plate subjected to a traction load is simulated using FEM code ABAQUS where the external boundaries are discretized by means of 80 elements per edge to finally collect the displacement of the border's 320 nodes, for the construction of the reduced model and the identification method is implemented in MATLAB. After a series of experiments, the following genetic parameters are chosen based on the accuracy of results: Population size: 100, Crossover rate: 0.8, Mutation rate: 0.01.
Since the proposed method relays on the nodal displacement, all boundary nodes are considered as sensor point. We consider data from sensors by obtaining the displacement results from the nodes chosen as sensors point. Fig. 2 depicts an example of the controlled plate using 8 sensors.
For the placement of the sensors, we found that better results are acquired when they are dispersed uniformly on both left and right sides, respecting the same distance between every sensor point.

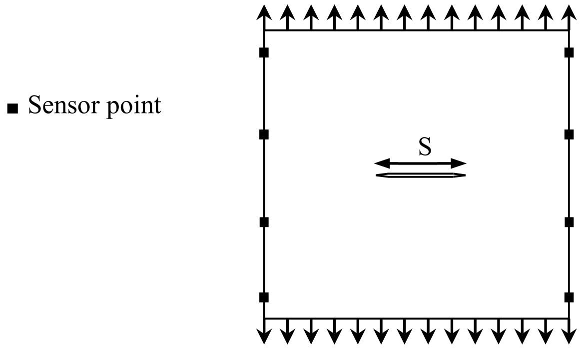

Fig. 2.cracked plate with sensor.

B. Model reduction

The existence of a crack changes the behavior of the plate when put under traction, therefore the response of the structure, which is also affected by the changes of the crack length. Benefiting from this fact, the boundary displacement of the structure is measured using sensors, and based the inverse crack size estimation on it.

POD in this stage has been used to build a model relating the crack length with boundary displacement based on the displacement values collected from 13 scenarios of crack length *s* belongs to the range 0 (no crack) to 12 mm. To test the accuracy of the new model, we compared the results issued from a crack parameter which is not included to snapshot data with equivalent results from FE model. The Fig. 3 displays the boundary displacement field of the FEM and POD models for the example of crack size equal to 2 mm, and Fig. 4 depicts the absolute value of the deference between those two fields.

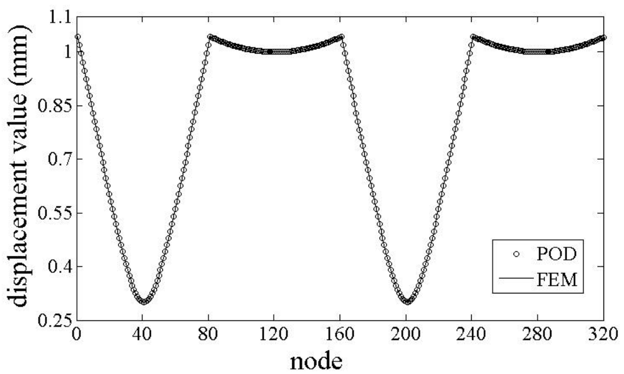

Fig. 3. Boundary displacement field comparison calculated by FEM and POD.

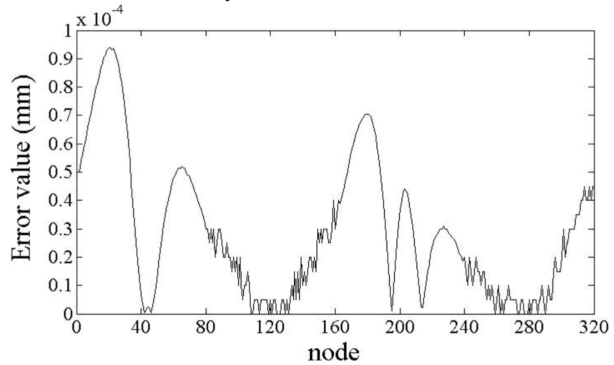

Fig. 4.Efficiency of the proper orthogonal model.

The reduced model provides very precise results as shown in the next two figures. Where the Figure 3 demonstrates identical results of the FEM and the POD models, and the Figure 4 shows that the error magnitude is very small where the major error is in the order of 0.0001 mm.

C. Crack size estimation

The inverse problem solved by GA, minimizing the cost function which is the deference between sensor's displacement caused by the crack we want to estimate its size and the one proposed by GA using a Population size equal 100.

Table I presents the normalized crack size estimation results for different application by means of sensor's number. Sensors number represents the quantity of sensors used knowing that it is spread on the left and right side of the plate. The first experience uses the data of all the nodes, later smaller sensor number is considered. Precession of 10^{-06} of cost function is chosen as the algorithm's stopping criteria. The approach could estimate the crack size presenting high accuracy, even with a very low number of sensors, and shows that the inverse calculation on the reduced model is very practical data for crack identification problem.

TABLE I: NORMALIZED RESULTS AND SOLUTION INFORMATION

Number of sensors	generations	Best fitness	Normalized result of size
230	3	2,551E-06	0,9999784
24	2	1,022E-06	1,0000473
12	2	4,439E-06	1,0002098
8	2	3,126E-07	0,9999664
6	2	9,881E-07	0,9999212
4	5	4,984E-06	0,9995465

A small number of generations is needed due to the large population size used, which proven in previous calculations to be faster that smaller population number. It is noted that large number of sensors dons not mean greater result quality and that the number of sensors equal to 8 gives the best estimation.

D. Noise

In order to study the stability of the crack size identification algorithm to measurements uncertainty, some level of perturbation has been added to the exact input displacement from the 8 sensor point's example. The noise is modelled by the White Gaussian law with fixed standard deviations. Fig. 5, 6 and 7 shows the convergence to the results of three noise levels: 1%, 5% and 10% respectively, illustrating the performance of the algorithm thought 5 applications in each level. A number of 10 generations is taken as a stopping criterion.

The variations obtained in the crack identity are in good agreement with the noise levels. It is noted that 1% of noise level does not affect the exactness of the results as shown in Figure 5.Giving an excellent average standard deviation equal to 0.003 at convergence, this is lower than the imposed perturbations rate of 0.01.

The results of noise level 5% is satisfactory, since the algorithm still can approximate the crack length asthe average standard deviation for convergence is 0.042 which did not exceed 5%.Recognizing that, for the same configuration, the accuracy increases with the number of sensors. The perturbation for noise level 10% in the application of 8 sensors, the average standard deviation for convergence is high (0.163).

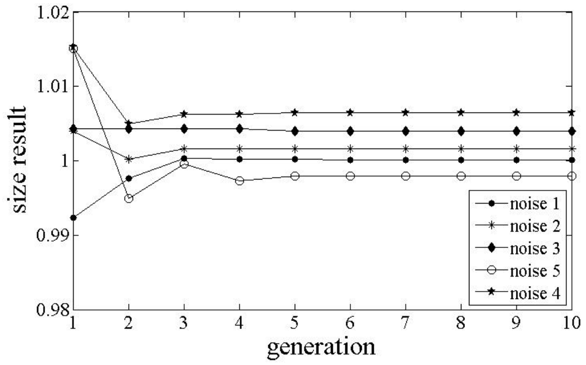

Fig. 5.Crack size estimation in noise level 1%

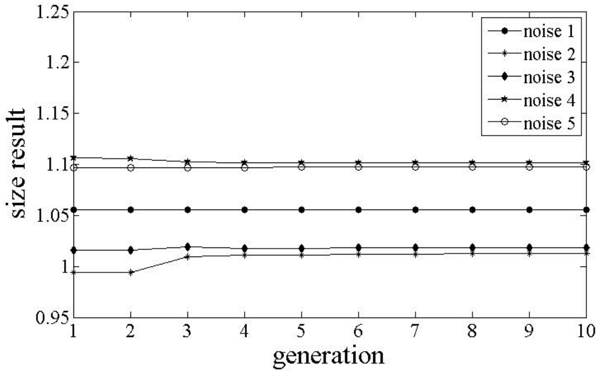

Fig. 6.Crack size estimation in noise level 5%

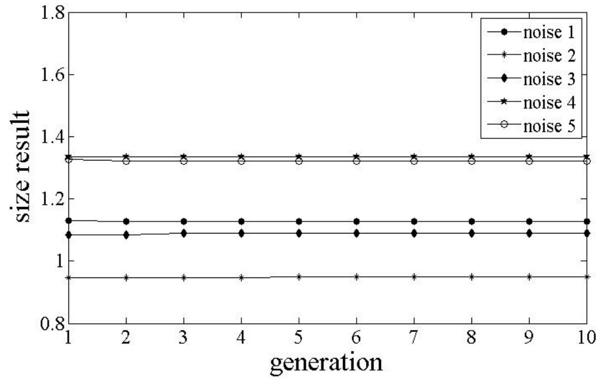

Fig. 7. Crack size estimation in noise level 10%.

V. CONCLUSION

In This numerical study, we presented model reduction for crack size identification based on boundary displacement measurements, after the finite element model of the structure was created for different crack lengths, a reduced model based on POD-RBF method was extracted and the results issued from both models was compared to insure the efficiency of the reduced model. Crack size was investigated based on boundary displacement data using the genetic algorithm for the inverse calculation, the results have clearly shown that the developed algorithm is capable of predicting crack size accurately, and prove its effectiveness even with a very low number of sensors.

The proposed study have demonstrated a high stability after a white noise was introduced to the input data, simulating measurement uncertainty.

The employment of the GA for the optimization task helps avoid imitations problems typical for the classical optimization methods and POD-RBF produced an accurate reduced model of the system, providing a low computational cost.

This method is extendable to experimental diagnostic study, due to the very few experiments required and the small number of sensor points required.

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