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SOME VIABLE MODELS FOR EXTRA DIMENSIONAL UNIVERSE

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Abstract

Some viable models in a 5D space-time are presented and Friedman like equations are also obtained. A dynamical study is also investigated.

Keywords: ***Extra dimension, Dynamical study***

Résumé

Quelques modèles viables dans espace-temps a 5D ont été présentés et des équations du type Friedman ont été obtenues. Une étude dynamique a été aussi investie.

Mots clés : ***dimension supplémentaires, étude dynamique.***

ملخص

بعض النماذج الحيوية في فضاء زمكان ذو 05 أبعاد قد طرح ومعادلة فريدمان المناسبة قد استنتجت. دراسة ديناميكية قد استعملت.

الكلمات المفتاحية : بعد إضافي، دراسة ديناميكية.

I. INTRODUCTION

In 1919, Theodor Kaluza developed a fundamental description to unify the electromagnetism and gravitation forces by introducing extra-dimensions in General Relativity[1]. the Standard Model can not describe the gravitation because of its high energy scale (10^{15}Gev), which leads us to look for a new physics.

By using the fifth dimension and according to Kaluza-Klein, the start was with a pure five dimensional gravitation but all the fields have to be independents of this extra-dimension and they can be written as a function of four-dimensional fields where the Maxwell equations are hidden in Einstein equation.

In this case, Kaluza theory preserves the geometry of General Relativity but the electromagnetic fields are added as a vibration in the five-dimensional space.

In 1926, Oskar Klein succeeded to explain why we can not perceive the additional dimension. He has considered that the five-dimensional fields are independent from the extra-dimension, which must be compactified. This means that it has a topology of a circle. for example a cylinder with a radius of the order of Plank length (it is extremely small).

The recent observations indicate that our universe is in a large scale in accelerated expansion. This was first observed from high red shift supernova Ia [1 ,7], and confirmed later by cross checking from the cosmic microwave background radiation [8,9]. The expansion rate was explained in the cosmological standard model by adding dark energy , which has a negative pressure . However, the nature of dark energy as well as dark matter is yet unknown , as long as the solution is not yet obtained in the context of the standard General Relativity. This leads to suggest a five dimensional model. Mohammedi gives an alternative explanation to dark energy responsible for the accelerated expansion of the universe by incorporating extra dimensions into Friedmann-Robertson-Walker (FRW) cosmology [10].

In this paper, we concentrate on some cosmological models with just one extra dimension and look for exact solutions as well make a general dynamical study to understand the stability and behavior of the general solutions.

II. FRW UNIVERSE WITH ONE EXTRA DIMENSION

The metric of a 5D space-time with a 4D spherical symmetric universe, isotropic and homogenous has the following form [11]:

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\varphi^2)\right] - A^2(t)dy^2 \quad (1)$$

Where A(t) is a scale factor of the extra-dimension, y is the fifth coordinate, k=-1,0,1 depending on the type of the 3D space geometry. By using the metric F.R.W and the perfect fluid stress-energy tensor, the 5D, FRW field equations are of the form

$$\rho = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} + 3\frac{\dot{R}\dot{A}}{RA}$$
$$p = -[2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} + \frac{\ddot{A}}{A} + 2\frac{\dot{R}\dot{A}}{RA}]$$
$$p_5 = -3\left(\frac{\ddot{R}}{R} + \frac{k}{R^2} + \frac{\dot{R}^2}{R^2}\right) \quad (2)$$

where a dot denotes a time derivative, and p, ρ *and* p_5 represent the energy density, pressure in 4D and 1D extra dimension spaces respectively.

We consider a flat space-time (k=0) with an expansion speed in extra dimension is constant (A=0), then, take into account the fact that the universe fluid is perfect ($p = w\rho$) we will get the following equation:

$$\dot{H} + \frac{3}{2}(1+w)H^2 + \left(\frac{2+3w}{2}\right)\left(\frac{c}{ct+c_0}\right)H\frac{\dot{A}}{A} = 0 \quad (3)$$

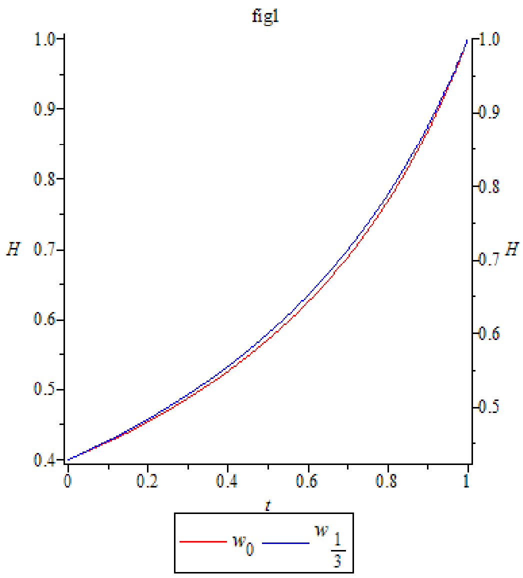
where c, c_0 are integration constants and H is the Hubble parameter. The form of the equation is

$$\widehat{H}(t) = \frac{(2+3w)}{-3(\hat{t}-1)(1+w)+(2+3w)} \quad (4)$$

Where

$$H_0 t_0 = \tau = 1, \frac{t}{t_0} = \hat{t}, \widehat{H} = \frac{H}{H_0}$$

Now we obtain the following fig1



$$\frac{dH}{dt} = H^2(1-q) \Rightarrow q = \frac{\frac{dH}{dt}+H^2}{-H^2},$$

notice that $H \succ 0$ (see fig1), then q<0. We deduce that the universe is in accelerated expansion.

III. DYNAMICAL STUDY I

We will write the Friedmann equations as a function of Hubble parameter H_R, ρ and H_A according the first Friedmann equation we find:

$$\dot{H}_A = \left(\frac{2\gamma+1}{3} - w\right)\rho - H^2{}_A - 3H_R H_A$$
$$\dot{H}_R = -(1+\gamma)\frac{\rho}{3} - H^2{}_R + H_R H_A \quad (5)$$
$$\dot{\rho} = -[3H_R(1+w) + H_A(1+\gamma)]\rho$$

the analysis leads to the following cases:

if $w, \gamma > 0$, we find the critical point $\rho = 0,\ H_R = 0, H_A = 0$, which correspond to a flat and static space for 4D universe and for 1D extra dimensional space.

If $\gamma = -1$ *and* $w = -1$, one has the following critical points: $\cdot\ \rho = 0, H_R = 0, H_A = 0$, which correspond to a flat and static space for $4D$ universe and for $1D$ extra dimensional space.

$\cdot\ H_R = 0, H_A = 1.101$, such that , it corresponds to a static space for $4D$ universe and an accelerated $1D$ extra dimensional space. Figure(2) displays the phase portrait for critical point $\{(H_R, H_A) = (0,1.101)\}$,such that we have a "saddle node point".

$\cdot H_R = 0.545, H_A = 0.545$, such that $\rho = 6{H_A}^2 = 1.787$, it corresponds to flat space and accelerated for $4D$ universe and $1D$ extra dimensional space, figure(3) displays the phase portrait for critical point $\{(H_R, H_A) = (0.545,0.545)\}$ which is "stable nodal sink".

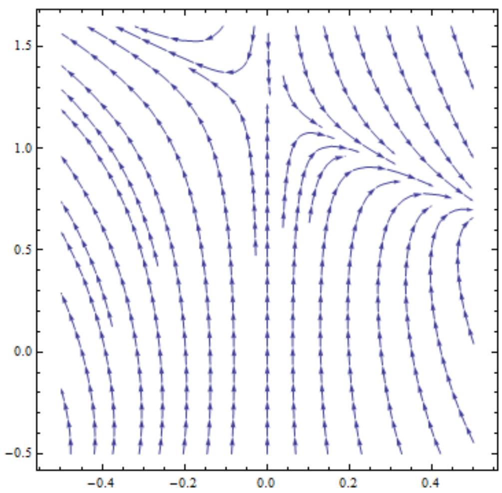


figure (2)

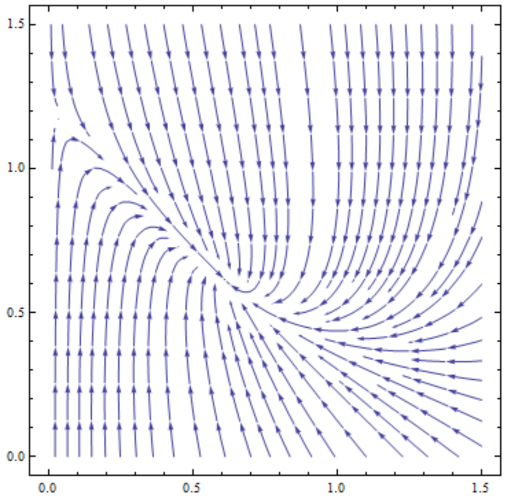


figure (3)

IV. FRIEDMAN EQUATION WITH SHEAR VISCOSITY

we consider $p = p + h(t)H_R$, equations of Friedman become

$$\rho = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} + 3\frac{\dot{R}\dot{A}}{RA}$$

$$\rho = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} + 3\frac{\dot{R}\dot{A}}{RA}$$

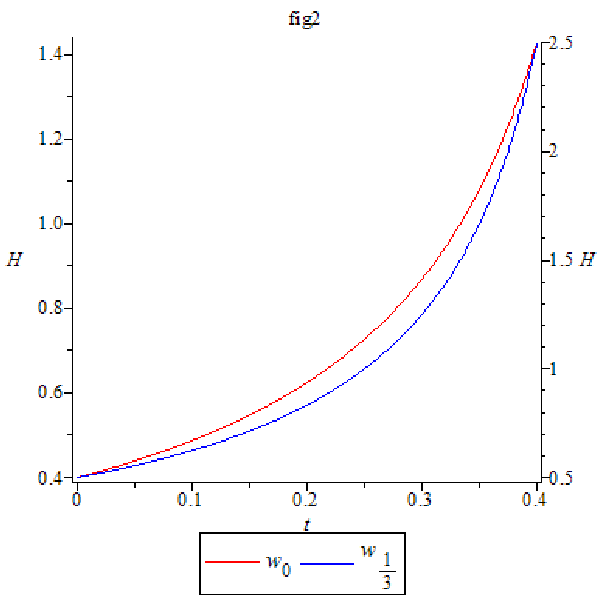
$$p = -\left[2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} + \frac{\ddot{A}}{A} + 2\frac{\dot{R}\dot{A}}{RA}\right] - \mathrm{h(t)}H_R \qquad (6)$$

$$p_5 = -3\left(\frac{\ddot{R}}{R} + \frac{k}{R^2} + \frac{\dot{R}^2}{R^2}\right)$$

We consider $h(t) = \alpha H$, and by using the equation p=wρ we find the expression:

$$\hat{H}(t) = \frac{(2+3w)}{-9(\hat{t}-1)(1+w)+2(2+3w)} \qquad (7)$$

Then, we obtain the following figure:



$\hat{H} > 0$ and $\frac{d\hat{H}}{dt} > 0$ $\rightarrow q < 0,$ there is an accelerated expansion

V. DYNAMICAL STUDY II

In the same way we find these dynamical equations

$$\dot{H}_A = \left(\frac{2\gamma+1}{3} - w\right)\rho - H^2{}_A - 3H_R H_A - \alpha H_R$$

$$\dot{H}_R = -(1+\gamma)\frac{\rho}{3} - H^2{}_R + H_R H_A \qquad (8)$$

$$\dot{\rho} = -[3H_R(1+w) + H_A(1+\gamma)]\rho - 3\alpha H^2{}_R$$

taking in account finally, we obtain these critical points

1) $H_R = -0.5\gamma^2\frac{\alpha}{2+\gamma^2-3\gamma w}$, $H_A = 0.5\frac{\alpha\gamma(2+\gamma)}{2+\gamma^2-3\gamma w}$

$\rho = -\left(\frac{1.5\alpha^2\gamma^3}{(2+\gamma^2-3\gamma w)^2}\right)$

2) $H_R = 0$, $H_A = 0$, $\rho = 0$

VI. DISCUSSION

The critical points are defined such that:

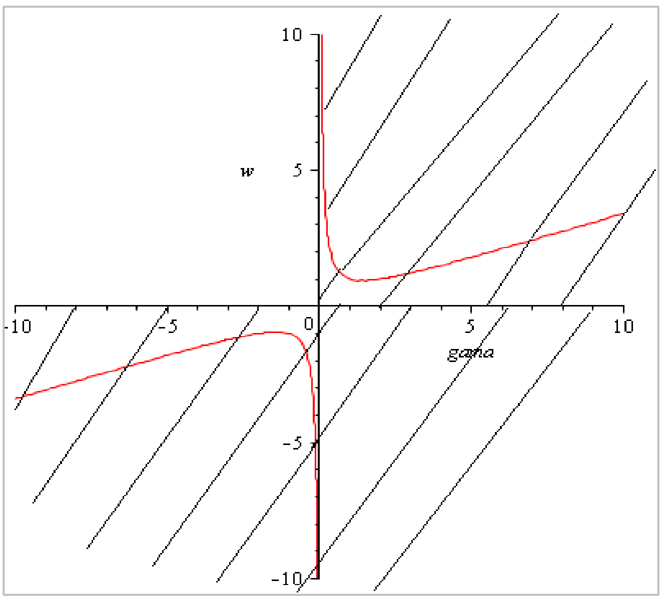
$$2 + \gamma^2 - 3\gamma w \neq 0$$

the region $(w < 0)$ give us value negative of pressure (dark energy).

For an accelerated expansion in 4 dimensions it must

For the first point $\rightarrow \begin{cases} \alpha < 0 \\ 2 + \gamma^2 - 3\gamma w > 0 \end{cases}$

for positive values of the energy density we must have $\gamma < 0$. Figure 3 displays the allowed values of w and γ.



Example :

For $\gamma = -1, w = 1, \alpha = -1$, and in order that the eigenvalues are defined, we must have $(w > -1)$. This leads to a phase portrait of a (Nodal Sink) type.

For the second point

$$\gamma > 0 \rightarrow \begin{cases} \alpha > 0 \\ 2 + \gamma^2 - 3\gamma w < 0 \end{cases}$$

Or

$$\begin{cases} \alpha < 0 \\ 2 + \gamma^2 - 3\gamma w > 0 \end{cases}$$

This is impossible because it give us $\rho < 0$

VII. CONCLUSION

In this work we have studied a model of 1D extra dimension, and have considered that our universe has a viscous . We tried to find exact solutions, and make a dynamical study for the general case. The obtained results indicate that the FRW model with viscous fluid is viable and give an accelerated expansion without dark energy.

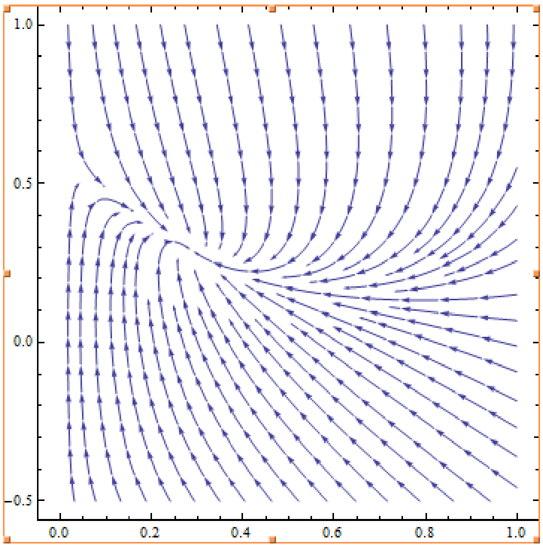


figure 7

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