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# BETA DECAY HALF-LIVES AND RATES OF $^{134\text{-}136}$SN NUCLEI

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## Abstract

In astrophysical environment, allowed Gamow-Teller (GT) transitions and space phase factors play an important role in determination of transition rates and half-lives, particularly for β-decay in presupernova evolution of massive stars. The estimation of these half-lives in neutron rich nuclei is needed in astrophysics for the understanding of supernovae explosions and the processes of nucleosynthesis, principally the r-process, and in the experimental exploration of the nuclear landscape. Their determination in agreement with experimental results is a challenging problem for nuclear theorists. In this work, the total β-decay half-lives and rates of 134-136Sn nuclei at different temperatures are calculated using various interactions developed in the light of recently available information on experimental binding energies and low-lying spectra of Sn, Sb and Te isotopes in 132Sn mass region. The calculation has been realized using Oxbash code in the frame work of the nuclear shell model. With these interactions, one can observe that the effective half-lives increase and the total decay rates decrease with increasing temperature. A deviation of half-lives starts at around 0.2 MeV and satures above 10 MeV, but the half-lives limit values are slightly different for all interactions.

***Keywords***: β decay half-lives and rates, Gamow-Teller, Oxbash code, 132Sn region.

## Résumé

Dans l'environnement astrophysique, les transitions permises de Gamow-Teller (GT) et les facteurs d'espace de phase jouent un rôle important dans la détermination des taux de transition et des demi-vies, en particulier pour la désintégration β dans l'évolution des étoiles massives des supernova. L'estimation de ces demi-vies dans les noyaux riches en neutrons est nécessaire en astrophysique pour la compréhension des explosions de supernovae et des processus de nucléosynthèse principalement dans le processus r, et dans l'exploration expérimentale de la charte nucléaire. Leur détermination en accord avec les résultats expérimentaux est un problème difficile pour les théoriciens nucléaires. Dans ce travail, les demi-vies totales et les taux de transition des noyaux 134-136Sn sont calculés en fonction de la température à l'aide de différentes interactions développées sur la base d'information récente sur les énergies de liaison expérimentales et les spectres des isotopes Sn, Sb et Te dans la région de masse 132Sn. Les calculs ont été réalisés au moyen du code Oxbash dans le cadre du modèle en couches nucléaires. Avec ces interactions, on observe que les demi-vies effectives augmentent et les taux de décroissance diminuent avec l'accroissement de la température. La déviation des demi-vies commence à environ 0,2 MeV et sature au dessus de 10 MeV, mais les valeurs limites des demi-vies sont légèrement différentes.

***Mots clés*** *: Désintégration β-, demi-vies et taux de transition, Gamow-Teller, code Oxbash, région de 132Sn.*

## ملخص.

**في البيئة الفلكية، الانتقالات المسموحة *GT) Gamow-Teller*) وعوامل طور الفضاء تلعب دورا هاما في تحديد معدلات الانتقال وأنصاف العمر، ولا سيما في التفكك *β* لتطور النجوم الضخمة في *supernova* . تقدير ا نصاف العمر في الانوية الغنية بالنيوترونات ضروري في الفيزياء الفلكية لفهم انفجارات *supernova* وعمليات الاصطناع النووي بشكل رئيسي في الظاهرة *r*، وفي الاستكشاف التجريبي على الخريطة النووية. تقديراتهم على اتفاق مع النتائج التجريبية مشكلة صعبة بالنسبة للنظرين النوويين. في هذا العمل يتم حساب انصاف العمر الكلية ومعدلات الانتقال في الانوية 134-136*Sn* بدلالة درجة الحرارة باستخدام تفاعلات مختلفة وضعت على اساس معلومات حديثة تقوم على طاقات الربط التجريبية والأطياف للنظائر *Sn, Sb*و*Te* في المنطقة 132*Sn*. تم إجراء الحسابات في اطار النموذج الطبقي بواسطة برنامج البنية النووية *Oxbash* مع هذه التفاعلات، لوحظ أن انصاف العمر الفعالة تزداد ومعدلات التفكك تنخفض مع زيادة درجة الحرارة. يبدأ الانحراف لأنصاف العمر بحوالي *0.2MeV* ويدرك التشبع ابتداء من 10*MeV*، ولكن قيم التشبع لأنصاف العمر مختلفة ببطء.**

***الكلمات المفتاحية***: التفكك *β* -انصاف العمر ومعدلات التفكك ، *Gamow-Teller*، البرنامج *Oxbash*، المنطقة 132*Sn*.

## Introduction:

The neutron-rich nuclei with few valence nucleons above the doubly closed $^{132}$Sn core are interesting to extract empirical NN interaction and test the theoretical description of the nuclear structure shell model in this mass region [1]. The study of structure properties of these nuclei aims to gathering new data on decay of *Sn* isotopes beyond the magic $^{132}Sn$ nucleus. These are of a great interest for modeling r-process [2], and comprehension the element abundances in the universe [3]. For *r-process* nucleosynthesis, $\beta^-$ decay of neutron rich nuclei becomes important when the timescale of neutron capture is comparable to that of the photodisintegration in the vicinity of the neutron shell gaps N= 50, 82 and 126 [4]. Beside, these neutron closed shells, the *r-process* comes closest to the line of β stability and falls on the waiting point isotopes, where the β-decay half-lives are the longest in the r-process path [5].

In the waiting point approximation, an $(n,\gamma)$ and $(\gamma,n)$ thermal equilibrium is assumed to be established in the nuclei inside an isotopic chain. Only β-decay half-lives and neutron binding energies are needed [5,6]

## Calculations

Large basis calculations are carried out by means of *Oxbash* nuclear structure code [7], in the framework of the shell model. In these calculations, the Z50N82 valence space consisting of $\pi(1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2})$ and $\nu(1h_{9/2}, 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}, 1i_{13/2})$ orbitals above the $^{132}Sn$ core is used with *kh5082* [8]*, cw5082* [8]*, cwg* [9] *, smpn* [10]*, kh3, cwΔ5082* [11] and *khΔ5082* [12] (1+2)-body Hamiltonians.

In the two latest ones, we carry out some modifications based on the *N-N* pairing interaction [13]. The *p-n* tbme, corresponding to eight excited states $0^-$, $1^-$, $2^-$, $3^-$, $4^-$, $5^-$, $6^-$, and $7^-$ of the original *kh5082* interaction [8], have been modified using the renormalization factor 0.74. For *n-n tbme*, we modified those corresponding to $0^+$ and $2^+$, $4^+$, $6^+$, $8^+$ excited states, using 0.48 and 0.6 renormalization factors respectively [14]. While, the *p-p* tbme modification correspond to $0^+$, $2^+$, $4^+$ and $6^+$ excited states, with 1.08 renormalization factor [10].These renormalization factors, reflecting the reduction of pairing in first excited states, were adjusted to experimental data in the *Sn*, *Sb* and *Te* isobars. The proton and neutron *SPE* are taken from Ref [15]. In the present work, an estimation of the depressed energies effect on $\beta^-$ decay rates of the exotic even *Sn* isotope ($^{134\text{-}136}Sn$) above the $^{132}Sn$ core have been calculated. In order to obtain the necessary *ft* values corresponding to the decay of thermally populated excited states of the mother to the excited states of the daughter nucleus, the calculation of reduced transition probabilities are needed. In the case of neutron rich nuclei, only Gamow-Teller transitions can occur. Indeed, the allowed Fermi transitions in the isobaric analogue states ($\Delta T=0$) are located at an excitation energy higher than that of the ground state of the mother nucleus, outside the energetic window $Q_\beta$. So, it is impossible to observe Fermi transitions in the side of neutron rich nuclei. The excitation energies and the transition densities of $^{134\text{-}136}Sn$ and $^{134\text{-}136}Sb$ nuclei are calculated using cited interactions, in order to evaluate $B(GT)$ values required in the calculation of beta decay rates. It is known that the thermal population of excited nuclear levels becomes more important with increasing temperature and lower excitation energy. In the situation of pre-supernovae, the temperature of nuclei is so high that the beta decay rate of a nucleus in this astrophysical environment depends on it. Also, one can express it by this formula [16,17]:

$$\lambda = \frac{\ln 2}{\kappa} \sum_i \frac{(2J_i + 1)e^{\left(\frac{-E_i}{kT}\right)}}{G_i(z, A, T)} \sum_j B_{ij}\phi_{ij} \qquad (1)$$

where the sums in *i* and *j* run over states in the mother and daughter nuclei respectively. The constant κ=6250 s [18 ,19], and $G_i$ denote the partition function of the mother nucleus defined as,

$$G_i(z, A, T) = \sum_i (2J_i + 1)e^{-E_i/kT} \qquad (2)$$

Here, $B_{ij}$ are the reduced transition probabilities given as a function of Gamow-Teller and Fermi transition probabilities by

$$B_{ij} = B_{ij}(GT) + B_{ij}(F) \qquad (3)$$

Gamow Teller $B_{ij}(GT)$ and Fermi $B_{ij}(F)$ transition probabilities are defined as [17, 19]:

$$B_{ij}(GT) = \left(\frac{g_A}{g_V}\right)^2_{bare} \frac{\left| \left\langle J_f \left\| \sum_i \sigma(i) t_-(i) \right\| J_i \right\rangle \right|^2}{2J_i + 1}$$

$$B_{ij}(F) = \frac{1}{2J_i + 1} \left| \left\langle J_f \left\| \sum_i t_-(i) \right\| J_i \right\rangle \right|^2 \qquad (4)$$

here $t_-(i)$ and $\sigma(i)$ stand for the isospin and spin vectors of the $i^{th}$ nucleon. $J_f$ and $J_i$ denote respectively the final and initial angular momenta, and $g_A$, $g_V$ are vector and axial-vector coupling constants such as [19]:

$$\left(\frac{g_A}{g_V}\right)_{bare} = 1.25 \qquad (5)$$

The last factor in Eq. (1), $\phi_{ij}$, is the phase space integral for which the approximation method is described in [17,20].

In a transition $\beta^-$, the values of the reduced transition probabilities from the state of the mother to the state of the daughter are used to determine the value of *ft* which is given as[19]:

$$ft = \frac{\kappa}{B_{ij}(GT) + B_{ij}(F)} \qquad (6)$$

## Results and Discussion

### III-1 Spectra

Several data are accumulated in the tin 132 region, exceptionally the single particle energies (SPE). The $^{134}Sb$ nucleus have one proton and one neutron in addition to the tin core. Its low lying proton-neutron states have the configuration ($\pi\ 1g_{7/2}\nu\ 2f_{7/2}$). The $0^-$, $1^-$, $2^-$ and $3^-$ excited states are observed in the $\beta^-$ decay of the $^{134}Sn$ *[ 21]*, while the state $7^-$ is populated by means of $\beta^- n$ decay in $^{135}Sn$ [22]. The spectrum of isotope $^{134}Sn$ have been observed in prompt γ-radiation fission fragments from $^{248}$Cm. The first three excited states were interpreted as members of the $\nu(2f_{7/2}{}^2)$ multiplet [23].

The isotope $^{136}Sb$ had been first observed as a β-*n* delayed precursor produced in thermal neutron induced fission of $^{235}U$, and it has been produced in the projectile fission of $^{138}U$ at the relativistic energy of 750 *MeV/u* on $^{9}Be$ target [24].

Very recently, experiments were carried out at the RIKEN Radioactive Isotope Beam Factory (RIBF) [25, 26] to study the neutron rich isotopes of Sn. In one of the experiments the first $2^+$ excited state in the neutron-rich tin isotope $^{136}$Sn has been identified at 682(13) keV by measuring γ-rays in coincidence with the one proton removal channel from $^{137}$Sb [25]. The calculated energetic spectra (low energies) in comparison with the experimental one of the parent and daughter nuclides are illustrated in Fig.1 and 2.

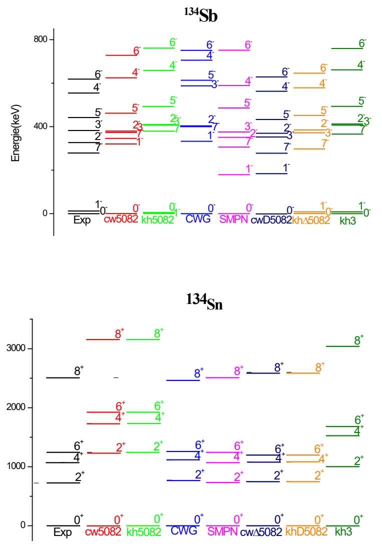

Fig 1: Calculated energetic spectrum using *kh5082*, *cw5082*, *cwg*, *smpn*, *kh3*, *cwΔ5082* and *khΔ5082* in comparison with experimental for $^{134}Sn$ and $^{134}Sb$ nuclei.

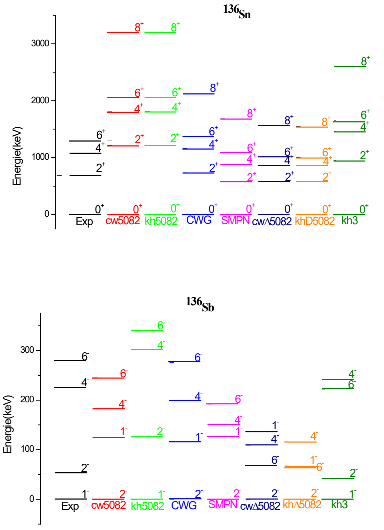

Fig 2: Calculated energetic spectrum using kh5082, cw5082, cwg, smpn, kh3, cwΔ5082 and khΔ5082 in comparison with experimental for 136Sn and 136Sb nuclei

These figures show that the getting results using the various interactions are in good agreement with the experimental data for $^{134}$Sn and $^{134}$Sb nuclei: the khΔ5082 interaction. However, for 136Sn and $^{136}$Sb nuclei, CWG and kh3 interactions reproduced well the experimental spectrum respectively.

### III-2 Half lives

The relevant information, for these isotopes incorporated for modifications in the decay rates that result from inclusion of excited states due to thermal excitations, are shown in Table 1. The figures 3 and 4 show relative energies of $^{134\text{-}136}$Sn and $^{134\text{-}136}$Sb nuclei respectively and $\beta^-$ decays between different levels.

Table 1: Relevant information used the calculations.

| Mother | Exp$_{\text{Half-life}}$ (s) | $Q_{Value}$ MeV | Mother (*Sn*) tates | Daughter (*Sb*) States |
|---|---|---|---|---|
| $^{134}$Sn | 1.05 | 7.37 | $2_1^{+,}\ 2_2^+$ | $3_1^+, 1_1^+, 2_1^+, 2_2^+, 3_2^+$ |
| $^{136}$Sn | 0.25 | 8.37 | $2_1^{+,}\ 2_2^+$ | $3_1^+, 1_1^+, 2_1^+, 2_2^+, 3_2^+$ |

The selection rules for GT transitions only allow transitions from single particle $\nu 1h_{9/2}$ orbital to $\pi 1h_{11/2}$ orbital in this model space. But the wave function compositions of the relevant low lying states in these isotopes of *Sn* and *Sb* have very small contribution from the shell model configurations involving these orbitals. So the calculated allowed GT strengths are generally very small.

We have also calculated the Gamow-Teller strengths and the half-lives in the temperature range from T= 0.01 to 100 *MeV* (fig.3 and 4).

While varying the temperature in the nuclear field going from 10 *keV* to 100 *MeV*, one can observe that for the interactions, the effective half-life increases with increasing temperature for $^{134\text{-}136}Sn$.

However, the total rate decreases with increasing temperature. The rate partial of decay starting from the fundamental state ($0^+$) is quite fast and those starting from the excited states $2_1^+$ and $2_2^+$ are of two orders of magnitude slower for ($kT=1MeV$), as the beta decay from ground state to ground state of daughter is forbidden, but it is quite fast. In the range 0.01 *MeV* to 0.2 *MeV*, $T_{1/2}$ is constant at ~ 1.05 s ($^{134}Sn$) or ~ 0.25 s ($^{136}Sn$) corresponding with that obtained in laboratory measurements. Beyond 0.2 *MeV* (~$10^9 K$), there is a deviation of this value for all interactions used. The deviation starts at the same temperature and the saturation is reached above 10 *MeV*. The limit values in $^{134}Sn$ isotope varied between 3s and 9s while in $^{136}Sn$ isotope they have a value around 2.5 s.

## CONCLUSION

In this paper, we calculate the excitation energies, beta decay half-lives and transition rates for A=134-136 isobars with two and four valence particles in addition to the $^{132}Sn$ ($^{134\text{-}136}Sn$ and $^{134\text{-}136}Sb$ nuclei). The calculations are carried out in the framework of the shell model by means of *Oxbash* nuclear structure code, using *kh5082*, *cw5082*, *cwg*, *smpn*, *kh3*, *cwΔ5082* and *khΔ5082* interactions. The new experimental values of the single particle energies were used. The getting results using the various interactions are in good agreement with the experimental data in the case of $^{134}Sn$ *and* $^{134}Sb$ nuclei : the khΔ5082 interaction. While for the case of $^{136}Sn$ *and* $^{136}Sb$ nuclei, CWG and kh3 interactions reproduced well the experimental spectrum respectively. With these interactions, the deviation and saturation of half-lives start respectively at around 0.2 *MeV* and above 10 *MeV*. The limit values in $^{134}Sn$ *and* $^{136}Sn$ isotopes are 3s and 2.5 s respectively.

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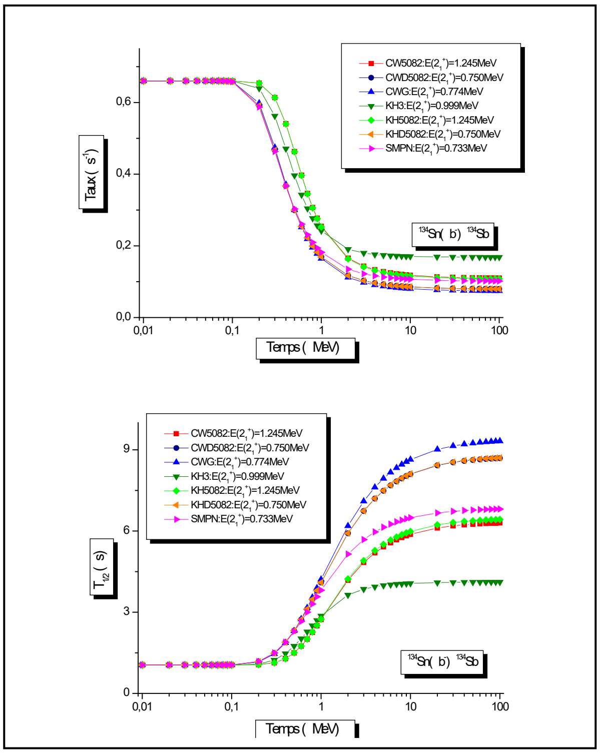

Fig 3: β decay rates and half-lives as a function of temperature for $^{134}Sn$

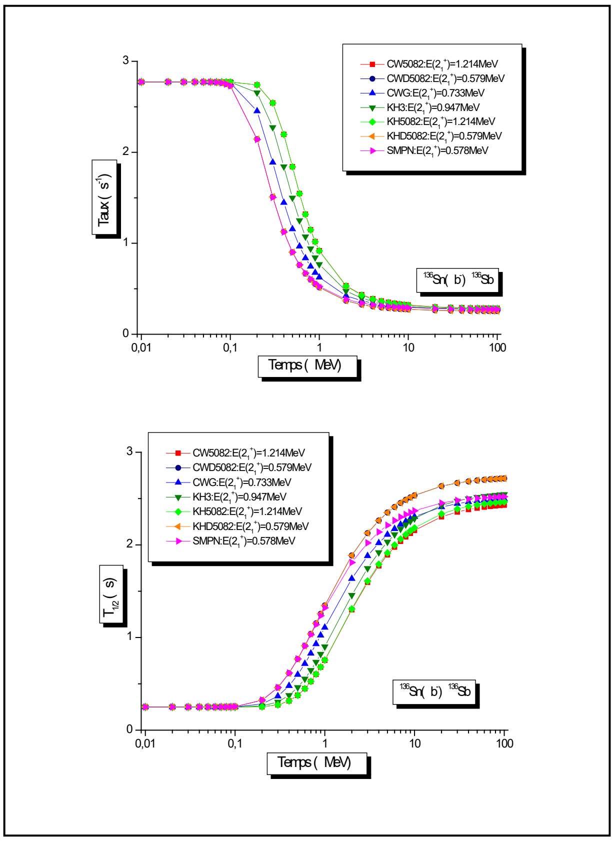

Fig 4: β decay rates and half-lives as a function of temperature for $^{136}Sn$

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